

**Chapter 8 Review**Name KEY

1. Consider the following functions and determine the following properties.

i)  $y = \left| -\frac{1}{2}x + 2 \right|$

a) Determine the x and y intercepts for the function.

x-int:  $y=0$

y-int:  $x=0$

$0 = \left| -\frac{1}{2}x + 2 \right|$

$y = \left| -\frac{1}{2}(0) + 2 \right|$

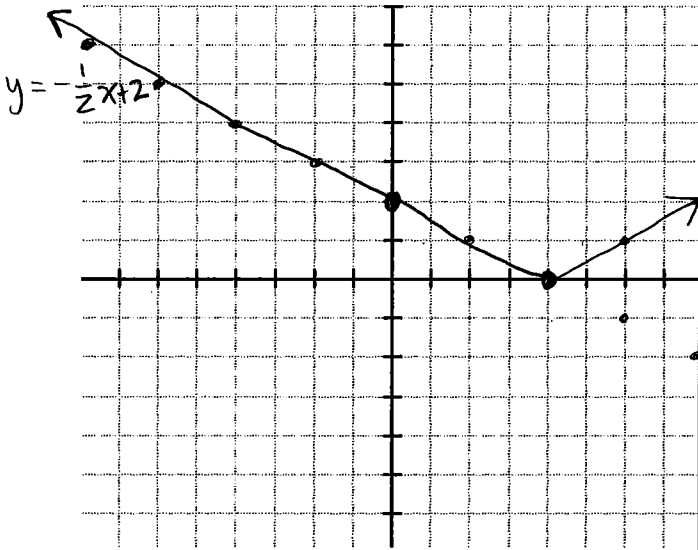
$0 = -\frac{1}{2}x + 2$

$y = 2$

$-2 = -\frac{1}{2}x$

$x = 4$

b) Sketch a graph of the function.



c) State the domain and range.

$D: \{x \mid x \in \mathbb{R}\}$

$R: \{y \mid y \geq 0, y \in \mathbb{R}\}$

d) Express the equation as a piecewise function.

$$y = \begin{cases} -\frac{1}{2}x + 2 & \text{if } x \leq 4 \\ -(-\frac{1}{2}x + 2) & \text{if } x > 4 \end{cases}$$

ii)  $y = |(x+1)^2 - 4|$

a) Determine the x and y intercepts for the function.

x-int:  $y=0$

y-int:  $x=0$

$0 = |(x+1)^2 - 4|$

$y = |(0+1)^2 - 4|$

$4 = (x+1)^2$

$y = |1 - 4|$

$\pm\sqrt{4} = x+1$

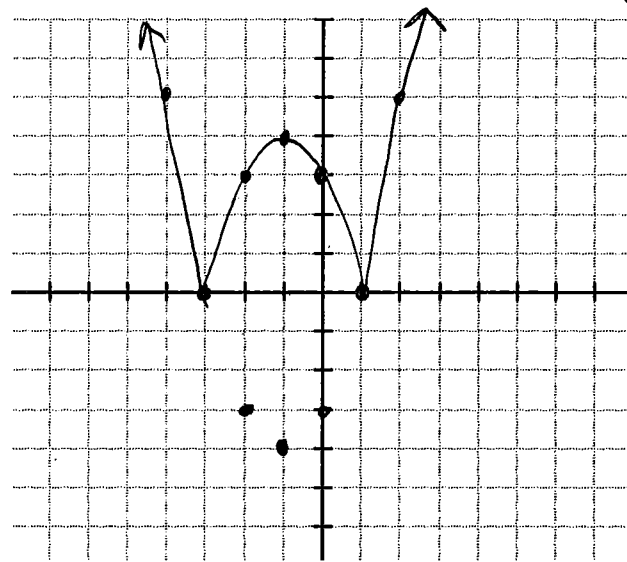
$y = |-3|$

$-1 \pm 2 = x$

$x = -3, x = 1$

$y = 3$

b) Sketch a graph of the function.



c) State the domain and range.

$D: \{x \mid x \in \mathbb{R}\}$

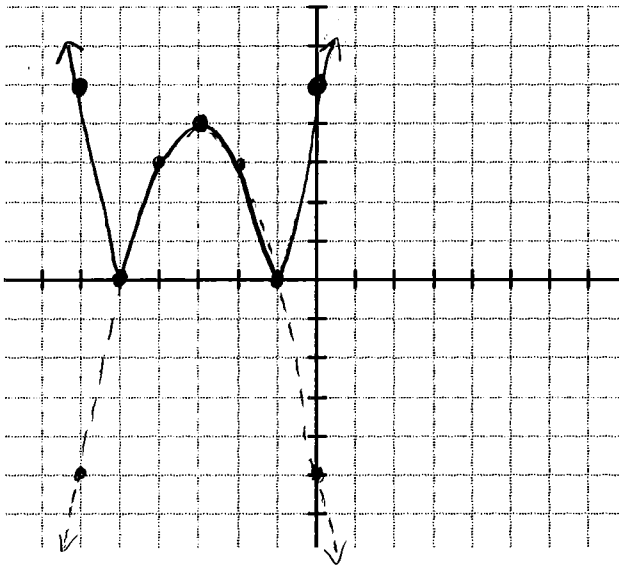
$R: \{y \mid y \geq 0, y \in \mathbb{R}\}$

d) Express the equation as a piecewise function.

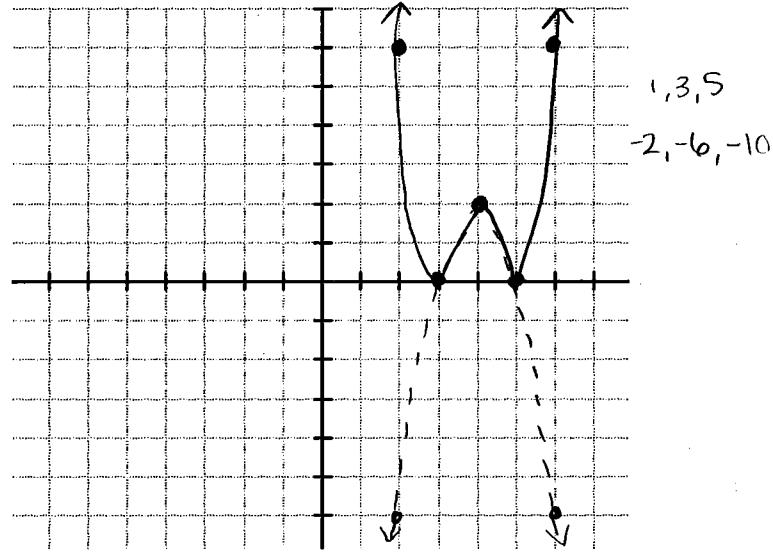
$$y = \begin{cases} (x+1)^2 - 4 & \text{if } x \leq -3 \text{ and } x \geq 1 \\ -[(x+1)^2 - 4] & \text{if } -3 < x < 1 \end{cases}$$

2. Graph the following absolute value functions.

iii)  $y = |-(x + 3)^2 + 4|$   $\leftarrow 3 \uparrow 4$



iv)  $y = |-2(x - 4)^2 + 2|$   $\rightarrow 4 \uparrow 2$



v)  $y = |x^2 + 4x + 3|$

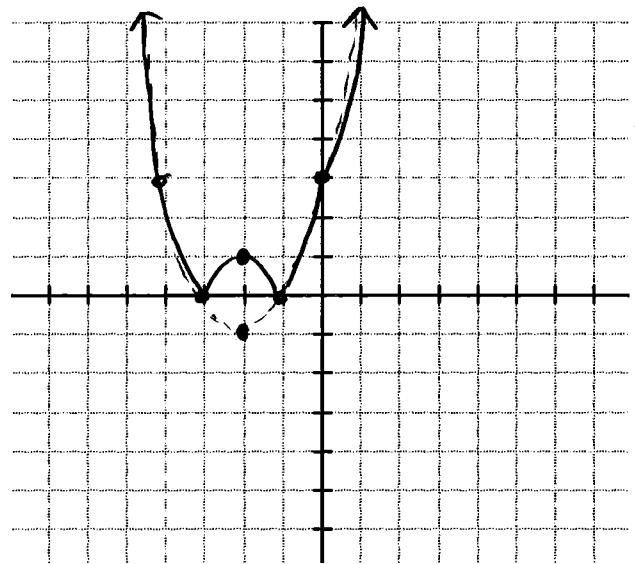
$y = (x^2 + 4x) + 3$

$y = (x^2 + 4x + 4 - 4) + 3$

$y = (x + 2)^2 - 1$

$\leftarrow 2 \downarrow 1$

$4(\frac{1}{2}) = 2$   
 $2^2 = 4$



3. Solve the following absolute value equations algebraically

a)  $|- \frac{x}{2} + 3| = 4$

$2 \times (- \frac{x}{2} + 3) = (4)^{\pm 2}$

$-x + 6 = 8$

$-2 = x$

$-(- \frac{x}{2} + 3) = 4$

$2 \times (\frac{x}{2} - 3) = (4)^{\pm 2}$

$x - 6 = 8$

$x = 14$

check:

$x = 14$

$|- \frac{14}{2} + 3| = 4$

$|-7 + 3| = 4$

$|-4| = 4$

$4 = 4$

✓

check:

$x = -2$

$|- \frac{-2}{2} + 3| = 4$

$|1 + 3| = 4$

$4 = 4$

✓

Solutions:  $x = -2, x = 14$

b)  $|\frac{2}{3}x - 1| = 2x - 5$

$3 \cdot (\frac{2}{3}x - 1 = 2x - 5)$

~~$2x - 3 = 6x - 15$~~   
 ~~$-7x + 15 - 2x + 15$~~

$12 = 4x$

$x = 3$

check:

$|\frac{2}{3}(3) - 1| = 2(3) - 5$

$|2 - 1| = 6 - 5$

$1 = 1 \checkmark$

**Solution:  $x = 3$**

d)  $|x + 5| = x^2 + 6x + 5$

$x + 5 = x^2 + 6x + 5$   
 ~~$-x - 5 - x - 5$~~

$0 = x^2 + 5x$

$0 = x(x + 5)$

$x = 0, x = -5$

check:  $x = 0$

$|0 + 5| = 0 + 6(0) + 5$

$5 = 5 \checkmark$

$x = -5$

$|-5 + 5| = (-5)^2 + 6(-5) + 5$

$0 = 25 - 30 + 5$

$0 = 0 \checkmark$

**Solutions:  $x = 0, x = -5$**

~~$-(\frac{2}{3}x - 1) = 2x - 5$~~

$3 \cdot (-\frac{2}{3}x + 1 = 2x - 5)$

~~$-2x + 3 = 6x - 15$~~   
 ~~$+2x + 15 + 2x + 15$~~

$18 = 8x$

$x = \frac{9}{4} \rightarrow \text{reject}$

check:

$|\frac{2}{3}(\frac{9}{4}) - 1| = 2(\frac{9}{4}) - 5$

$|\frac{3}{2} - \frac{2}{2}| = \frac{9}{2} - \frac{10}{2}$

$\frac{1}{2} \neq -\frac{1}{2}$

check:  $x = -2$  ↑  
reject

$|-2 + 5| = (-2)^2 + 6(-2) + 5$

$3 = 4 - 12 + 5$

$3 \neq -3$

c)  $|3x + 6| + 3 = 6$

$3x + 6 + 3 = 6$

$3x + 9 = 6$

$3x = -3$

$x = -1$

check:

$|3(-1) + 6| + 3 = 6$

$|-3 + 6| + 3 = 6$

$|3| + 3 = 6$

$6 = 6 \checkmark$

**Solutions:  $x = -1, x = -3$**

e)  $x + 1 = |x^2 - 2x - 3|$

$x + 1 = x^2 - 2x - 3$

~~$-x - 1 - x - 1$~~

$0 = x^2 - 3x - 4$

$0 = (x - 4)(x + 1)$

$x = 4, x = -1$

check:  $x = 4$

$4 + 1 = |4^2 - 2(4) - 3|$

$5 = |16 - 8 - 3|$

$5 = 5 \checkmark$

$x = -1$

$-1 + 1 = |(-1)^2 - 2(-1) - 3|$

$0 = |1 + 2 - 3|$

$0 = 0 \checkmark$

**Solutions:  $x = 4, x = 2, x = -1$**

~~$-(3x + 6) + 3 = 6$~~

~~$-3x - 6 + 3 = 6$~~

~~$-3x - 3 = 6$~~

~~$-3x = 9$~~

~~$x = -3$~~

check:

$|3(-3) + 6| + 3 = 6$

$|-9 + 6| + 3 = 6$

$|-3| + 3 = 6$

$3 + 3 = 6$

$6 = 6 \checkmark$

$x + 1 = -(x^2 - 2x - 3)$

~~$x + 1 = -x^2 + 2x + 3$~~   
 ~~$-x - 1 - x - 1$~~

$0 = -x^2 + x + 2$

$0 = x^2 - x - 2$

$0 = (x + 1)(x - 2)$

$x = 2, x = -1$

check:  $x = 2$

$2 + 1 = |2^2 - 2(2) - 3|$

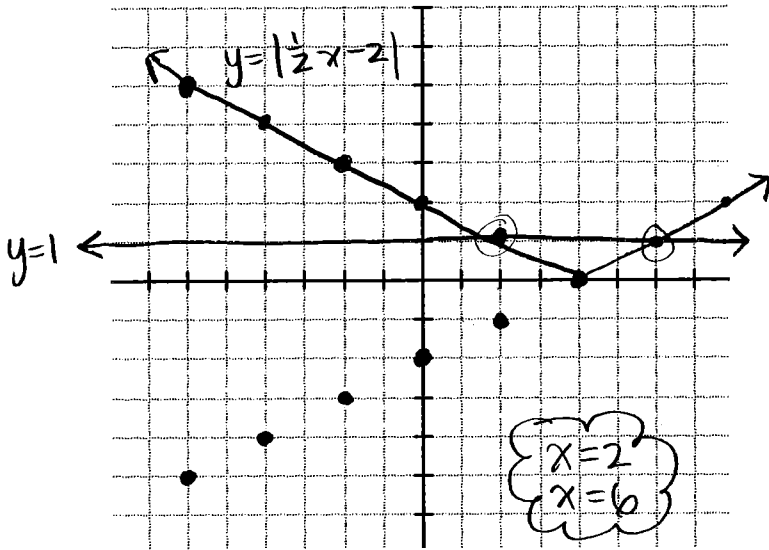
$3 = |4 - 4 - 3|$

$3 = |-3|$

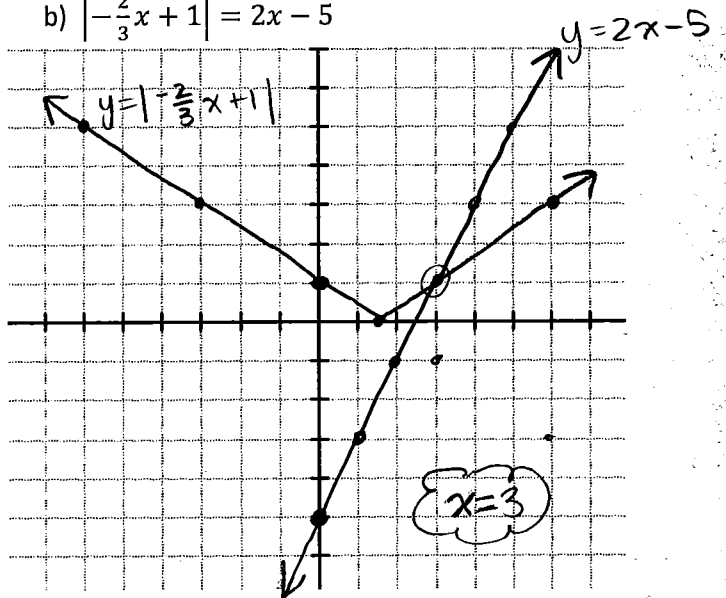
$3 = 3 \checkmark$

4. Solve the following absolute value equations graphically

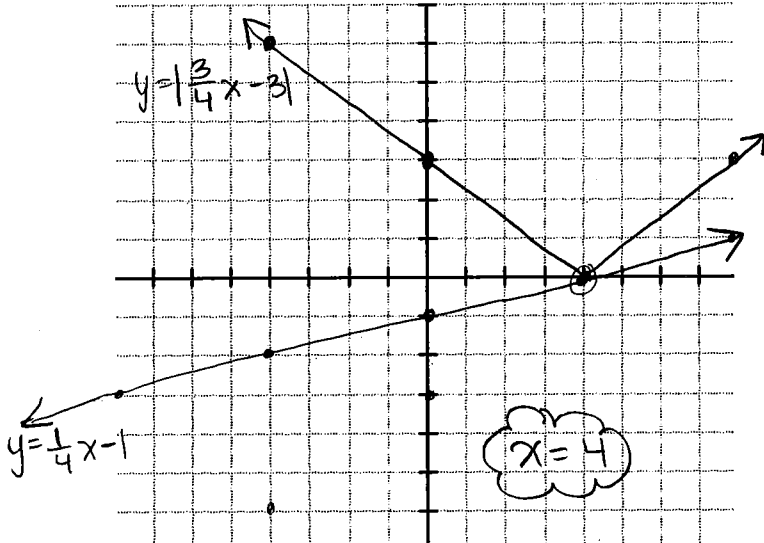
a)  $|\frac{1}{2}x - 2| = 1$



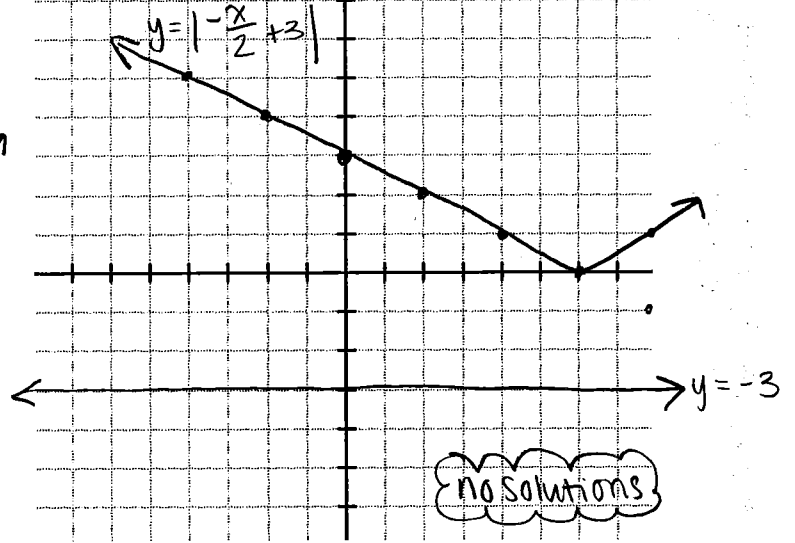
b)  $|\frac{2}{3}x + 1| = 2x - 5$



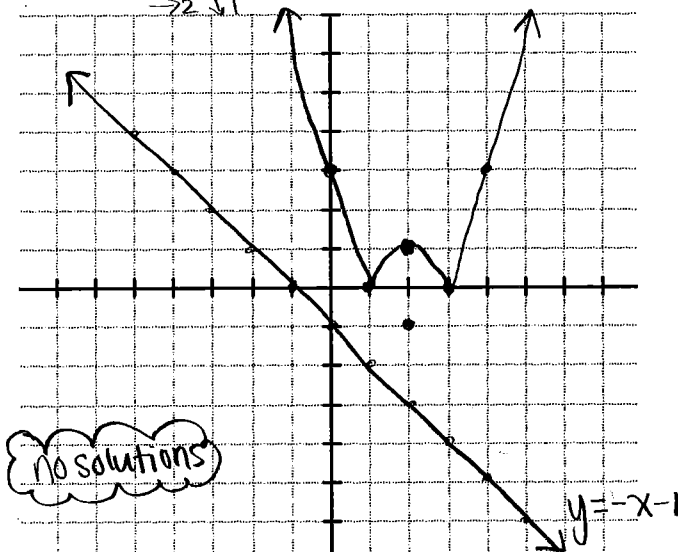
c)  $|\frac{3}{4}x - 3| = \frac{1}{4}x - 1$



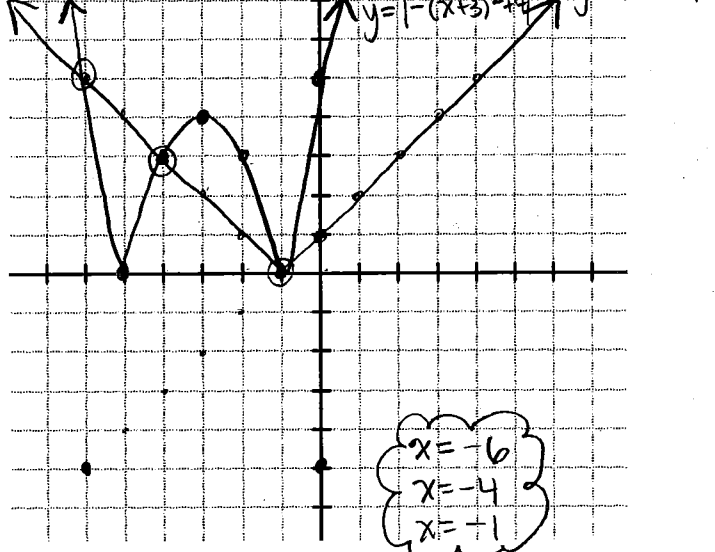
d)  $|\frac{-x}{2} + 3| = -3$



e)  $|(x-2)^2 - 1| = -x - 1$   
 $\rightarrow 2 \downarrow 1$



f)  $|\overset{\leftarrow 3 \uparrow 4}{-(x+3)^2 + 4}| = |x+1|$



5. Graph each of the following reciprocal functions and state the equation of the vertical & horizontal asymptotes and the domain & range.

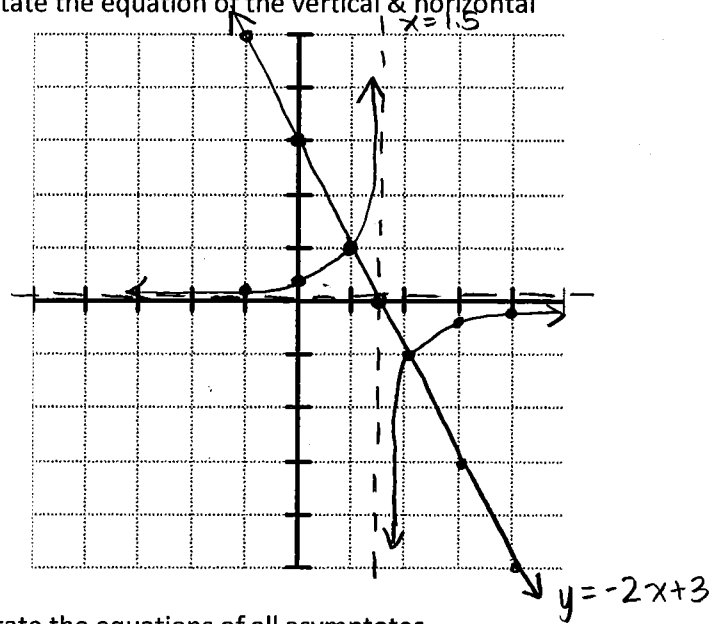
$$f(x) = \frac{1}{-2x+3}$$

Vertical Asymptote:  $x=1.5$

Horizontal Asymptote:  $y=0$

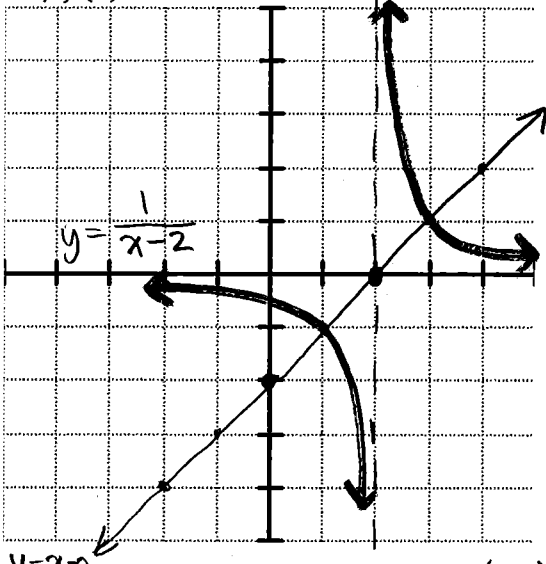
Domain:  $\{x \mid x \neq 1.5, x \in \mathbb{R}\}$

Range:  $\{y \mid y \neq 0, y \in \mathbb{R}\}$

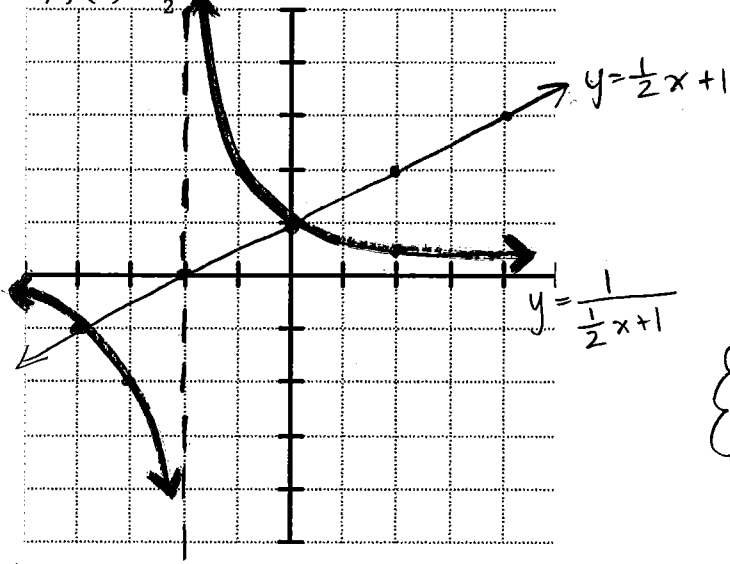


6. Graph the following functions and their reciprocals. State the equations of all asymptotes.

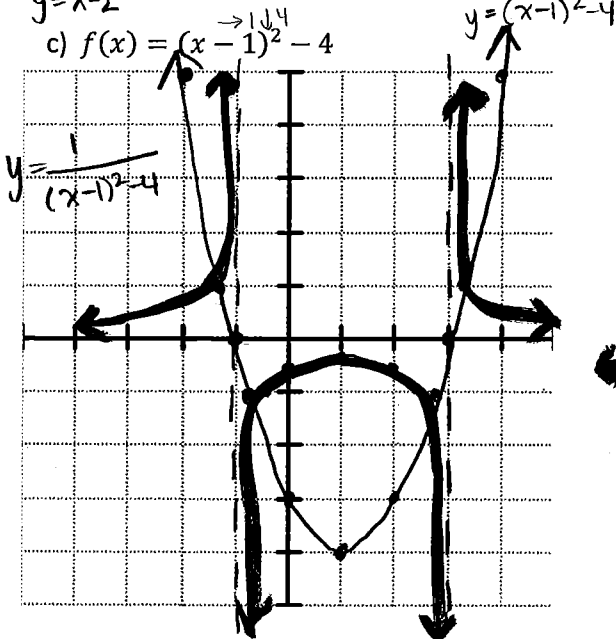
a)  $f(x) = x - 2$



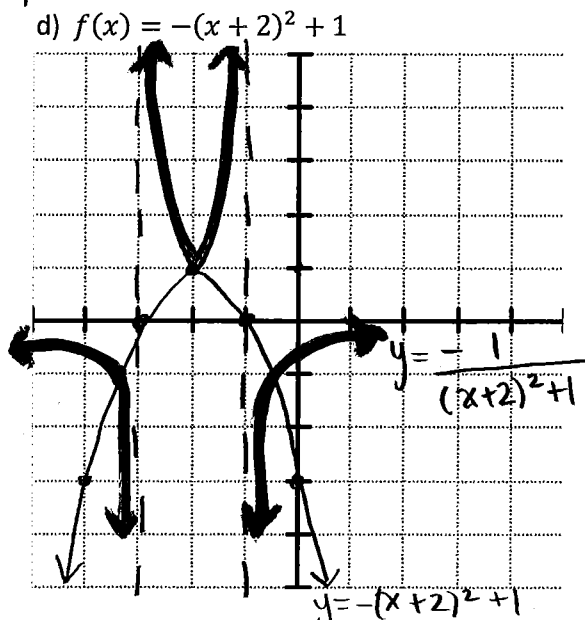
b)  $f(x) = \frac{1}{2}x + 1$



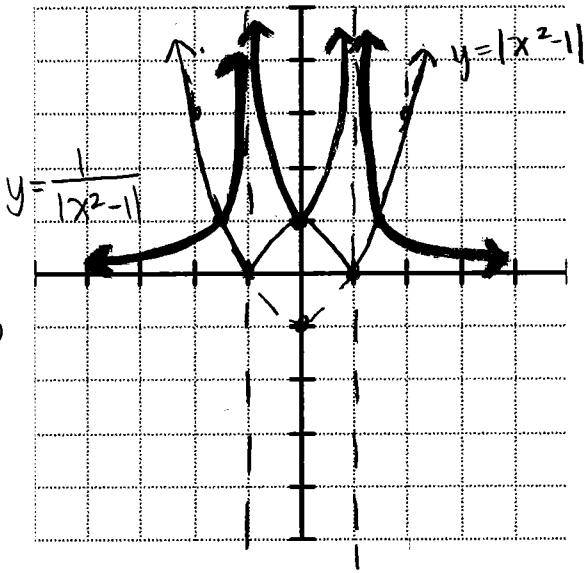
c)  $f(x) = (x-1)^2 - 4$



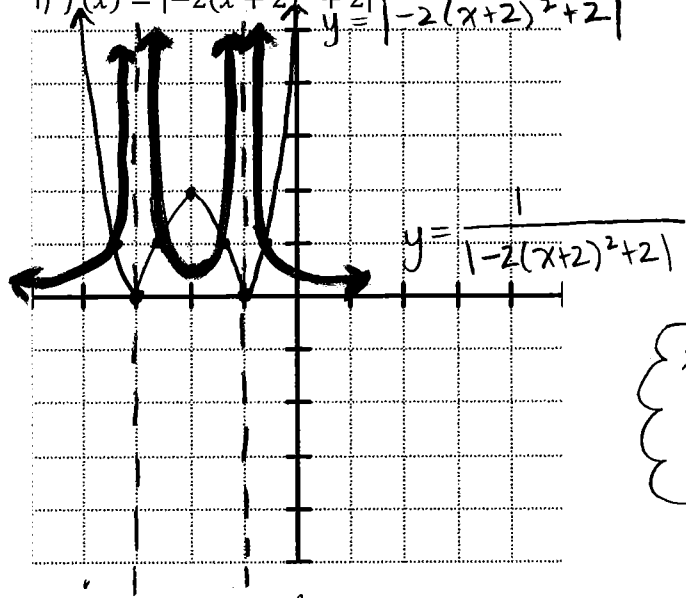
d)  $f(x) = -(x+2)^2 + 1$



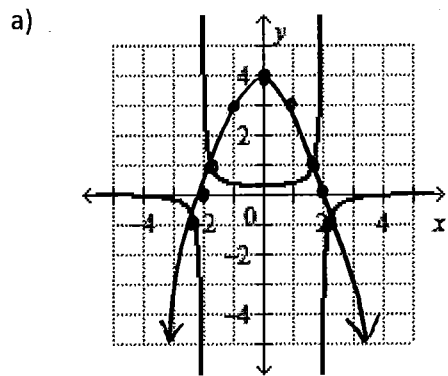
e)  $f(x) = |x^2 - 1|$



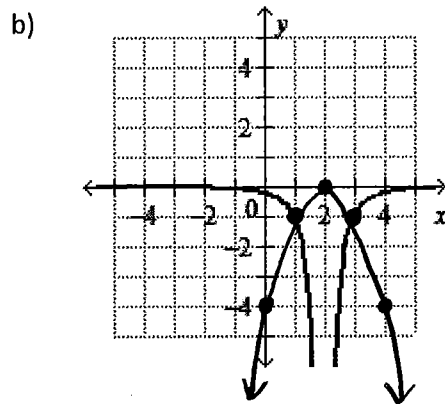
f)  $f(x) = |-2(x+2)^2 + 2|$



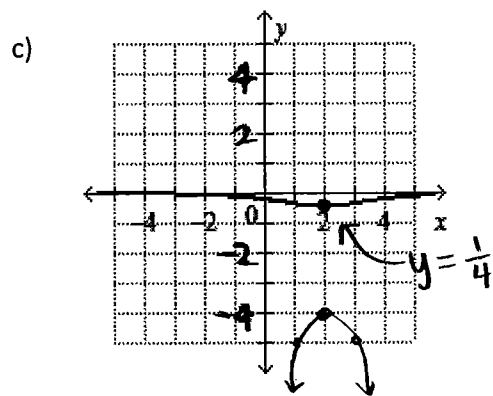
7. Given the graph of the following reciprocal function determine the equation of the original function.



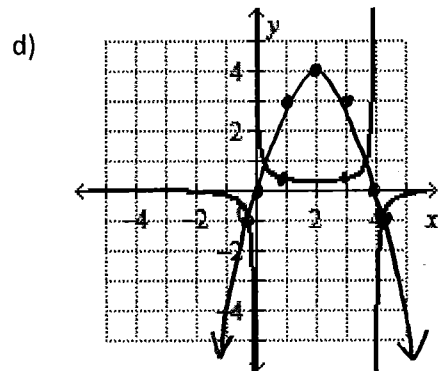
Original Function:  $y = -x^2 + 4$



Original Function:  $y = -(x-2)^2$



Original Function:  $y = -(x-2)^2 - 4$



Original Function:  $y = -(x-2)^2 + 4$