**Chapter 3 Review**                     Name (KEY) _____

1. Explain the difference between what is being asked in the following questions and complete the questions.

| a) Factor - just factor, can't | b) Solve by Factoring → "solve". The $=$ sign means |
|---|---|
| $x^2 + 5x + 6$ find $x$ | $x^2 + 7x + 6 = 0$   we can find $x$ |
| $= (x+3)(x+2)$ | $(x+1)(x+6) = 0$ |
| | $\downarrow$        $\downarrow$ |
| | $x = -1$      $x = -6$ |

2. What is an extraneous root? Give an example.

An extraneous root is a root to the equation, but not a solution to the problem.

3. What is the discriminant? What does the discriminant tell you? Give examples.

The expression $b^2 - 4ac$. The discriminant tells you what type of possible solution you have (2, 1, or no roots) (the radicand in the quadratic formula)

| If $b^2 - 4ac = C$ | If $b^2 - 4ac \geq 0$ | If $b^2 - 4ac \leq 0$ |
|---|---|---|
| $\downarrow$ | $\downarrow$ | $\downarrow$ |
| 1 solution | 2 solutions | $\emptyset$ solutions |

4. Factor the following polynomials completely.

(a) $9(x-1)^2 - 100y^2$

$= (3(x-1))^2 - 100y^2$

$= (3(x-1) - 10y)(3(x-1) + 10y)$

$= (3x - 3 - 10y)(3x - 3 + 10y)$

(b) $\frac{1}{4}x^2 + \frac{1}{2}x - 6$

$= \frac{1}{4}x^2 + \frac{2}{4}x - \frac{24}{4}$

$= \frac{1}{4}(x^2 + 2x - 24)$

$= \frac{1}{4}(x+6)(x-4)$

(c) $0.1n^2 - 0.1n - 3$

$= 0.1(n^2 - n - 30)$

$= 0.1(n-6)(n+5)$

(d) $4(x+3)^2 + 8(x+3) - 5$    let $a = x+3$

$= 4a^2 + 8a - 5$          $mn = 4(-5) = -20$

$= 4a^2 + 10a - 2a - 5$                $10 \diagup \diagdown -2$

$= 2a(2a+5) - 1(2a+5)$

$= (2a-1)(2a+5)$

$= [2(x+3) - 1][2(x+3) + 5]$

$= (2x+5)(2x+11)$

5. Solve each equation by factoring.

(a) $x^2 + 7x + 10 = 0$

$$(x+5)(x+2) = 0$$

$$x+5=0 \quad or \quad x+2=0$$

$\boxed{x=-5}$ $\boxed{x=-2}$

(b) $x^2 - x = 6$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x-3=0 \qquad x+2=0$$

$\boxed{x=3}$ $\boxed{x=-2}$

(c) $8x^2 = 72x - 144$

$$\frac{8x^2 - 72x + 144 = 0}{8} \quad \frac{}{8}$$

$$x^2 - 9x + 18 = 0$$

$$(x-6)(x-3) = 0$$

$\boxed{x=6}$ $\boxed{x=3}$

(d) $5x^2 + 20 = -25x$

$$\frac{5x^2 + 25x + 20 = 0}{5} \quad \frac{}{5}$$

$$x^2 + 5x + 4 = 0$$

$$(x+4)(x+1) = 0$$

$\boxed{x=-4}$ $\boxed{x=-1}$

$mn = 4(3) = 12$

$6 \diagup \diagdown 2$

(e) $4x^2 + 8x + 3 = 0$

$$4x^2 + 6x + 2x + 3 = 0$$

$$2x(2x+3) + (2x+3) = 0$$

$$(2x+1)(2x+3) = 0$$

$$2x+1=0 \qquad 2x+3=0$$

$\boxed{x = -\frac{1}{2}}$ $\boxed{x = -\frac{3}{2}}$

(f) $2x^2 - 5x = 0$

$$x(2x-5) = 0$$

$\boxed{x=0}$ $2x-5=0$

$\boxed{x = \frac{5}{2}}$

6. Write a quadratic equation that has the following solutions.

(a) $-5, 7$

$$x=-5 \qquad x=7$$

$$x+5=0 \qquad x-7=0$$

$$(x+5)(x-7) = 0$$

$$x^2 - 7x + 5x - 35 = 0$$

$\boxed{x^2 - 2x - 35 = 0}$

(b) $2, \frac{4}{3}$

$$x=2 \qquad x=\frac{4}{3}$$

$$x-2=0 \qquad 3x-4=0$$

$$(x-2)(3x-4) = 0$$

$$3x^2 - 4x - 6x + 8 = 0$$

$\boxed{3x^2 - 10x + 8 = 0}$

**7. Solve each equation.**

(a) $8x^2 - 7 = 249$

$+7 \quad +7$

$x = \pm\sqrt{32}$

$\dfrac{8x^2}{8} = \dfrac{256}{8}$

$x = \pm\sqrt{16 \cdot 2}$

$x^2 = 32$

$\boxed{x = \pm 4\sqrt{2}}$

(b) $\sqrt{(x+5)^2} = \sqrt{49}$

$x + 5 = \pm 7$

$x = -5 \pm 7$

$x = -5 + 7 = \boxed{2}$

$x = -5 - 7 = \boxed{-12}$

(c) $\dfrac{2(x-2)^2}{2} = \dfrac{18}{2}$

$\sqrt{(x-2)^2} = \sqrt{9}$

$x - 2 = \pm 3$

$x = 2 \pm 3$

$x = 2 + 3 = \boxed{5}$

$x = 2 - 3 = \boxed{-1}$

(d) $\sqrt{\left(x - \dfrac{7}{5}\right)^2} = \sqrt{\dfrac{36}{25}}$

$x - \dfrac{7}{5} = \pm\dfrac{6}{5}$

$x = \dfrac{7}{5} \pm \dfrac{6}{5}$

$x = \dfrac{7}{5} + \dfrac{6}{5} = \boxed{\dfrac{13}{5}}$

$x = \dfrac{7}{5} - \dfrac{6}{5} = \boxed{\dfrac{1}{5}}$

**8. Solve each equation by completing the square.**

(a) $\dfrac{3x^2 - 12x + 9}{3} = \dfrac{0}{3}$

$\dfrac{1}{2}(-4) = -2$

$(-2)^2 = 4$

$x^2 - 4x + 3 = 0$

$(x^2 - 4x + 4) - 4 + 3 = 0$

$(x-2)(x-2) - 1 = 0$

$(x-2)^2 - 1 = 0$

$\sqrt{(x-2)^2} = \sqrt{1}$

$x - 2 = \pm 1$

$x = 2 + 1$

$\boxed{x = 3}$

$x = 2 - 1$

$\boxed{x = 1}$

(b) $x^2 - 12x + 31 = 0$

$x^2 - 12x + 36 - 36 + 31 = 0$

$(x^2 - 12x + 36) - 5 = 0$

$\sqrt{(x-6)^2} = \sqrt{5}$

$x - 6 = \pm\sqrt{5}$

$\boxed{x = 6 \pm \sqrt{5}}$

$\dfrac{1}{2}(-12) = -6$

$(-6)^2 = 36$

(c) $\dfrac{-4x^2 + 24x - 21}{-4} = \dfrac{0}{-4}$

$\dfrac{1}{2}(-6) = -3$

$(-3)^2 = 9$

$x^2 - 6x + \dfrac{21}{4} = 0$

$x^2 - 6x + 9 - 9 + \dfrac{21}{4} = 0$

$(x^2 - 6x + 9) - \dfrac{36}{4} + \dfrac{21}{4} = 0$

$\sqrt{(x-3)^2} = \sqrt{\dfrac{15}{4}}$

$x - 3 = \pm\dfrac{\sqrt{15}}{2}$

$\boxed{x = 3 \pm \dfrac{\sqrt{15}}{2}}$

(d) $\left(\dfrac{1}{4}x^2 + x - \dfrac{7}{2}\right) = 0 \times 4$

$x^2 + 4x - 14 = 0$

$x^2 + 4x + 4 - 4 - 14 = 0$

$(x^2 + 4x + 4) - 18 = 0$

$\sqrt{(x+2)^2} = \sqrt{18}$

$x + 2 = \pm\sqrt{18}$

$x = -2 \pm \sqrt{9 \cdot 2}$

$\boxed{x = -2 \pm 3\sqrt{2}}$

$\dfrac{1}{2}(4) = 2$

$(2)^2 = 4$

**9. Solve each equation with the quadratic formula.**

(a) $4x^2 - 3x - 27 = 0$

$a = 4 \quad b = -3 \quad c = -27$

$x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-27)}}{2(4)}$

$x = \dfrac{3 \pm \sqrt{9 + 432}}{8}$

$x = \dfrac{3 \pm \sqrt{441}}{8}$

$x = \dfrac{3 + 21}{8} = \boxed{3}$

$x = \dfrac{3 - 21}{8} = \boxed{-\dfrac{9}{4}}$

(b) $x^2 - 10x + 22 = 0$

$a = 1 \quad b = -10 \quad c = 22$

$x = \dfrac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(22)}}{2(1)}$

$x = \dfrac{10 \pm \sqrt{100 - 88}}{2}$

$x = \dfrac{10 \pm \sqrt{12}}{2}$

$x = \dfrac{10 \pm 2\sqrt{3}}{2} \rightarrow \boxed{x = 5 \pm \sqrt{3}}$

$b^2 - 4ac$

10. Use the discriminant to determine the number of solutions to each question.

(a) $2x^2 - 9x + 4 = 0$

$(-9)^2 - 4(2)(4)$

$= 81 - 32$

$= 49$    $\therefore$ 2 solutions

(b) $-6x^2 + 7x - 5 = 0$

$(7)^2 - 4(-6)(-5)$

$= 49 - 120$

$= -71$    $\therefore$ no solutions

(c) $-6x^2 - 3x + 9 = 0$

$(-3)^2 - 4(-6)(9)$

$= 9 + 216$

$= 225$

$\therefore$ 2 solutions

(d) $-x^2 - 6x - 9 = 0$

$(-6)^2 - 4(-1)(-9)$

$= 36 - 36$

$= 0$

$\therefore$ 1 solution

11. Solve the following.

(a) $x(x-1) = \dfrac{2}{x} \cdot x$

$x(x-1) = 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\quad\downarrow \qquad\qquad \downarrow$

$x = 2 \qquad\quad x = -1$

(b) $x(2x-3) + 4(x+1) = 2(3+2x)$

$2x^2 - 3x + 4x + 4 = 6 + 4x$

$\qquad\qquad -4x - 6 \;-6\; -4x$

$2x^2 - 3x - 2 = 0$

$2x^2 - 4x + x - 2 = 0$

$2x(x-2) + (x-2) = 0$

$(2x+1)(x-2) = 0$

$\quad\downarrow \qquad\qquad \downarrow$

$x = -\dfrac{1}{2} \qquad x = 2$

$mn = 2(-2) = -4$

$\begin{array}{cc} \swarrow & \searrow \\ -4 & 1 \end{array}$

(c) $-2 - 3(x+1)^2 = -50$

$+2 \qquad\qquad\qquad +2$

$\dfrac{-3(x+1)^2}{-3} = \dfrac{-48}{-3}$

$\sqrt{(x+1)^2} = \sqrt{16}$

$x + 1 = \pm 4$

$x = -1 \pm 4$

$\swarrow \qquad\qquad \searrow$

$x = -1 + 4 \qquad x = -1 - 4$

$x = 3 \qquad\qquad x = -5$

(d) $2(x+3)^2 - 11(x+3) + 15 = 0$    let $a = x+3$

$2a^2 - 11a + 15 = 0$

$2a^2 - 5a - 6a + 15 = 0$

$a(2a-5) - 3(2a-5) = 0$

$(a-3)(2a-5) = 0$

$[(x+3)-3][2(x+3)-5] = 0$

$(x)(2x+6-5) = 0$

$(x)(2x+1) = 0$

$\quad\downarrow \qquad\qquad \searrow$

$x = 0 \qquad\quad x = -\dfrac{1}{2}$

$mn = 2(15)$

$= 30$

$\begin{array}{cc} \swarrow & \searrow \\ -5 & -6 \end{array}$

(e) $\frac{1}{4}x^2 + \frac{1}{2}x = 1$

$4 \times \left(\frac{1}{4}x^2 + \frac{1}{2}x - 1\right) = 0 \times 4$

$x^2 + 2x - 4 = 0$

$x = \dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(-4)}}{2(1)}$

$x = \dfrac{-2 \pm \sqrt{20}}{2}$ → $\boxed{x = -1 \pm \sqrt{5}}$

$x = \dfrac{-2 \pm 2\sqrt{5}}{2}$

(f) $\sqrt{2x-7} + 5 = x$

$\left(\sqrt{2x-7}\right)^2 = (x-5)^2$
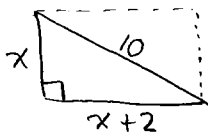
$2x - 7 = (x-5)(x-5)$

$0 = x^2 - 5x - 5x + 25 - 2x + 7$

$0 = x^2 - 12x + 32$

$0 = (x-4)(x-8)$

↓          ↓

$\boxed{x=4}$      $\boxed{x=8}$

**11.** The diagonal of a rectangle is 10 cm. The length is 2 cm longer than the width. Determine the width.



$a^2 + b^2 = c^2$

$x^2 + (x+2)^2 = 10^2$

$x^2 + (x+2)(x+2) = 100$

$x^2 + x^2 + 2x + 2x + 4 = 100$
$\qquad\qquad\qquad -100 \quad -100$

$\dfrac{2x^2 + 4x - 96}{2} = \dfrac{0}{2}$

$x^2 + 2x - 48 = 0$

$(x+8)(x-6) = 0$

$x = -8$ → width can't be negative

$x = 6$

$\boxed{\therefore \text{width} = 6\text{ cm}}$

**11.** The height of a golf ball, in yards, is $h(d) = -0.02d^2 + 2d$, where $d$ is the horizontal distance the ball has travelled, in yards, after being struck. Determine how far the ball travels before it first strikes the ground. (Note: Think of what the height is when it touches the ground).



height here is 0 yards

$h = -0.02d^2 + 2d$

$0 = -0.02d^2 + 2d$

$0 = -0.02d(d-100)$

↓          ↓

$d = 0$          $d = 100$

$\boxed{\therefore \text{the ball travels 100 m}}$