Unit 1: Sequences and Series

1. Which of the following is a geometric sequence?
   a) 2, 4, 6, ...
   b) 8, 12, 18, ...
   c) 10, 30, 90, ...
   d) 12, 8, 4, ...
2. Which of the above sequences is (are) arithmetic?
3. Find the 18th term of the sequence: 11, 8, 5, 2, ...
4. For each of the sequences, write an expression for the general term t_n:
   a) 4, 7, 10, 13, 16, ...
   b) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$
5. Which term of the sequence 3, 7, 11, 15, ... has a value of 111?
6. In a grocery store cans of tomato juice are displayed in a pyramid containing 15, 14, 13, ..., 3 cans in each row. How many cans are displayed?
7. Determine the 9th term of the sequence: $-\frac{1}{8}, \frac{1}{2}, -2, 8, \ldots$
8. Find the first term if $S_n = -\frac{91}{2}$ and $r = -3$.
9. Find the sum of the series: $10 + 6 + 2 + \cdots + -34$.
10. Alison's new business has $10 000 in sales the first year. Sales are expected to increase by a constant amount each year. How much is this constant yearly increase in sales if the total sales for the first five years is $500 000?
11. Determine the common ratio 'r' for a geometric sequence of positive terms with $t_1 = 4$ and $t_3 = 9$.
12. How many terms are in the sequence: -16, -12, -8, ... , 384?
13. Find the 3rd term of the geometric series with $t_2 = 30$ and $S_3 = -35$.
14. What value of x in $x + 1, 2x + 3, x^2 - 5, \ldots$ will form an arithmetic sequence?
15. Find the 7th term if $S_7 = 756$ and $S_6 = 457$.

Answers: 1. b,c, 2. a,c, 3. -40, 4a) $t_n = 3n + 1$, b) $t_n = \frac{1}{n}, n \neq 0$, 5. 28, 6. 117, 7. -8192, 8. $\frac{1}{4}$, 9. -144, 10. $45 000, 11. $\frac{3}{2}$, 12. 101, 13. -20 or -45, 14. 5, 15. 293

Unit 2: Absolute Value and Radicals

Part 1: Evaluate:

1. $|-6|$ 2. $|9| - |-8|$ 3. $|-3(2-4)^2 + 8|$ 4. $|4x^2 + 3x - 7|$ for x= -2.

Part 2: Entire Radical and Mixed Radicals

Convert the following into entire radicals: 1. $4\sqrt{3}$ 2. $a\sqrt[3]{a}$ 3. $2b\sqrt[3]{5b^2}$ 4. $-\frac{1}{3}\sqrt[3]{\frac{2}{3}}$

Convert the following into mixed radicals: 5. $\sqrt{24}$ 6. $\sqrt[3]{c^5}$ 7. $\sqrt{32y^7}$ 8. $\sqrt[3]{\frac{-125}{72}}$

Part 3: Simplify and identify the values for which the variables of each radical is defined.

1. $\sqrt{18}+4\sqrt{2}$ 2. $-\sqrt{8}+4\sqrt{7}-\sqrt{28}+\sqrt{18}$ 3. $\sqrt{4d}-\sqrt{32d}$

4. $\sqrt{48ab^4}+b^2\sqrt{36a}+\sqrt{75ab^4}-6a^2\sqrt{18b}$ 5. $\sqrt[3]{27b}+\sqrt[3]{16b^4}-\sqrt[3]{-64b}$

6. $(-2\sqrt{3x})(5\sqrt{8})$ 7. $4\sqrt{3}(6\sqrt{5}-5\sqrt{3})$ 8. $(6\sqrt{2}-7)(9\sqrt{8}+6)$

9. $2\sqrt[3]{16k}(\sqrt[3]{4k}+5\sqrt[3]{24})$ 10. $\frac{4\sqrt{125}}{\sqrt{5}}$ 11. $\frac{-4}{2\sqrt[3]{7p}}$ 12. $\frac{2}{-5+4\sqrt{8}}$ 13. $\frac{2\sqrt{8}+3\sqrt{5}}{\sqrt{2}+\sqrt{20}}$

14. $\frac{5\sqrt{3}-\sqrt{11}}{\sqrt{7}}$ 15. $\frac{6-3\sqrt{5}}{4\sqrt{3}+\sqrt{6}}$ 16. $\frac{\sqrt{3}+\sqrt{7}}{\sqrt{3}-\sqrt{7}}$

Part 4: Solve and verify:

1. $4+\sqrt{2x-1}=15$ 2. $x-\sqrt{4-x}=-8$ 3. $4+\sqrt{3x}=\sqrt{6x+1}+3$

4. $\sqrt{-4x+7}=\sqrt{-3x+8}$ 5. $3\sqrt{x-6}=4\sqrt{2x-3}$ 6. $4\sqrt{x+2}-7=1$

Answers:

Part 1: 1. 6, 2. 1, 3. 4, 4. 3. Part 2: 1.$\sqrt{48}$, 2.$\sqrt[3]{a^4}$, 3. $\sqrt[3]{40b^5}$, 4. $-\sqrt[3]{\frac{2}{81}}$, 5. $2\sqrt{6}$, 6.$c\sqrt[3]{c^2}$, 7. $4y^3\sqrt{2y}$, 8.$-\frac{5}{2}\sqrt[3]{\frac{1}{9}}$
Part 3:1. $7\sqrt{2}$, 2. $2\sqrt{7}+\sqrt{2}$, 3. $2\sqrt{d}-4\sqrt{d}$, d≥ 0, 4. $9b^2\sqrt{3a}-6b^2\sqrt{a}-18a^2\sqrt{2b}$, a≥0,b≥0,
5 .$7\sqrt[3]{b}+2b\sqrt[3]{2b}; b\geq 0$, 6. $-20\sqrt{6x}, x\geq 0$, 7. $24\sqrt{15}-60$, 8. 174-90$\sqrt{2}$, 9. $8\sqrt[3]{k^2}+40\sqrt[3]{6k}, k\geq 0$,
10. 20, 11. $-2\frac{\sqrt[3]{49p^2}}{7p}$, p> 0, 12. $\frac{15+24\sqrt{2}}{103}$, 13. $\frac{22+5\sqrt{10}}{18}$, 14. $\frac{5\sqrt{21}-\sqrt{77}}{7}$, 15. $\frac{24\sqrt{3}-6\sqrt{6}-12\sqrt{15}+3\sqrt{30}}{42}$,
16. $\frac{-5-2\sqrt{21}}{2}$, Part 4: 1. 61, 2. -12,-5, 3. 0, $\frac{4}{3}$, 4. -1 , 5. No real numbers , 6. 2

Unit 3: Solving Quadratic Equations

Factor the following:

1. $81x^2-49y^2$ 2. $2x^2-15x+18$ 3. $9y^2-24y+16$ 4. $1.5x^2+5.5x-2$ 5. $\frac{5}{3}-\frac{7}{3}x-2x^2$

6. $16(2x-5)^2-25(y+3)^2$ 7. $8(3x-1)^2+10(3x-1)-3$ 8. $4x^2+12x+9-16y^2$

9. $64x^2-16a^2+24a-9$ 10. $(4a-3)^2-(3a+1)^2$

Solve the following using any method:

12. $x^2-7x+10=0$ 13. $3x^2-17x=6$ 14. $2x(x-4)+9x=4x+35$ 15. $\sqrt{2x^2+9}+3=2x$

16. $\frac{x^2}{2}+\frac{17x}{6}=1$ 17. $3x^2-4x-1=0$ 18. $(2x+3)^2+2=0$ 19. $-2x^2+\frac{3}{2}x-\frac{5}{6}=0$

20. $1.5x^2 = 1.2x + 4$ 21. $3x = \sqrt{9x + 28} - 4$

Solve by completing the square:

22. $x^2 - 3x + 1 = 0$ 23. $\frac{1}{4}x^2 + x - 3 = 0$

Determine whether each of the following have one, two, or no real roots

24. $2x^2 - 3.6x - 1.5 = 0$ 25. $-2x^2 + 4x - 5 = 0$

26. Consider the equation: $6x^2 - 5x + k = 0$. Determine the values of k if the equation has

a) no real roots b) one real root or c) two real roots

Answers:

1. $(9x + 7y)(9x - 7y)$, 2. $(2x - 3)(x - 6)$, 3.$(3y - 4)^2$, 4. $0.5(3x - 1)(x + 4)$, 5. $\frac{-1}{3}(2x - 1)(3x + 5)$, 6. $(8x - 5y - 35)(8x + 5y - 5)$, 7. $(6x + 1)(12x - 5)$, 8. $(2x + 3 - 4y)(2x + 3 + 4y)$, 9. $(8x + 4a - 3)(8x - 4a + 3)$, 10. $(a - 4)(7a - 2)$, 12. 2, 5, 13. $\frac{-1}{3}$, 6 , 14. $\frac{-7}{2}$, 5, 15. 0, 6, 16. $\frac{1}{3}$, -6, 17. $\frac{2 \pm \sqrt{7}}{3}$, 18. No real roots, 19. No real roots, 20. $\frac{6 \pm 2\sqrt{159}}{15}$, 21. $\frac{-5 \pm \sqrt{73}}{6}$, 22. $\frac{3 \pm \sqrt{5}}{2}$, 23. 2,-6, 24. 2 roots, 25. no real roots, 26.a) $k > \frac{25}{24}$,b) $k = \frac{25}{24}$,c) $k < \frac{25}{24}$

Unit 4: Analyzing Quadratic Functions

Graph each of the following functions. State the x and y intercepts, whether the quadratic has a maximum or a minimum, what the max or min value is, the equation of the axis of symmetry, the direction of opening , the domain and the range of each of the following:

1. $y = 2(x - 3)^2 - 4$ 2. $y = -\frac{1}{2}(x + 3)^2 + 2$ 3. $y = -2x^2 + 16x + 9$ 4. $y = 3x^2 - 12x + 5$

Determine the zeros of: 5. $y = x^2 - 2x - 15$ 6. $y = 5x^2 - 7x - 6$

Determine the roots of: 7. $16 = 10c^2 - 36c$ 8. $7x + 4 = 2x^2$

Write an equation of the quadratic function for:

9. A quadratic function that has a vertex (-2, 6) that opens up and is congruent to $y = 4x^2$

10. A quadratic function that has a vertex (3, -8) and x-intercepts: 1 and 5.

11. A quadratic function that has a vertex (6, 3) and a y-intercept: -8.

12. A quadratic function with zeros -4 and 6, passing through (2, 3).

13. A student is charged \$10 for a gym membership . 2000 students purchase the membership. Each increase by \$1 in membership, 100 fewer students signed up. What membership fee would provide maximum revenue for the gym?

Answers: 1. x- int;1,5 y-int;14, min value of -4 occurs when x=3, vertex:(3,-4), AOS: x=3,opens up, D $\{x|x \in R\}$, R$\{y|y \geq -4, y \in R\}$, 2. x-int.-1,-5 y- int:$\frac{-5}{2}$, max value of 2 occurs when x=-3, vertex: (-3,2), AOS: x=-3, opens down, D $\{x|x \in R\}$, R$\{y|\ y \leq 2, y \in R\}$, 3. x-int:$\frac{8\pm\sqrt{82}}{2}$, y-int: -23, max value of 9 occurs when x=4, vertex: (4,9), AOS: x=4, opens down, D $\{x|x \in R\}$, R$\{y \leq 9, y \in R\}$, 4. X-int: $\frac{6\pm\sqrt{21}}{3}$, y-int: 5, min value of -7 occurs when x=2, vertex: (2,-7), AOS: x=2, opens up, , D $\{x|x \in R\}$, R$\{y|y \geq -7\}, y \in R$, 5. 5,-3 , 6. $\frac{-3}{5}, 2$, 7. $-\frac{2}{5}, 4$, 8. $-\frac{1}{2}, 4$, 9. $y = 4(x+2)^2 + 6$ 10. $y = 2(x-3)^2$-8, 11. $y = \frac{-11}{36}(x-6)^2 + 3$, 12. $y = \frac{-1}{8}(x+4)(x-6)$, 13. Max Rev. \$22500 when the membership price is \$15,

Unit 5: Graphing Inequalities

Solve each one variable inequality:

1. $(x-5)(x-8) > 0$ 2. $(x+3)(x-4) \leq 0$ 3. $(6x+5)(4x-1) \geq 0$

4. $x^2 + x - 12 > 0$ 5. $2x^2 - 15x + 28 < 0$ 6. $7x^2 \leq 35x$

Create an inequality that has each solution:

7. $\{x|-9 < x < 3, x \in R\}$ 8. $\{x|x \leq -4 \text{ or } x \geq 3, x \in R\}$

Graph each linear inequality. Give the coordinates of 2 points that satisfy the inequality:

9. $5x + 3y > 12$ 10. $3x - 2y \leq -9$ 11. $y < 4x - 3$ 12. $5x - 4y \geq 24$

Graph each quadratic inequality. Give the coordinates of 2 points that satisfy the inequality:

13. $y \leq -(x-2)^2 + 4$ 14. $y > (x+3)^2$ 15. $y \geq x^2 + 8x + 19$ 16. $y < 2x^2 - 12x + 13$

Solve each system of equations graphically:

17. $y = x^2 + 1$ 18. $y = -2(x-3)^2 + 5$

$y = 3x + 1$ $y = (x-3)^2 + 2$

Solve each system of equations algebraically:

19. $y = x^2 - 16$ 20. $y = x - 16$ 21. $y = 3(x-3)^2 + 1$ 22. $y = x^2 + 2x - 6$

$x + y = 4$ $y = x^2 + x$ $y = -(x-3)^2 + 5$ $y = -2x^2 - 4x + 3$

Answers:

1. $x < 5$ and $x > 8$ 2. $-3 \leq x \leq 4$ 3. $x \leq -\frac{5}{6}$ and $x \geq \frac{1}{4}$ 4. $x < -4$ and $x > 3$ 5. $\frac{1}{2} < x < 4$
6. $0 \leq x \leq 5$ 7. $(x+9)(x-3) < 0$ 8. $(x+4)(x-3) \geq 0$

19. $x = -5, 4$ 20. no solutions 21. $x = 4, 2$ 22. $x = -3, 1$

Unit 6: Trigonometry

1. In ΔXYZ, XY = 5, XZ = $\sqrt{50}$, YZ = $\sqrt{75}$, ∠X=90°. Find *cos* Y.
2. In ΔABC, AB = 17 cm, BC = 8 cm, AC = 15cm and ∠C = 90°. Only one of the following ratios is correct, the correct ratio is: a) $\sin A = \frac{8}{15}$ b) $\cos A = \frac{8}{17}$ c) $\sin B = \frac{15}{17}$ d) $\tan B = \frac{8}{15}$ e) $\tan A = \frac{8}{17}$
3. If $\tan\theta = \frac{5}{12}$, then $\cos\theta =$
4. Solve the triangle ΔDEF if ∠E=90°, EF=36, and DF=45.
5. In which quadrants is the tangent function negative?
6. What is the reference angle of --235°?
7. In what quadrant does the terminal side of angle 835° lie?
8. Given point P(-3, 5) is on the terminal arm of angle θ.

a) draw a diagram showing θ in standard position
b) find sin θ, cos θ, and tan θ. (leave answers in simplest radical form)
c) Find θ. Express θ to the nearest degree.

9. Solve for θ to the nearest degree if $0 \le \theta < 360°$.
   a) $\sin\theta = 0.570$ b) $\tan\theta = -0.782$ c) $\tan\theta = 4.5257$
10. Solve $2\cos\theta - 1 = 0$ to the nearest degree for $0° \le \theta \le 180°$.
11. Solve $\sin\theta = -0.723$ for θ to the nearest degree; $0° \le \theta \le 360°$
12. Solve $8\cos\theta + 7 = 0$ for θ to the nearest degree; $0° \le \theta \le 360°$
13. In ΔABC, AB = 9 cm, AC = 13 cm, ∠ABC = 113°. To calculate the measure of ∠BCA, what is the most appropriate method for solving the problem? (∠B) (∠C)
14. What values must be known to use the Sine Law in ΔDEF?

a) d, e, ∠F, b) e, f, ∠D c) ∠D, ∠E, ∠F d) d, e, f e) d, e, ∠D

15. A possible Cosine Law for ΔDEF is:
   a) $e^2 = d^2 + f^2 - 2df\sin E$ b) $f^2 = d^2 + e^2 - 2ed\cos E$ c) $d^2 = e^2 + f^2 - 2ef\cos D$
16. In ΔMNO, find side m if ∠M = 46°, n = 48 cm, and o = 32 cm.
17. Find the measure of angle Q in ΔPQR if p = 101, q = 136, and r =162.
18. In ΔYZA, ∠Y = 36°, y = 24, and z = 34, find angle Z.

Answers:

1. 55° 2. C 3. $\frac{12}{13}$ 4. DE=27, ∠D=53°, ∠F=37° 5. II and III 6. 35° 7. II 8. a) b) $\sin\theta = \frac{5}{\sqrt{34}}$, $\cos\theta = -\frac{3}{\sqrt{34}}$, $\tan\theta = -\frac{5}{3}$ c) θ = 121° 9. a) 35° or 145° b) 142° or 322° c) 0.08° or 180.08°
10. θ = 45° 11. 223° or 317° 12. 151° or 209° 13. sine law 14. E 15. C 16. 35 cm
17. 57° 18. 56° or 124°

Unit 7: Rational Expressions

* circled questions are challenge questions *

Part A: Simplify each expression, identify all non-permissible values.

1) $\frac{48x^3y^5}{-8x^2y} \cdot \frac{-10xy^2}{12x^2y}$ 2) $\frac{27x}{63x^2+54x}$ 3) $\frac{x^2+6x}{x^2+7x+6}$ 4) $\frac{x^2-4x-45}{x^2-25}$

5) $\frac{9x^3y^2}{5(a+1)} \cdot \frac{20(a+1)}{3xy}$ 6) $\frac{6x^2+11x-35}{6x^2+12x-40}$ 7) $\frac{3x-6}{4x^2+8x} \div \frac{6}{8x+16}$

8) $\frac{x^2-64}{x^2-8x+16} \cdot \frac{7x^2-28x}{5x^2+40x}$ 9) $\frac{5x^2-7x+2}{x^3-3x^2+2x} \cdot \frac{x^2-2x}{3x^2+2x-1}$

Part B: Simplify each sum or difference, identify all non-permissible values.

1) $\frac{x-5}{7x}+\frac{x+3}{7x}$ 2) $\frac{1}{a+1}+\frac{a}{a+1}$ 3) $\frac{2x-4}{x-1}-\frac{x+7}{x-1}$ 4) $\frac{3x-1}{3}+\frac{x+4}{5}$ 5) $\frac{8}{7x}-\frac{3}{2x}$

6) $\frac{2}{3}-\frac{8}{3x^2}+\frac{7}{x}$ 7) $\frac{5}{x+4}-\frac{6}{x-2}$ 8) $\frac{6x-1}{x-4}+\frac{2x-5}{x-2}$ 9) $4-\frac{3x}{x-7}$

(10) $\frac{8y}{5y-2}+\frac{3y}{3y+2}$ (11) $\frac{7x}{2x-14}+\frac{2x}{3x-21}$ (12) $\frac{7}{x^2-7x+12}+\frac{5}{x-4}-\frac{9}{x-3}$

(13) $\frac{2}{x^2-7x-12}-\frac{3}{x^2+3x-18}+\frac{6}{x^2+2x-24}$ (14) $\frac{4x^2-9x+2}{4x^2+3x-1}-\frac{3x^2+4x-4}{3x^2-8x+4}$

Part C: Solve each of the following rational equations

1) $\frac{3x}{5}=\frac{2}{x}+\frac{3x+2}{5}$ 2) $\frac{3x-2}{2}=4$ 3) $\frac{2x+1}{3x-2}=\frac{4x+3}{6x-5}$

4) $\frac{1}{2+x}+\frac{4}{2x-1}=1$ 5) $x+\frac{6}{x+5}=2$ (6) $\frac{2x}{2x-1}-\frac{x+1}{x+3}=\frac{3x+1}{2x-1}$

Answers:

PART A 1. $5y^5$ 2. $\frac{3}{7x+6}$ 3. $\frac{x}{x+1}$ 4. $\frac{x-9}{x-5}$ 5. $12x^2y$ 6. $\frac{x+7}{2(x+4)}$ 7. $\frac{x-2}{x}$ 8. $\frac{7(x+8)}{5(x-4)}$ 9. $\frac{5x-2}{(3x-1)(x+1)}$

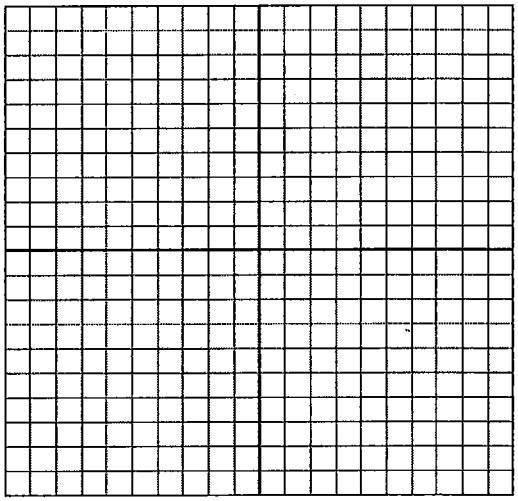
PART B 1. $\frac{2x-2}{7x}$ 2. 1 3. $\frac{x-11}{x-1}$ 4. $\frac{18x+7}{15}$ 5. $-\frac{5}{14}$ 6. $\frac{2x^2+21x-8}{3x^2}$ 7. $\frac{-x-34}{(x+4)(x-2)}$ 8. $\frac{2(4x^2-13x+11)}{(x-4)(x-2)}$ 9. $\frac{x-28}{x-7}$

PART C 1. -5 2. $\frac{10}{3}$ 3. $\frac{1}{5}$ 4. $-\frac{3}{2}$ and 3 5. -4 and 1 6. $-\frac{2}{3}$ and -1

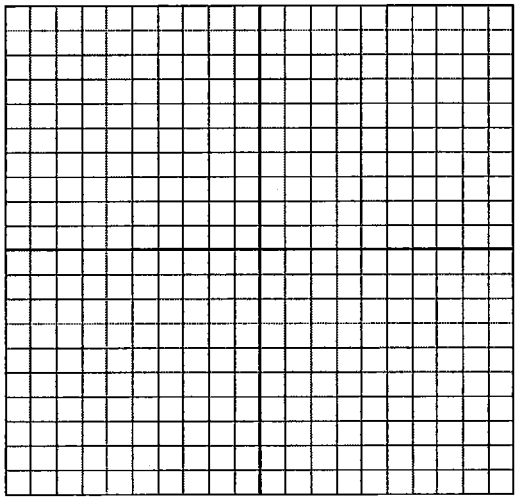
Unit 8: Absolute Value & Reciprocal Functions

1. Sketch a graph of

a) $y = |4x + 2|$



b) $y = |(x - 2)(x + 3)|$



2. Write each absolute value function in piecewise notation:
   a) $y = |-x - 5|$
   b) $y = |(x-2)(x+3)|$

3. Solve by graphing:
   a) $3 = |2x - 3|$
   b) $|x^2 - x| = 6$

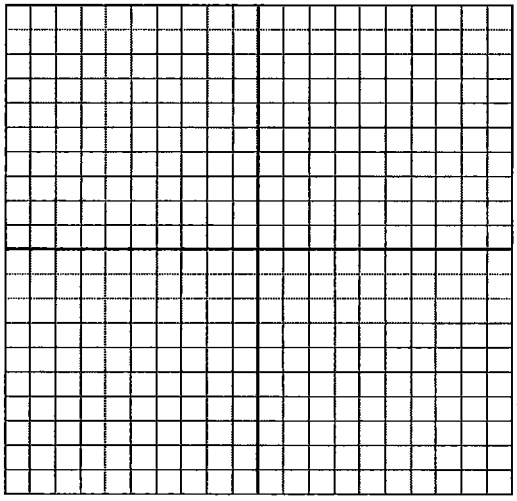
4. Use algebra to solve each equation.
   a) $y = |(x-1)^2 - 2|$
   b) $y = \frac{1}{2}|3x - 5|$

5. Given: $y = -x - 4$

a) Graph this function.

b) Graph the reciprocal of this function.

Clearly draw in any asymptotes with dashed lines.

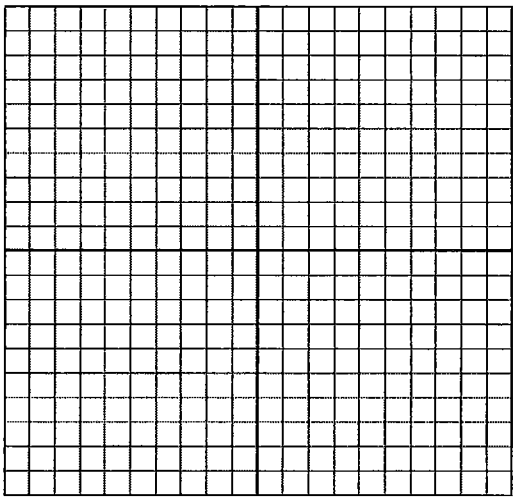


6. Given: $y = x^2 - 9$

a) Graph this function.

b) Graph the reciprocal of this function.

Clearly draw in any asymptotes with dashed lines.



7. What is the equation of the graph shown?

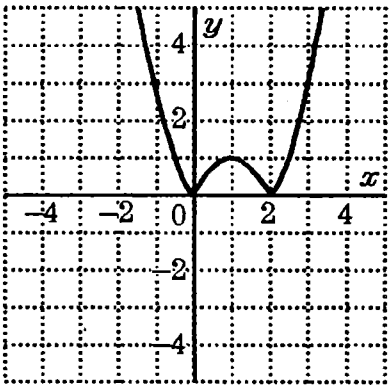
a) $y = |x^2 - 2x|$

b) $y = \dfrac{1}{x^2 - 2x}$

c) $y = x^2 - 2x$

d) $y = |x^2 + 2x|$

e) $y = x^2 + 2x$



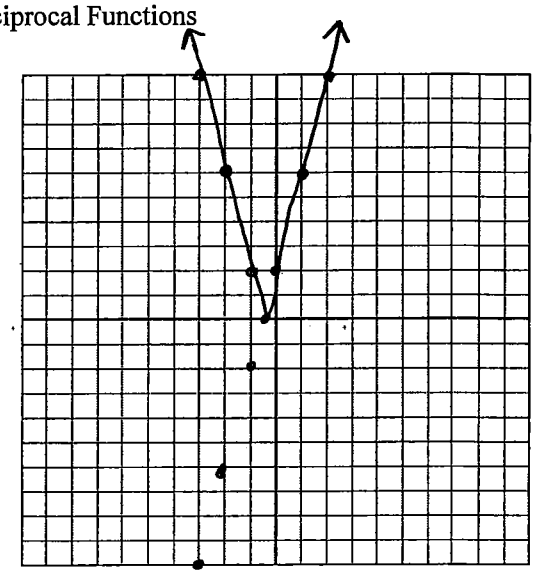
Answers: see next page

ANSWERS:

Unit 8: Absolute Value & Reciprocal Functions

1. Sketch a graph of

a) $y = |4x+2|$



b) $y = |(x-2)(x+3)|$ → $x = 2$, $x = -3$

$\frac{1}{2}(1) = 0.5$
$\Rightarrow 0.5^2 = 0.25$

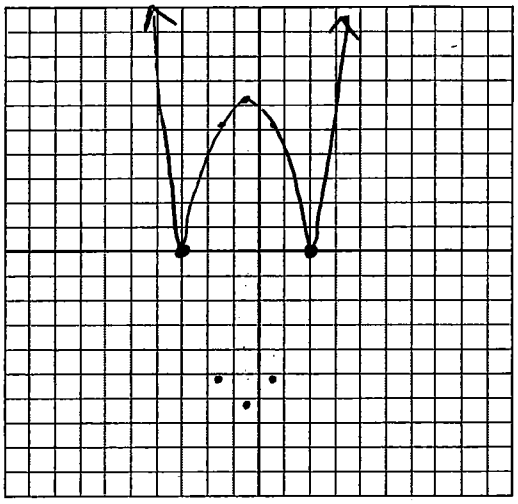
$y = |x^2 - 2x + 3x - 6|$

$y = |x^2 + x - 6|$

$y = |x^2 + x + 0.25 - 0.25 - 6|$

$y = |(x+0.5)^2 - 6.25|$

$\leftarrow 0.5 \downarrow 6.25$



2. Write each absolute value function in piecewise notation: (from #1)

a) $y = |4x+2|$ → $y = \begin{cases} 4x+2 \text{ if } x \geq -0.5 \\ -(4x+2) \text{ if } x < -0.5 \end{cases}$

b) $y = |(x-2)(x+3)|$ → $y = \begin{cases} (x-2)(x+3) \text{ if } x \leq -3 \text{ and } x \geq 2 \\ -[(x-2)(x+3)] \text{ if } -3 < x < 2 \end{cases}$

3. Solve using algebra:

a) $3 = |2x-3|$ → $x = 3$, $x = 0$

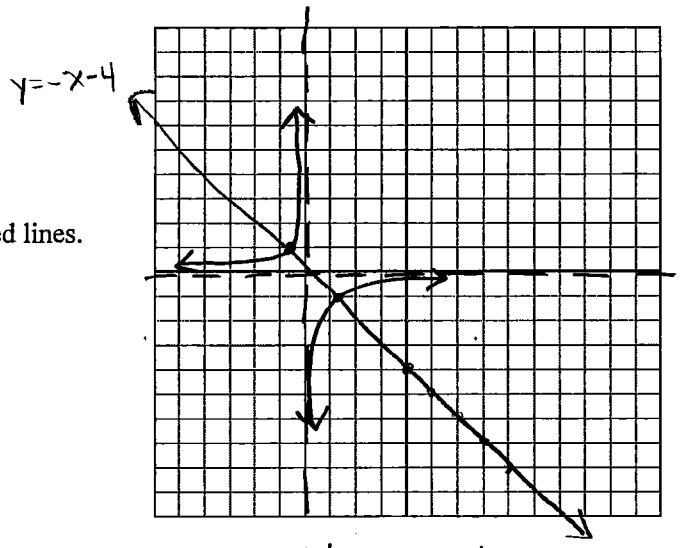
b) $|x^2 - x| = 6$ → $x = 3$, $x = -2$

5. Given: $y = -x - 4$

a) Graph this function.

b) Graph the reciprocal of this function.

Clearly draw in any asymptotes with dashed lines.

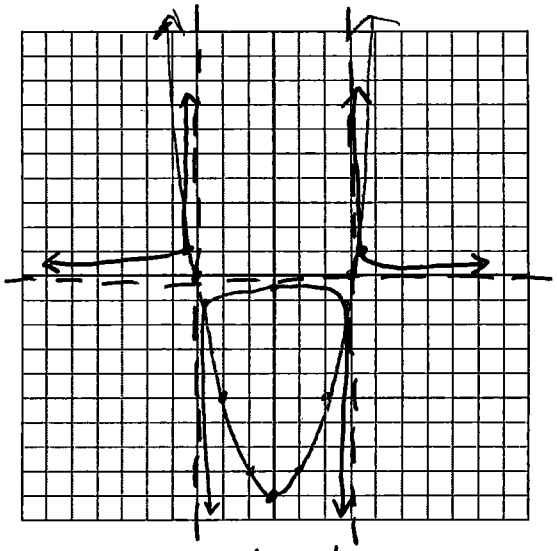


6. Given: $y = x^2 - 9$

a) Graph this function.

b) Graph the reciprocal of this function.

Clearly draw in any asymptotes with dashed lines.



7. What is the equation of the graph shown?

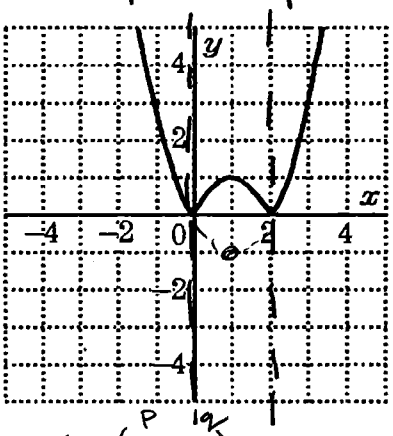
a) $y = |x^2 - 2x|$

b) $y = \dfrac{1}{x^2 - 2x}$

c) $y = x^2 - 2x$

d) $y = |x^2 + 2x|$

e) $y = x^2 + 2x$



vertex (1, -1)

$y = (x-1)^2 - 1$

$y = (x^2 - x - x + 1) - 1$

$y = x^2 - 2x$