

### Rational Expressions

Simplify each expression.

$$1) \frac{2 \sqrt{2} n^2}{3 \sqrt{18} n^3}$$

$$= -\frac{2}{3n}$$

$$2) \frac{40x^2}{20x^3 \cdot 2}$$

$$= \frac{2}{x^2}$$

$$3) \frac{6p^2 - 30p}{5 - p}$$

$$= \frac{6p(p-5)}{-(p-5)}$$

$$= -6p$$

$$4) \frac{5 \sqrt{40k^2} - \sqrt{16k}}{5 \sqrt{40k}}$$

$$= \frac{5k - 2}{5}$$

$$5) \frac{n^2 - n - 30}{n^2 - 12n + 36}$$

$$= \frac{(n-6)(n+5)}{(n-6)(n-6)}$$

$$= \frac{n+5}{n-6}$$

$$6) \frac{x^2 + 19x + 90}{x^2 + 13x + 36}$$

$$= \frac{(x+9)(x+10)}{(x+9)(x+4)}$$

$$= \frac{x+10}{x+4}$$

7)  $\frac{7m^2 + 60m + 32}{3m^2 + 29m + 40}$

$7(32) = 224$   
 $56 \cdot 4$   
 $3(40) = 120$   
 $24 \cdot 5$

$$= \frac{7m^2 + 56m + 4m + 32}{3m^2 + 24m + 5m + 40}$$

$$= \frac{7m(m+8) + 4(m+8)}{3m(m+8) + 5(m+8)}$$

$$= \frac{(7m+4)(m+8)}{(3m+5)(m+8)}$$

$$= \frac{7m+4}{3m+5}$$

$$8) \frac{5r^2 - 26r - 24}{12 + 28r - 5r^2}$$

$$= \frac{5r^2 - 30r + 4r - 24}{12 + 30r - 2r - 5r^2}$$

$$= \frac{5r(r-6) + 4(r-6)}{-5r^2 + 30r - 2r + 12}$$

$$= \frac{5r(r-6) + 4(r-6)}{-5r(r-6) - 2(r-6)}$$

$$= \frac{5r+4}{-5r-2} \text{ or } \frac{-5r-4}{5r+2}$$

$$5(-24) = -120$$

$$\begin{array}{r} / \quad \backslash \\ -30 \quad 4 \end{array}$$

$$-5(12) = -60$$

$$\begin{array}{r} / \quad \backslash \\ 30 \quad -2 \end{array}$$

$$9) \frac{\cancel{n+1}}{\cancel{(n-10)}\cancel{(n+1)}} \cdot \frac{10\cancel{n}\cancel{(n-10)}}{10}$$

$$= n$$

$$11) \frac{\cancel{10}b^2}{\cancel{b+10} \cdot 9\cancel{10}}$$

$$= \frac{b}{9}$$

$$\begin{aligned} & \xrightarrow{-(x^2-7x-16)} 13) \frac{4x}{18+7x-x^2} \cdot \frac{x^2-2x-63}{x^2-49} \\ &= \frac{4x}{-(\cancel{x-9})(x+2)} \cdot \frac{(\cancel{x-9})(\cancel{x+7})}{(x-7)(\cancel{x+7})} \\ &= -\frac{4x}{(x+2)(x-7)} \end{aligned}$$

$$15) \frac{k^2-19k+90}{k^2-10k+25} \div \frac{k^2-2k-80}{k+8}$$

$$= \frac{(k-9)(\cancel{k-10})}{(k-5)(k-5)} \times \frac{(\cancel{k+8})}{(\cancel{k-10})(\cancel{k+8})}$$

$$= \frac{k-9}{(k-5)^2}$$

$$10) \frac{\cancel{x+6}}{(x-7)(\cancel{x+4})} \cdot \frac{(x+8)(\cancel{x-4})}{\cancel{x+6}}$$

$$= \frac{x+8}{x-7}$$

$$12) \frac{1}{3v} \cdot \frac{4v^3-32v^2}{4v^2}$$

$$= \frac{4\cancel{v}^2(v-8)}{3v \cdot 4\cancel{v}^2}$$

$$= \frac{v-8}{3v}$$

$$14) \frac{n^2+18n+80}{3n+24} \cdot \frac{3n-24}{8n^2+80n}$$

$$= \frac{(\cancel{n+8})(\cancel{n+10})}{3(\cancel{n+8})} \cdot \frac{3(n-8)}{8n(\cancel{n+10})}$$

$$= \frac{n-8}{8n}$$

$$16) \frac{12a^2-12a}{a-6} \div \frac{18a^2-18a}{9a^2-54a}$$

$$= \frac{12a(\cancel{a-1})}{(\cancel{a-6})} \times \frac{9\cancel{a}(a-1)}{2 \cdot 18\cancel{a}(a-1)}$$

$$= \frac{12a}{2}$$

$$= 6a$$

$$17) \frac{15b-3}{5b-1} \div \frac{3b-12}{80b}$$

$$= \frac{\cancel{3}(5b-1)}{\cancel{5b-1}} \times \frac{80b}{\cancel{3}(b-4)}$$

$$= \frac{80b}{b-4}$$

$$19) \frac{2u}{8u^3v^2} - \frac{u-4v}{8u^3v^2}$$

$$= \frac{2u - u + 4v}{8u^3v^2}$$

$$= \frac{u+4v}{8u^3v^2}$$

$$21) \frac{5x}{x-3} + \frac{6}{6x} \quad \text{LCD: } 6x(x-3)$$

$$= \frac{5x}{(x-3)} \left( \frac{6x}{6x} \right) + \frac{6}{6x} \left( \frac{x-3}{x-3} \right)$$

$$= \frac{30x^2 + 6x - 18}{6x(x-3)}$$

$$= \frac{5x^2 + x - 3}{x(x-3)}$$

$$23) \frac{3b}{b+4} - \frac{2b}{b-5} \quad \text{LCD: } (b+4)(b-5)$$

$$= \frac{3b}{b+4} \left( \frac{b-5}{b-5} \right) - \frac{2b}{b-5} \left( \frac{b+4}{b+4} \right)$$

$$= \frac{3b^2 - 15b - 2b^2 - 8b}{(b+4)(b-5)}$$

$$= \frac{b^2 - 23b}{(b+4)(b-5)} \quad \text{or} \quad \frac{b(b-23)}{(b+4)(b-5)}$$

$$18) \frac{3x^2 - 13x - 30}{25x^2 + 45x} \div \frac{24x^2 + 40x}{25x^2 + 45x}$$

$3(-30) = -90$   
 $-18 \quad 5$

$$= \frac{3x^2 - 18x + 5x - 30}{5x(5x+9)} \times \frac{5x(5x+9)}{8x(3x+5)}$$

$$= \frac{3x(x-6) + 5(x-6)}{8x(3x+5)}$$

$$= \frac{(3x+5)(x-6)}{8x(3x+5)}$$

$$= \frac{x-6}{8x}$$

$$20) \frac{5x-4y}{8x^3} + \frac{x+3y}{8x^3}$$

$$= \frac{5x-4y+x+3y}{8x^3}$$

$$= \frac{6x-y}{8x^3}$$

$$22) \frac{3}{3r+1} + \frac{6}{r-1} \quad \text{LCD: } (3r+1)(r-1)$$

$$= \frac{3}{3r+1} \left( \frac{r-1}{r-1} \right) + \frac{6}{r-1} \left( \frac{3r+1}{3r+1} \right)$$

$$= \frac{3(r-1)}{(3r+1)(r-1)} + \frac{6(3r+1)}{(3r+1)(r-1)}$$

$$= \frac{3r-3+18r+6}{(3r+1)(r-1)} = \frac{21r+3}{(3r+1)(r-1)} \quad \text{or} \quad \frac{3(7r+1)}{(3r+1)(r-1)}$$

$$24) \frac{n+6}{n-5} + \frac{6}{2n} \quad \text{LCD: } 2n(n-5)$$

$$= \frac{n+6}{n-5} \left( \frac{2n}{2n} \right) + \frac{6}{2n} \left( \frac{n-5}{n-5} \right)$$

$$= \frac{2n^2 + 12n + 6n - 30}{2n(n-5)}$$

$$= \frac{2n^2 + 18n - 30}{2n(n-5)}$$

$$= \frac{n^2 + 9n - 15}{n(n-5)}$$

Solve each equation. Remember to check for extraneous solutions.

25)  $\frac{1}{4x} = \frac{2}{x} - 1$  LCD:  $4x$

$$\frac{1}{4x} = \frac{2}{x} \left(\frac{4}{4}\right) - 1 \left(\frac{4x}{4x}\right)$$

$$\frac{1}{4x} = \frac{8}{4x} - \frac{4x}{4x}$$

$$1 = 8 - 4x$$

$$-8 -8$$

$$-7 = -4x$$

check

$$\frac{1}{4\left(\frac{7}{4}\right)} = \frac{2}{7/4} - 1$$

$$\frac{1}{7} = \frac{8}{7} - \frac{7}{7}$$

$$\boxed{x = \frac{7}{4}}$$

26)  $\frac{1}{v^2} = \frac{1}{3v^2} + \frac{v+3}{3v^2}$  LCD:  $3v^2$

$$\frac{1}{v^2} \left(\frac{3}{3}\right) = \frac{1}{3v^2} + \frac{v+3}{3v^2}$$

$$\frac{3}{-3} = \frac{1}{-1} + \frac{v+3}{-3}$$

$$\boxed{-1 = v}$$

check

$$\frac{1}{(-1)^2} = \frac{1}{3(-1)^2} + \frac{(-1)+3}{3(-1)^2}$$

$$1 = \frac{1}{3} + \frac{2}{3}$$

$$1 = 1$$

✓

28)  $\frac{2}{n^2-3n} = \frac{1}{n} + \frac{1}{n^2-3n}$  LCD:  $n^2-3n$

$$\frac{2}{n^2-3n} = \frac{1}{n} \left(\frac{n-3}{n-3}\right) + \frac{1}{n^2-3n}$$

$$\frac{2}{+3-1} = \frac{n-3}{+3-1} + \frac{1}{-1}$$

$$\boxed{4 = n}$$

check

$$\frac{2}{4^2-3(4)} = \frac{1}{4} + \frac{1}{4^2-3(4)}$$

$$\frac{2}{4} = \frac{1}{4} + \frac{1}{4} \rightarrow \frac{2}{4} = \frac{2}{4}$$

30)  $\frac{6k+36}{k} = \frac{k-1}{k^2+5k} + \frac{6}{k^2+5k}$  LCD:  $k^2+5k$

$$\frac{6k+36}{k} \left(\frac{k+5}{k+5}\right) = \frac{k-1}{k^2+5k} + \frac{6}{k^2+5k}$$

$$6k^2+30k+36k+180 = \frac{k-1}{-5} + \frac{6}{-5}$$

$$6k^2+65k+175=0$$

$$6k^2+30k+35k+175=0$$

$$6k(k+5)+35(k+5)=0$$

$$(6k+35)(k+5)=0$$

$$\boxed{k = -\frac{35}{6}}$$

$$k = -5$$

reject

check:  $k = -5$

$$\frac{6(-5)+36}{-5} = \frac{-5-1}{(-5)^2+5(-5)} + \frac{6}{(-5)^2+5(-5)}$$

$$\frac{6}{-5} = \frac{-6}{0} + \frac{6}{0}$$

↳ DNE

27)  $\frac{6}{a^2+a} = \frac{1}{a^2+a} + \frac{1}{a+1}$  LCD:  $a^2+a$

$$\frac{6}{a^2+a} = \frac{1}{a^2+a} + \frac{1}{a+1} \left(\frac{a}{a}\right)$$

$$6 = 1 + a$$

$$-1 -1$$

$$\boxed{5 = a}$$

check

$$\frac{6}{5^2+5} = \frac{1}{5^2+5} + \frac{1}{5+1}$$

$$\frac{6}{30} = \frac{1}{30} + \frac{1}{6 \times 5}$$

$$\frac{6}{30} = \frac{1}{30} + \frac{5}{30}$$

$$\frac{6}{30} = \frac{6}{30}$$

29)  $\frac{x+4}{x} - \frac{6}{x-5} = \frac{1}{x-5}$  LCD:  $x(x-5)$

$$\frac{x+4}{x} \left(\frac{x-5}{x-5}\right) - \frac{6}{x-5} \left(\frac{x}{x}\right) = \frac{1}{x-5} \left(\frac{x}{x}\right)$$

$$x^2 - 5x + 4x - 20 - 6x = x$$

$$x^2 - 7x - 20 = x$$

$$x^2 - 8x - 20 = 0$$

$$(x-10)(x+2) = 0$$

$$\boxed{x = 10}$$

$$\boxed{x = -2}$$

check:  $x = 10$

$$\frac{10+4}{10} - \frac{6}{10-5} = \frac{1}{10-5}$$

$$\frac{7}{5} - \frac{6}{5} = \frac{1}{5}$$

$$\frac{1}{5} = \frac{1}{5} \checkmark$$

check:  $x = -2$

$$\frac{-2+4}{-2} - \frac{6}{-2-5} = \frac{1}{-2-5}$$

$$-\frac{1}{2} - \frac{6}{-7} = -\frac{1}{-7}$$

$$-\frac{7}{7} + \frac{6}{7} = -\frac{1}{7} \checkmark$$

$$6(175) = 1050$$

$$\begin{matrix} 30 & 35 \end{matrix}$$