

Chapter 2 Radicals

Name _____

Radicals are any expression involving the root sign: $\sqrt{\quad}$

coefficient \rightarrow $5\sqrt[3]{2}$ \leftarrow radicand
 index

- An entire radical has a coefficient of one or negative one.
 ex. $\sqrt{2}$, $-\sqrt[3]{7}$, ...
- A mixed radical has a coefficient other than one or negative one.
 ex. $2\sqrt{3}$, $5\sqrt[3]{11}$, ...

We can convert from **mixed** to **entire radicals** and **entire** to **mixed radicals**.

Ex. a) $\sqrt{98}$

$$= \sqrt{49 \cdot 2}$$

$$= 7\sqrt{2}$$

simplest form

b) $2\sqrt{7}$

$$= \sqrt{4} \sqrt{7}$$

$$= \sqrt{28}$$

c) $\sqrt[3]{24n^7}$

$$= \sqrt[3]{8 \cdot 3 \cdot (n \cdot n \cdot n) \cdot (n \cdot n \cdot n) \cdot n}$$

$$= 2nn\sqrt[3]{3n}$$

$$= 2n^2\sqrt[3]{3n}$$

We can **add or subtract like radicals** by adding or subtracting their coefficients.

Ex. a) $2\sqrt{5} - 3\sqrt{7} + \sqrt{5}$

$$= 3\sqrt{5} - 3\sqrt{7}$$

b) $2\sqrt{28} - 5\sqrt{7}$

$$= 2\sqrt{4 \cdot 7} - 5\sqrt{7}$$

$$= 2(2)\sqrt{7} - 5\sqrt{7}$$

$$= 4\sqrt{7} - 5\sqrt{7}$$

$$= -\sqrt{7}$$

We **multiply** radicals by multiplying coefficient by coefficient and radicand by radicand. We can **divide** radicals by dividing coefficient by coefficient and radicand by radicand. Answers must be in **simplest form**.

Ex. a) $4\sqrt{6} \cdot 3\sqrt{2}$

$$= 12\sqrt{12}$$

$$= 12\sqrt{4 \cdot 3}$$

$$= 12(2)\sqrt{3}$$

$$= 24\sqrt{3}$$

b) $\frac{18\sqrt{2}}{3\sqrt{14}}$

$$= 6\sqrt{\frac{2}{14}}$$

$$= 6\sqrt{\frac{1}{7}}$$

$$= \frac{6\sqrt{1}}{\sqrt{7}}$$

$$= \frac{6}{\sqrt{7}}$$

Rationalize the denominator

$$= \frac{6}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$$

$$= \frac{6\sqrt{7}}{\sqrt{49}}$$

$$= \frac{6\sqrt{7}}{7}$$

Multiplying binomials: Use FOIL (First, Outside, Inside, Last)

$$\begin{aligned}
 \text{Ex. } & (8\sqrt{6} + 2)(\sqrt{2} - 3) \\
 & = 8\sqrt{12} - 24\sqrt{6} + 2\sqrt{2} - 6 \\
 & = 8\sqrt{4 \cdot 3} - 24\sqrt{6} + 2\sqrt{2} - 6 \\
 & = 16\sqrt{3} - 24\sqrt{6} + 2\sqrt{2} - 6
 \end{aligned}$$

Rationalizing binomial denominators—we multiply by the **conjugate**.

○ The conjugate of $a + b$ is: $a - b$ and vice versa

$$\begin{aligned}
 \text{Ex. } & \frac{5}{(2-\sqrt{3})} \cdot \frac{(2+\sqrt{3})}{(2+\sqrt{3})} \\
 & = \frac{10 + 5\sqrt{3}}{4 + 2\sqrt{3} - 2\sqrt{3} - \sqrt{9}} \\
 & = \frac{10 + 5\sqrt{3}}{4 - 3} \\
 & = \frac{10 + 5\sqrt{3}}{1} \\
 & = 10 + 5\sqrt{3}
 \end{aligned}$$

Solving Radical Equations

1. Isolate the radical
2. Square both sides
3. Solve for 'x'
4. Check for extraneous roots

$$\begin{aligned}
 \text{Ex. } & \sqrt{x+1} + 3 = 5 \\
 & \quad \quad \quad -3 \quad -3
 \end{aligned}$$

$$\begin{aligned}
 & \sqrt{x+1} = 2 \\
 & (\sqrt{x+1})^2 = (2)^2 \\
 & x+1 = 4 \\
 & \quad \quad \quad -1 \quad -1 \\
 & x = 3
 \end{aligned}$$

Check:

$$\sqrt{x+1} + 3 = 5$$

$$\sqrt{3+1} + 3 = 5$$

$$\sqrt{4} + 3 = 5$$

$$2 + 3 = 5$$

$$5 = 5 \quad \checkmark$$