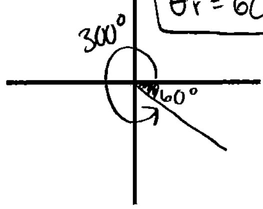
Trigonometry Review Name KEY

1. Find the reference angles of the following angles in standard position.

a) $300°$

$\theta_r = 360° - 300°$

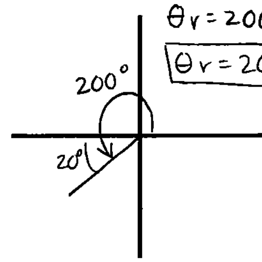
$\boxed{\theta_r = 60°}$



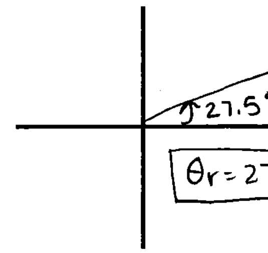
b) $200°$

$\theta_r = 200° - 180°$

$\boxed{\theta_r = 20°}$



c) $27.5°$



$\boxed{\theta_r = 27.5°}$

2. The point $P(3, -4)$ lies on the terminal arm of an angle θ, in standard position. Determine the exact trigonometric ratios for $\sin\theta$, $\cos\theta$, and $tan\,\theta$.

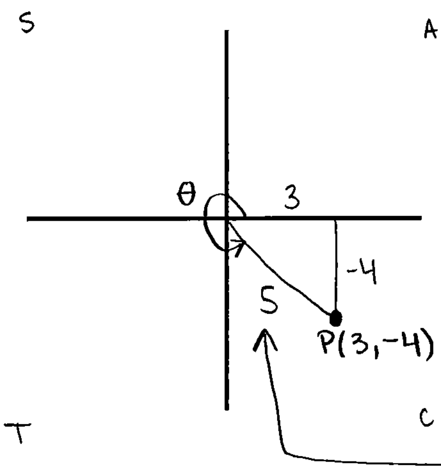
$3^2 + (-4)^2 = c^2$

$9 + 16 = c^2$

$25 = c^2$

$\sqrt{25} = c$

$5 = c$



$sin\,\theta = \frac{-4}{5}$

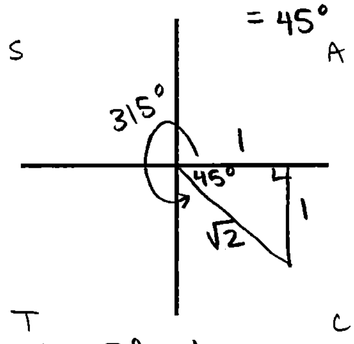
$\cos\theta = \frac{3}{5}$

$tan\,\theta = \frac{-4}{3}$

3. Determine the exact value of the following angles. Show work. No Calculators.

a) $\sin 315°$

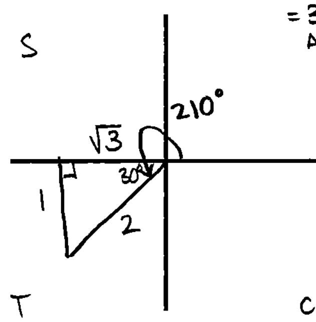
$\theta_r = 360° - 315°$
$= 45°$



$\sin 315° = -\frac{1}{\sqrt{2}}$

b) $\tan 210°$

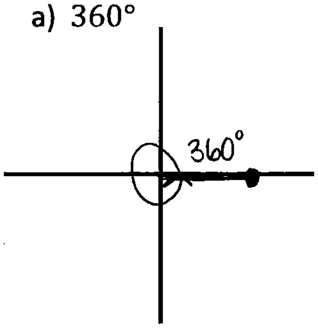
$\theta_r = 210° - 180°$
$= 30°$



$\tan 210° = \frac{1}{\sqrt{3}}$

4. Determine the exact value of the sine, cosine, and tangent values for each ratio.

a) 360°

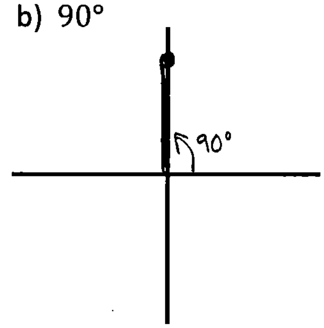


$\sin\theta = \frac{y}{r} = \frac{0}{1} = 0$

$\cos\theta = \frac{x}{r} = \frac{1}{1} = 1$

$\tan\theta = \frac{y}{x} = \frac{0}{1} = 0$

b) 90°



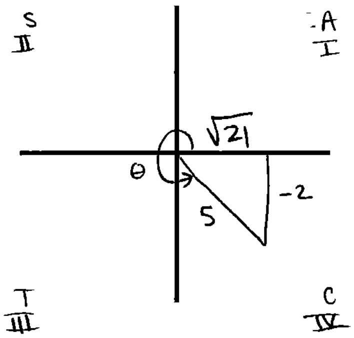
$\sin\theta = \frac{y}{r} = \frac{1}{1} = 1$

$\cos\theta = \frac{x}{r} = \frac{0}{1} = 0$

$\tan\theta = \frac{y}{x} = \frac{1}{0} = \text{DNE}$

5. Suppose θ is an angle in standard position with a terminal arm in quadrant IV, $\sin\theta = \frac{-2}{5}$ $\frac{\text{opp}}{\text{hyp}}$

What is the exact value of $\cos\theta$ and $\tan\theta$.



$\cos\theta = \frac{\sqrt{21}}{5}$

$\tan\theta = \frac{-2}{\sqrt{21}}$

$(-2)^2 + b^2 = 5^2$

$b^2 = 25 - 4$

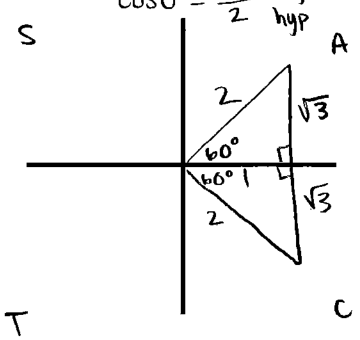
$b^2 = 21$

$b = \sqrt{21}$

6. Solve for θ.

a) $2\cos\theta - 1 = 0 \quad 0° \leq \theta < 360°$

$\cos\theta = \frac{1}{2}$ $\frac{\text{adj}}{\text{hyp}}$



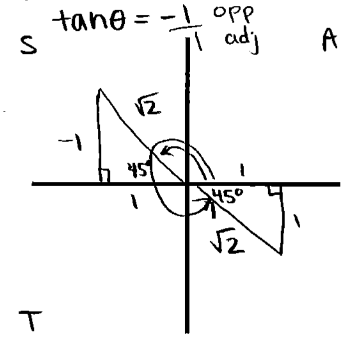
$\boxed{\theta = 60°}$

$\theta = 360° - 60°$

$= \boxed{300°}$

b) $\tan\theta + 1 = 0 \quad 0° \leq \theta < 360°$

$\tan\theta = \frac{-1}{1}$ $\frac{\text{opp}}{\text{adj}}$



$\theta = 180° - 45°$

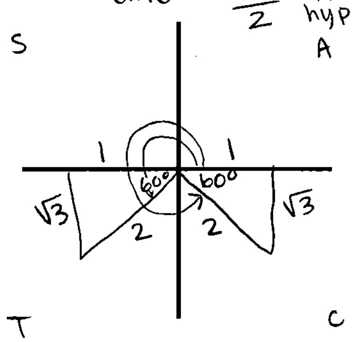
$= \boxed{135°}$

$\theta = 360° - 45°$

$= \boxed{315°}$

c) $2\sin\theta + \sqrt{3} = 0 \quad 0° \leq \theta < 360°$

$\sin\theta = -\frac{\sqrt{3}}{2}$ $\frac{\text{opp}}{\text{hyp}}$



$\theta = 180° + 60°$

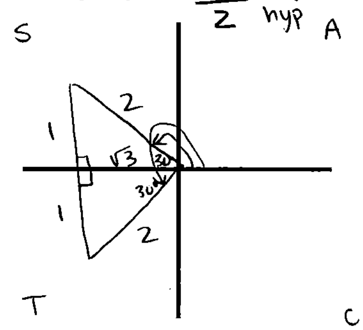
$= \boxed{240°}$

$\theta = 360° - 60°$

$= \boxed{300°}$

d) $2\cos\theta + \sqrt{3} = 0 \quad 0° \leq \theta < 360°$

$\cos\theta = -\frac{\sqrt{3}}{2}$ $\frac{\text{adj}}{\text{hyp}}$



$\theta = 180° - 30°$

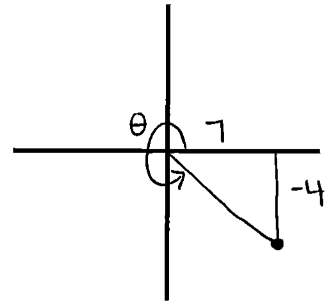
$= \boxed{150°}$

$\theta = 180° + 30°$

$= \boxed{210°}$

7. Point $P(7, -4)$ is on the terminal arm of an angle θ.

   a) Sketch the angle in standard position. $\tan\theta_r = \frac{-4}{7}$

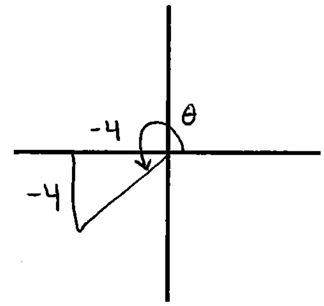


   b) State the reference angle. $\theta_r = \tan^{-1}\left(\frac{-4}{7}\right) = \boxed{30°}$

   c) State the angle θ. $\theta = 360° - 30° = \boxed{330°}$

8. Point $P(-4, -4)$ is on the terminal arm of an angle θ.

   a) Sketch the angle in standard position. $\tan\theta_r = \frac{-4}{-4}$



   b) State the reference angle. $\theta_r = \tan^{-1}(1) = \boxed{45°}$

   c) State the angle θ. $\theta = 180° + 45° = \boxed{225°}$

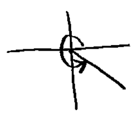
9. Without using a calculator, state whether each ratio is positive or negative.

S | A
T | C

   a) $\sin 80°$

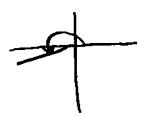
positive

   b) $\tan 345°$



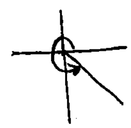
negative

   c) $\cos 181°$



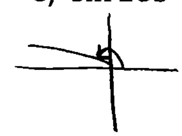
negative

   d) $\tan 280°$



negative

   e) $\sin 165°$

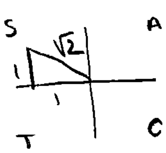


positive

10. Without using a calculator, determine whether each statement is true or false. Show work.

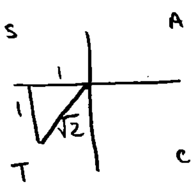
   a) $\cos 135° = \sin 225°$

180-135=45



$\cos 135° = -\frac{1}{\sqrt{2}}$

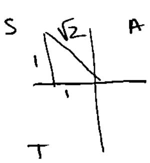
225-180 = 45



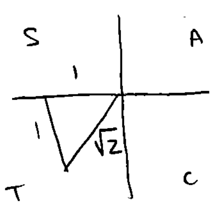
$\sin 225° = -\frac{1}{\sqrt{2}}$

TRUE

   b) $\tan 135° = \tan 225°$



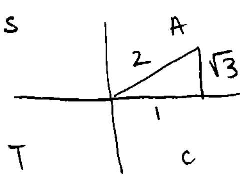
$\tan 135° = \frac{-1}{1} = -1$



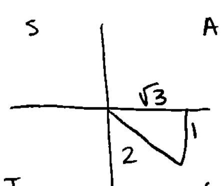
$\tan 225° = \frac{1}{1} = 1$

FALSE

   c) $\sin 60° = \cos 330°$



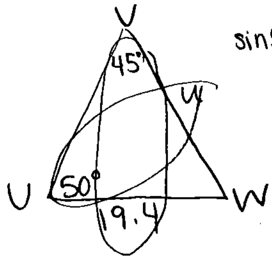
$\sin 60° = \frac{\sqrt{3}}{2}$



$\cos 330° = \frac{\sqrt{3}}{2}$

TRUE

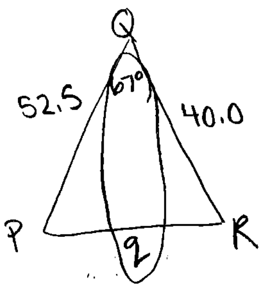
11. In ΔUVW, $\angle U = 50°$, $v = 19.4\ cm$, and $\angle V = 45°$.
    Determine the length of side u to the nearest tenth of a centimetre.



$$\sin 50° \times \frac{u}{\sin 50°} = \frac{19.4}{\sin 45°} \times \sin 50°$$

$$\boxed{u = 21.0\,cm}$$

12. In ΔPQR, $r = 52.5\ cm$, $p = 40.0\ cm$, and $\angle Q = 67°$.
    Determine the length of side q to the nearest tenth of a centimetre.



$$q^2 = 52.5^2 + 40^2 - 2(52.5)(40)\cos 67°$$

$$q^2 = 2715.17926$$

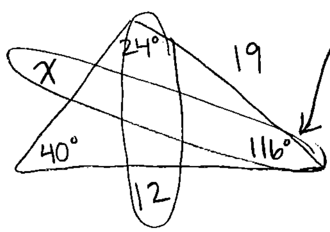
$$q = \sqrt{ans}$$

$$\boxed{q = 52.1\,cm}$$

13. Sketch a triangle that corresponds to the equation.
    Then determine the third angle and the third side length

$$\frac{12}{\sin 24} = \frac{19}{\sin 40}$$

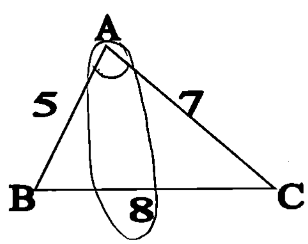
$$180° - 24° - 40°$$
$$= \boxed{116°}$$



$$\sin 116 \times \frac{x}{\sin 116°} = \frac{12}{\sin 24°} \times \sin 116$$

$$\boxed{x = 26.5}$$

14. In ΔABC below, determine the measure of $\angle A$



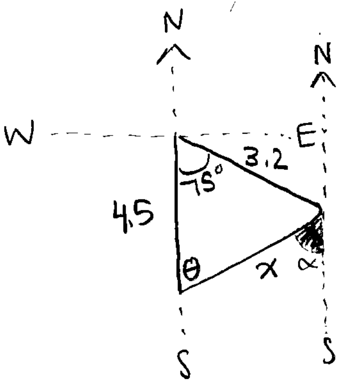
$$\cos A = \frac{5^2 + 7^2 - 8^2}{2(5)(7)}$$

$$\cos A = \frac{10}{70}$$

$$\angle A = \cos^{-1}\left(\frac{10}{70}\right)$$

$$\boxed{\angle A = 81.8°}$$

15. A kayak leaves Rankin Inlet, Nunavut, and heads due south for 4.5 km. At the same time, a second kayak travels in a direction $S75°E$ from the inlet for 3.2 km. In which direction, to the nearest degree, would the second kayak have to travel to meet the first kayak?



① Find x: $x^2 = 4.5^2 + 3.2^2 - 2(4.5)(3.2)\cos 75°$

$x^2 = 23.036$

$x = \sqrt{ans}$

$x = 4.8$ km

② Find θ:

$3.2 \times \dfrac{\sin\theta}{3.2} = \dfrac{\sin 75°}{4.8} \times 3.2$

$\sin\theta = 0.6439\ldots$

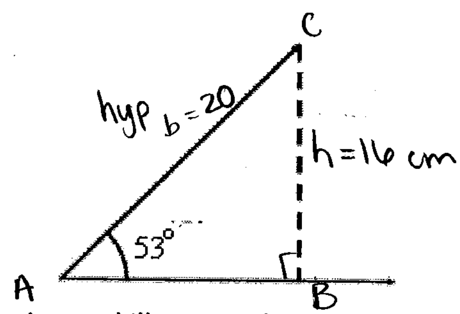
$\theta = \sin^{-1}(0.6439\ldots)$

$\theta = 40°$ $\boxed{\therefore \alpha = 40°}$

The kayaker must travel in a direction S40°W to meet the first kayaker.

① find side x
② find θ
③ θ is equal to α

16. Given $\angle A = 53°$ and $b = 20cm$



a) Determine the height of the triangle to the nearest tenth of a centimeter.

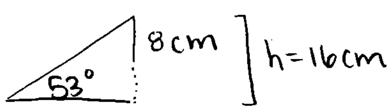
$\sin 53° = \dfrac{opp}{hyp}$

$20 \times \sin 53° = \dfrac{h}{20} \times 20$

$h = 15.97$

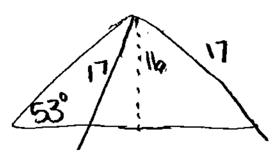
$\boxed{h = 16.0cm}$

b) Determine and illustrate the number of triangles that can be drawn if $a = 8cm$



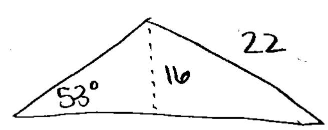
$\boxed{\text{Zero}}$

c) Determine and illustrate the number of triangles that can be drawn if $a = 17cm$



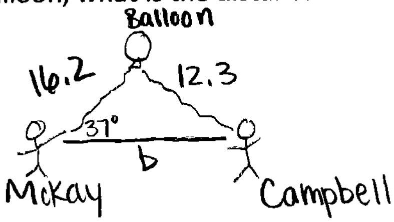
$\boxed{\text{two}}$

d) Determine and illustrate the number of triangles that can be drawn if $a = 22cm$



$\boxed{\text{One}}$

17. Mr. McKay and Mr. Campbell are keeping busy playing with a helium filled balloon in the field. There are two strings tied to the balloon and each man is holding one of the strings. The string of Mr. McKay's balloon is 16.2 feet long and forms an angle of 37° with the ground. Mr. Campbell's balloon string is 12.3 feet. Assuming that Mr. McKay and Mr. Campbell form a triangle in a vertical plane with the balloon, what is the distance between Mr. McKay and Mr. Campbell?



① Find $\angle C$:

$$16.2 \times \frac{\sin C}{16.2} = \frac{\sin 37^\circ}{12.3} \times 16.2$$

$\sin C = 0.7926\ldots$

$\boxed{\angle C = 52^\circ}$

② what is $\angle B$?

$180 - 52^\circ - 37^\circ = \boxed{91^\circ}$

③ what is side b?

$$\sin 91^\circ \times \frac{b}{\sin 91^\circ} = \frac{12.3}{\sin 37^\circ} \times \sin 91^\circ$$

$\boxed{b = 20.4\text{ ft}}$

④ can another triangle exist?
Check when $\angle C$ is obtuse
$\angle C = 180^\circ - 52^\circ$

$\boxed{\angle C = 128^\circ}$ there is a second triangle!

$\therefore \angle B = 180 - 128^\circ - 37^\circ = \boxed{15^\circ}$

side b: $\sin 15^\circ \times \frac{b}{\sin 15^\circ} = \frac{12.3}{\sin 37^\circ} \times \sin 15^\circ$

$\boxed{b = 5.3\text{ ft}}$

$\therefore$ They are either 20.4 ft or 5.3 ft a part.