Pre-Calculus 11

Rationals Unit Review Name ______________________

A rational expression is an algebraic fraction that could have a polynomial in the numerator and/or denominator.

Ex. $\frac{1}{x}$, $\frac{x}{5}$, $\frac{x+2}{x^2-4x+4}$, $\frac{2x-6}{5}$, etc.

A non-permissible value or restriction is any value of the variable that makes the denominator equal to zero.

$$\frac{2x}{x^2-x-20} = \frac{2x}{(x-5)(x+4)} \rightsquigarrow \neq 0$$

$x-5 \neq 0$ and $x+4 \neq 0$

$x \neq 5$ $\quad$ $x \neq -4$

Simplifying a rational expression means to cancel common factors from the numerator and denominator.

$2(-10) = -20$

$5, -4$

$$\frac{3x-6}{2x^2+x-10}$$

$$= \frac{3(x-2)}{\underbrace{2x^2+5x}\underbrace{-4x-10}} = \frac{3(x-2)}{x(2x+5)-2(2x+5)}$$

$$= \frac{3\cancel{(x-2)}}{\cancel{(x-2)}(2x+5)}$$

$$= \frac{3}{2x+5}$$

NPVs: $x \neq 2$, $x \neq -5/2$

Multiplying Rationals

$$\frac{x^2+7x+12}{x^2+4x+4} \cdot \frac{x^2-x-6}{x^2-9}$$

* Factor everything first, then cancel common factors before multiplying

$$= \frac{(x+4)\cancel{(x+3)}}{(x+2)\cancel{(x+2)}} \cdot \frac{\cancel{(x-3)}\cancel{(x+2)}}{\cancel{(x+3)}\cancel{(x-3)}}$$

NPVs: $x \neq -2$, $x \neq 3$, $x \neq -3$

$$= \frac{x+4}{x+2}$$

Dividing Rationals

$$\frac{x^2+15x+56}{x^2-3x-54} \div \frac{x^2+6x-16}{x^2+4x-12}$$

$$= \frac{(x+7)(x+8)}{(x-9)(x+6)} \div \frac{(x+8)(x-2)}{(x+6)(x-2)}$$

① Factor

② multiply by the reciprocal

$$= \frac{(x+7)\cancel{(x+8)}}{(x-9)\cancel{(x+6)}} \times \frac{\cancel{(x+6)}\cancel{(x-2)}}{\cancel{(x+8)}\cancel{(x-2)}}$$

③ cancel common factors & write out NPVs: $x \neq 9, -6, 2, -8$

$$= \frac{x+7}{x-9}$$

To add or subtract rational expressions we need a common denominator.

$$\frac{4}{x^2+5x+6} - \frac{5}{x^2-x-12}$$

① Factor

$$= \frac{4}{(x+3)(x+2)} - \frac{5}{(x-4)(x+3)}$$

② LCD: $(x+3)(x+2)(x-4)$

$$= \frac{4}{(x+3)(x+2)} \cdot \frac{(x-4)}{(x-4)} - \frac{5}{(x-4)(x+3)} \cdot \frac{(x+2)}{(x+2)}$$

$$= \frac{(4x-16)-(5x+10)}{(x+3)(x+2)(x-4)}$$

$$= \frac{-x-26}{(x+3)(x+2)(x-4)}$$

③ NPVs: $x \neq -3, -2, 4$

Solving Rational Equations

$$\frac{2x+3}{x+3}+\frac{1}{2}=\frac{x+1}{x-1}$$

① factor (can't in this example)

② LCD: $(2)(x+3)(x-1)$

③ drop the denominator, then expand & simplify

$$\frac{(2x+3)(2)(x-1)}{\cancel{(x+3)(2)(x-1)}}+\frac{(1)(x+3)(x-1)}{\cancel{(2)(x+3)(x-1)}}=\frac{(x+1)(x+3)(2)}{\cancel{(x-1)(x+3)(2)}}$$

$$2(2x+3)(x-1)+(x+3)(x-1)=2(x+1)(x+3)$$ * FOIL

$$2(2x^2-2x+3x-3)+(x^2+3x-x-3)=2(x^2+x+3x+3)$$

$$4x^2-4x+6x-6+x^2+3x-x-3=2x^2+2x+6x+6$$

$$5x^2+4x-9=2x^2+8x+6$$

$$-2x^2 \quad -8x \quad -6 \quad -2x^2 \quad -8x \quad -6$$

$3(-15)$
$=-45$
-9 $+5$

$$3x^2-4x-15=0$$

$$\underbrace{3x^2-9x}+\underbrace{5x-15}=0$$

$$3x(x-3)+5(x-3)=0$$

$$(3x+5)(x-3)=0$$

$$\downarrow \qquad \downarrow$$

$$x=-\frac{5}{3} \qquad x=3$$