Chapter 8 Review Worksheet Name KEY

1. Consider the following functions and determine the following properties.

i) $y = |-2x + 4|$

a) Determine the x and y intercepts for the function.

x-int: set $y = 0$

$0 = -2x + 4$

$2x = 4$

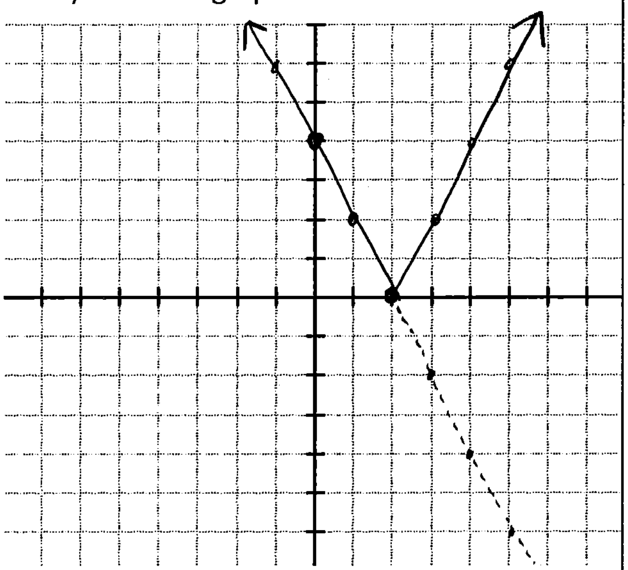
$\boxed{x = 2}$

y-int: set $x = 0$

$y = |-2(0) + 4|$

$\boxed{y = 4}$

b) Sketch a graph of the function.



c) State the domain and range.

D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

d) Express the equation as a piecewise function.

$$y = \begin{cases} -2x+4 & \text{if } x \leq 2 \\ -(-2x+4) & \text{if } x > 2 \end{cases}$$

ii) $y = |-2(x + 3)^2 + 2|$

a) Determine the x and y intercepts for the function.

x-int: $y = 0$

$0 = -2(x+3)^2 + 2$

$-2 = -2(x+3)^2$

$1 = (x+3)^2$

$\pm\sqrt{1} = x + 3$

$-3 \pm 1 = x \rightarrow \boxed{x = -4}, \boxed{x = -2}$

y-int: $x = 0$

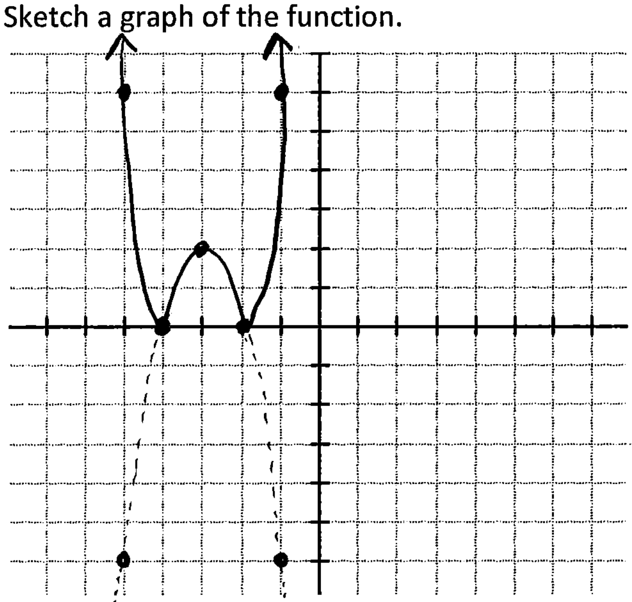
$y = |-2(0+3)^2 + 2|$

$y = |-2(9) + 2|$

$y = |-16|$

$\boxed{y = 16}$

b) Sketch a graph of the function.



c) State the domain and range.

D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

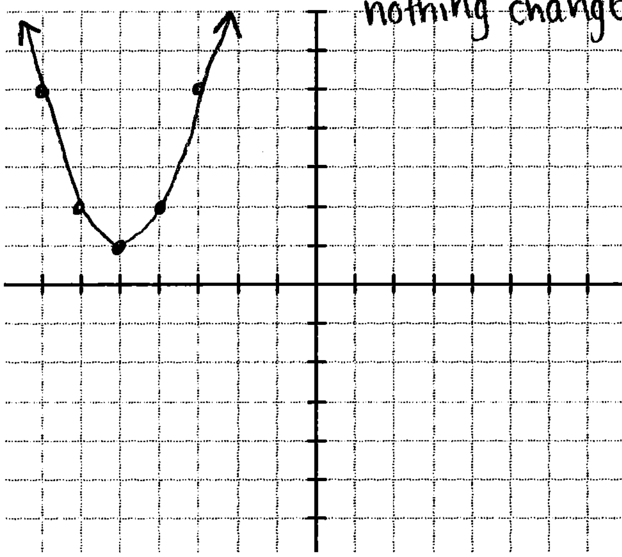
d) Express the equation as a piecewise function.

$$y = \begin{cases} -2(x+3)^2+2 & \text{if } -4 \leq x \leq -2 \\ -(-2(x+3)^2+2) & \text{if } -4 > x \text{ and } x > -2 \end{cases}$$

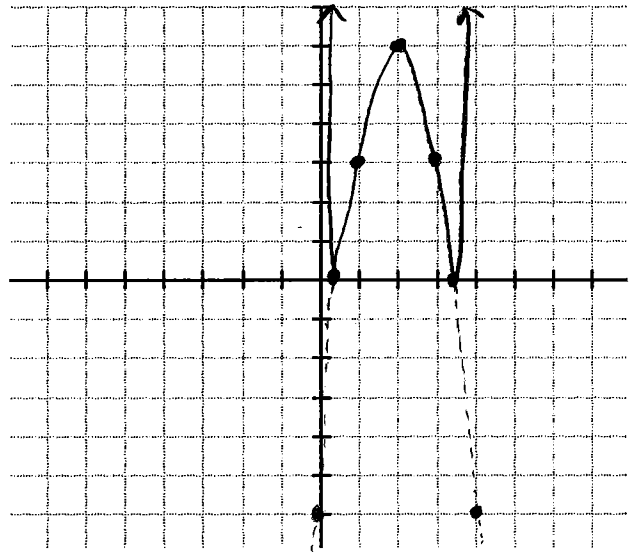
2. Graph the following absolute value functions.

Steps: 1, 3, 5
x -3 : -3, -9, -15

iii) $y = |(x + 5)^2 + 1|$ *all positive so nothing changes



iv) $y = |-3(x - 2)^2 + 6|$



v) $y = |-x^2 - 6x - 5|$

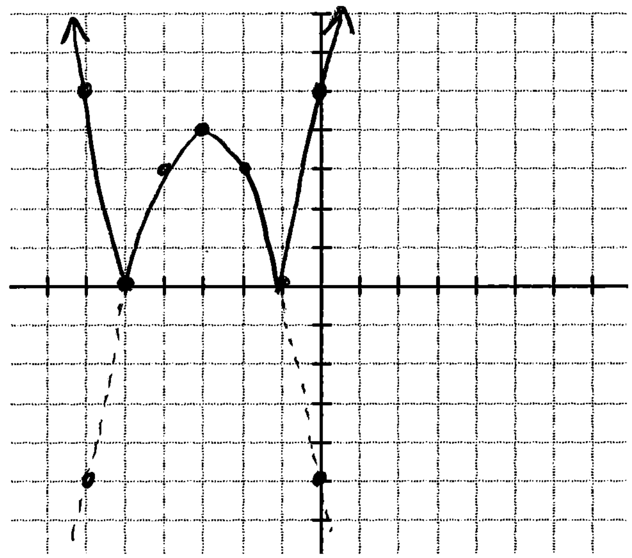
$\frac{1}{2}(6) = 3$
$\hookrightarrow 3^2 = 9$

$y = |-(x^2+6x)-5|$

$y = |-(x^2+6x+9-9)-5|$

$y = |-(x+3)^2+9-5|$

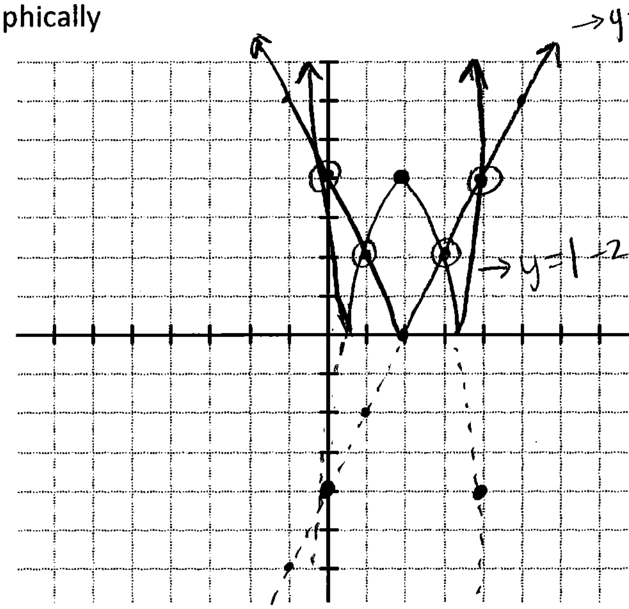
$y = |-(x+3)^2+4|$



3. Solve the following absolute value equations graphically

$y = |-2(x - 2)^2 + 4|$
$y = |2x - 4|$

$\boxed{\text{Solutions: } (0,4), (1,2), (3,2), (4,4)}$



4. Solve the following absolute value equations algebraically and graphically.

a) $|-4x + 6| = 2$

case #1:

$-4x + 6 = 2$

$\quad -6 \quad -6$

$-4x = -4$

$\frac{-4x}{-4} = \frac{-4}{-4}$

$x = 1$

case 2:

$-(-4x+6) = 2$

$4x - 6 = 2$

$\quad +6 \quad +6$

$4x = 8$

$\frac{4x}{4} = \frac{8}{4}$

$x = 2$

Check: $x = 1$

$|-4(1)+6| = 2$

$|-4+6| = 2$

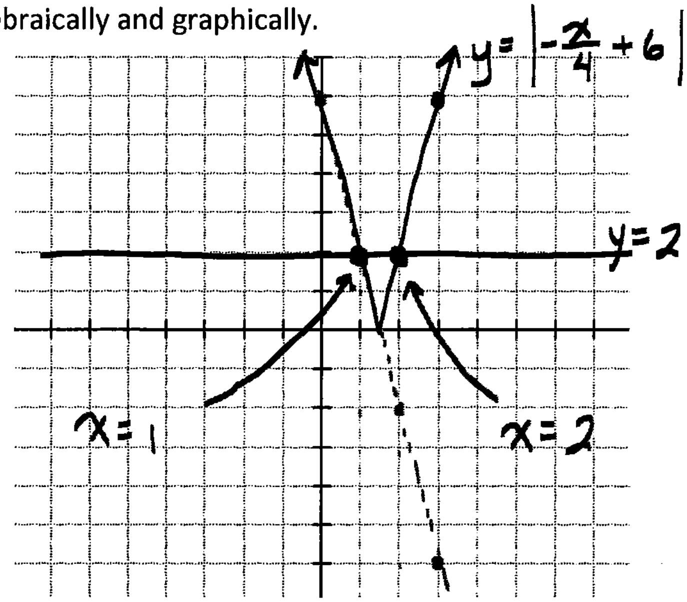
$|2| = 2$ ✓

Check: $x = 2$

$|-4(2)+6| = 2$

$|-8+6| = 2$

$|-2| = 2$ ✓



Solutions $x = 1, x = 2$

b) $\left|\frac{2}{3}x - 2\right| = x - 4$

case #1

$3\left(\frac{2}{3}x - 2 = x - 4\right)$

$2x - 6 = 3x - 12$

$-2x \quad -2x$

$-6 = x - 12$

$+12 \quad +12$

$6 = x$

case #2:

$-\left(\frac{2}{3}x - 2\right) = x - 4$

$3\left(-\frac{2}{3}x + 2 = x - 4\right)$

$-2x + 6 = 3x - 12$

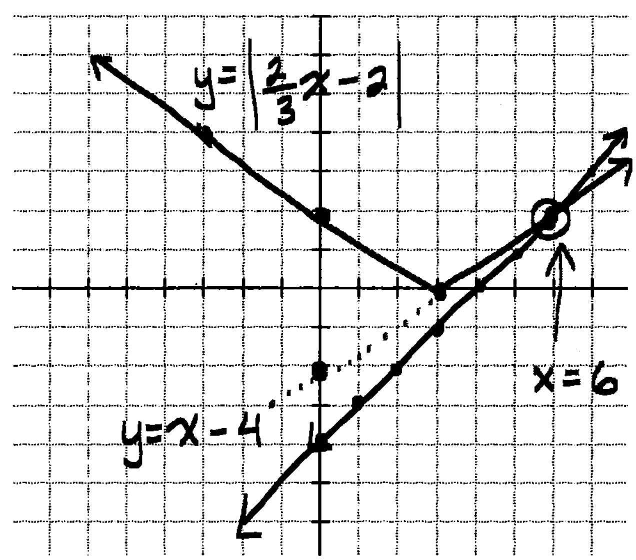
$+2x \quad +2x$

$6 = 5x - 12$

$+12 \quad +12$

$18 = 5x$

$x = \frac{18}{5}$



Check

$\left|\frac{2}{3}(6) - 2\right| = 6 - 4$

$|4 - 2| = 2$

$|2| = 2$ ✓

Check: $\left|\frac{2}{3}\left(\frac{18}{5}\right) - 2\right| = \frac{18}{5} - 4$

$\left|\frac{36}{15} - \frac{30}{15}\right| = \frac{18}{5} - \frac{20}{5}$

$\left|\frac{6}{15}\right| = \frac{-2}{5}$ ×

Solution $\boxed{x = 6}$

c) $|-x + 1| = x^2 - 6x + 9$

Case # 1

$-x+1 = x^2-6x+9$

$+x-1 \qquad +x-1$

$0 = x^2-5x+8$

$x = \dfrac{-(-5) \pm \sqrt{(-5)^2-4(1)(8)}}{2(1)}$

$x = \dfrac{5 \pm \sqrt{25-32}}{2}$

$x = \dfrac{5 \pm \sqrt{-7}}{2}$

Cannot take the square root of a negative → no Solution

Case # 2

$-(-x+1) = x^2-6x+9$

$x-1 = x^2-6x+9$

$-x+1 \qquad -x+1$

$0 = x^2-7x+10$

$0 = (x-5)(x-2)$

$x=5 \quad x=2$

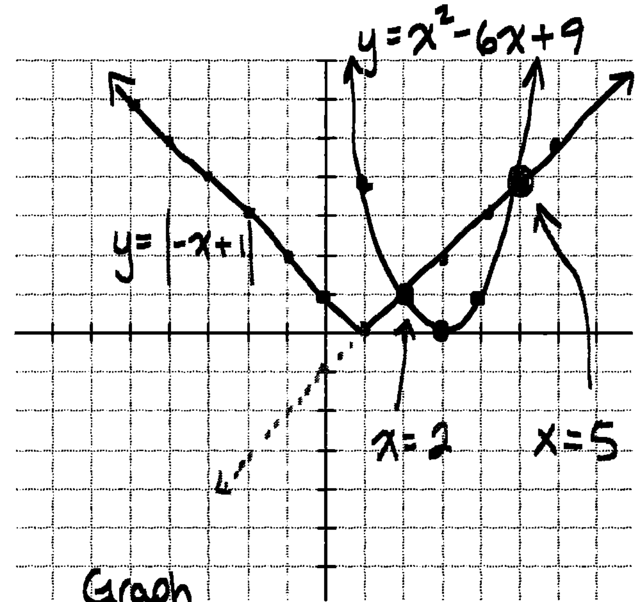
Check:

$|-5+1| = (5)^2-6(5)+9$

$4 = 4$ ✓

$|-2+1| = 2^2-6(2)+9$

$1 = 1$ ✓



Graph

$y = x^2-6x+9$

$y = (x^2-6x+9)-9+9$

$y = (x-3)^2$

d) $x + 4 = |-x^2-4x|$

Case#1:

$x+4 = -x^2-4x$

$+x^2+4x \qquad +x^2+4x$

$x^2+5x+4 = 0$

$(x+1)(x+4) = 0$

$x=-1 \quad x=-4$

Check $x=-1$

$-1+4 = |-(-1)^2-4(-1)|$

$3 = |-1+4|$

$3 = |3|$ ✓

$x=-4$

$-4+4 = |-(-4)^2-4(-4)|$

$0 = |-16+16|$

$0 = 0$ ✓

Case # 2:

$x+4 = -(-x^2-4x)$

$x+4 = x^2+4x$

$-x-4 \qquad -x-4$

$0 = x^2+3x-4$

$0 = (x+4)(x-1)$

$x=-4 \quad x=1$

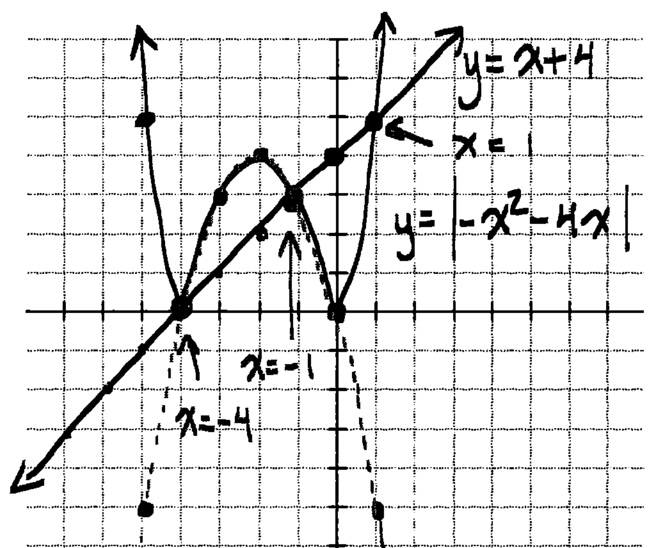
Check $x=1$

$1+4 = |-(1)^2-4(1)|$

$5 = |-1-4|$

$5 = |-5|$

$5 = 5$ ✓



Graph

$y = -x^2-4x$

$y = -(x^2+4x)$

$y = -(x^2+4x+4-4)$

$y = -(x^2+4x+4)+4$

$y = -(x+2)^2+4$

5. Graph each of the following reciprocal functions and state the equation of the vertical & horizontal asymptotes and the domain & range.

$$f(x) = \frac{1}{2x+4}$$

Vertical Asymptote:

$x = -2$

Horizontal Asymptote:

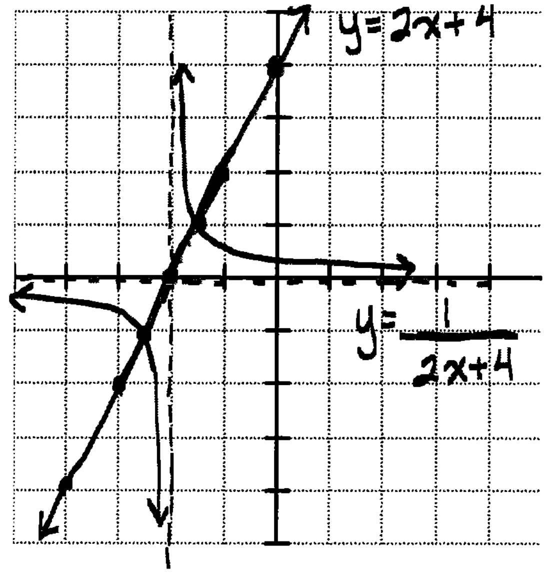
$y = 0$

Domain:

$D: \{x \mid x \neq -2, x \in \mathbb{R}\}$

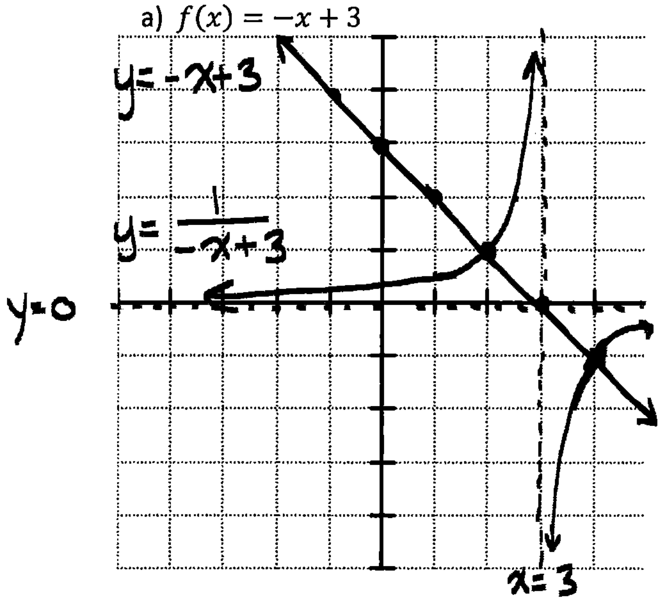
Range:

$R: \{y \mid y \neq 0, y \in \mathbb{R}\}$

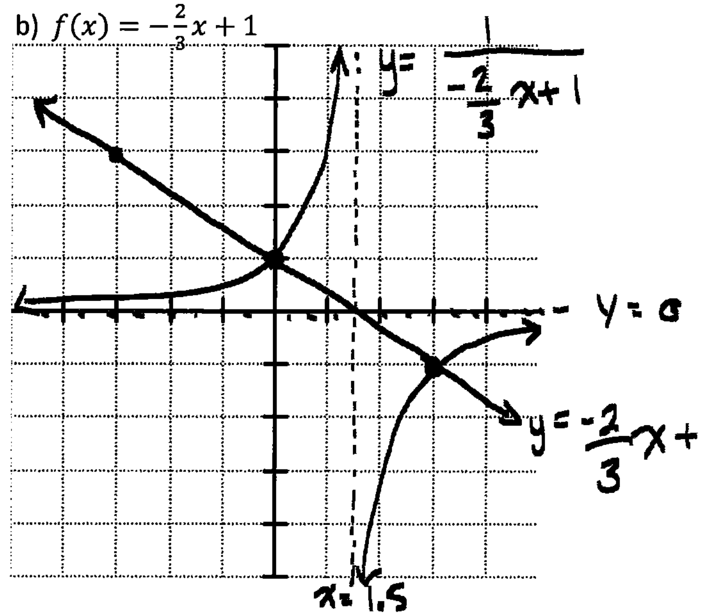


6. Graph the following functions and their reciprocals. State the equations of all asymptotes.

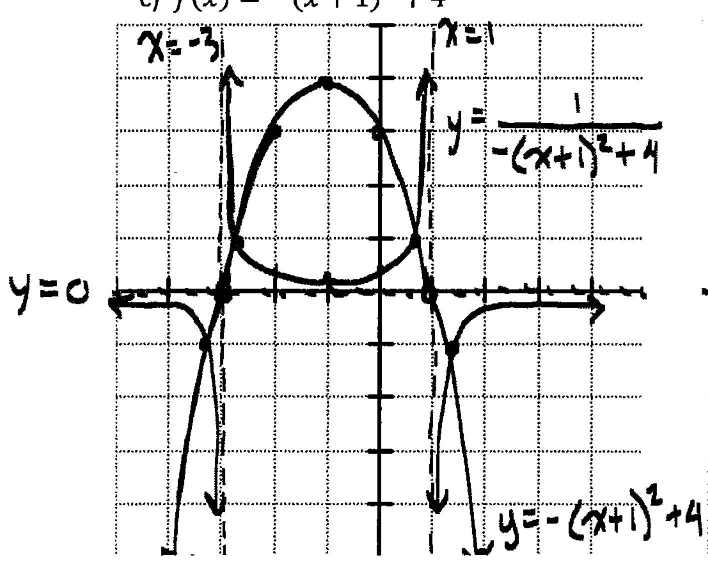
a) $f(x) = -x + 3$



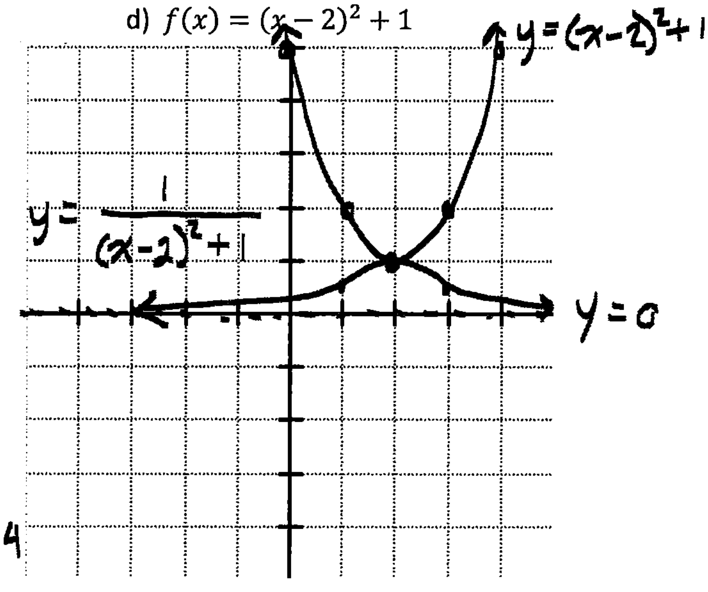
b) $f(x) = -\frac{2}{3}x + 1$



c) $f(x) = -(x+1)^2 + 4$

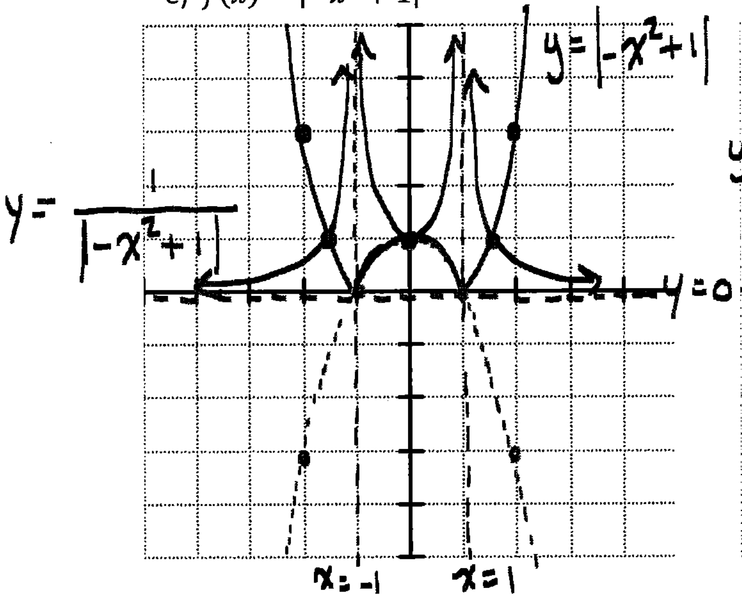


d) $f(x) = (x-2)^2 + 1$

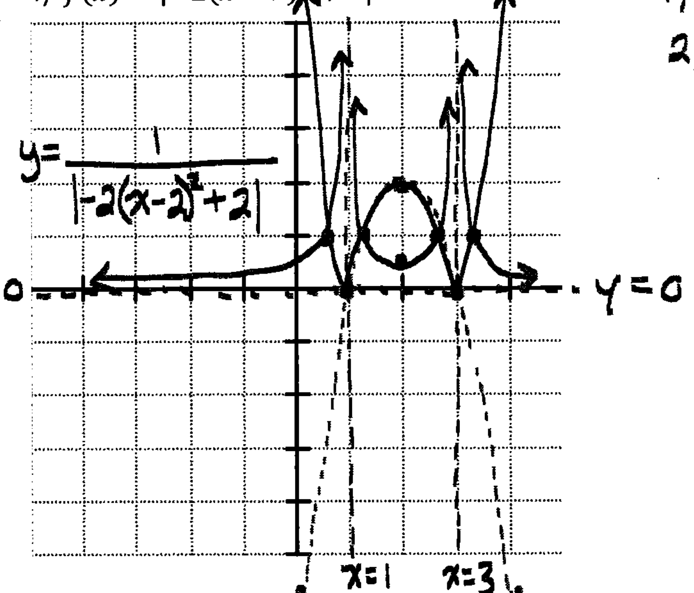


1, 3, 5
2, 6, 10

e) $f(x) = |-x^2 + 1|$

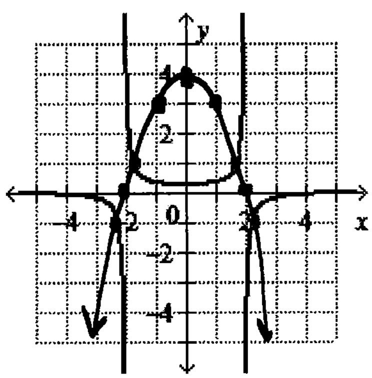


f) $f(x) = |-2(x-2)^2 + 2|$



7. Given the graph of the following reciprocal function determine the equation of the original function.

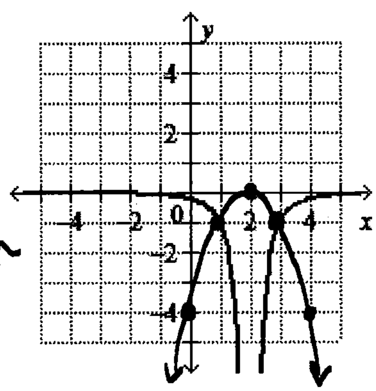
a)



Original Function: $y = -x^2 + 4$

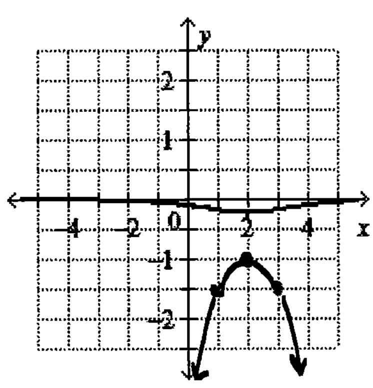
The red dots are the ones that are given to use from the reciprocal graph

b)



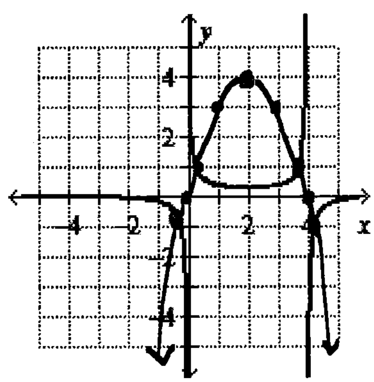
Original Function: $y = -(x-2)^2$

c)



Original Function: $y = -(x-2)^2 - 2$

d)



Original Function:

$y = -(x-2)^2 + 4$