

Radicals Review

Simplify.

$$\begin{aligned}
 1) \sqrt{216} \\
 &= \sqrt{4 \cdot 54} \\
 &= 2\sqrt{54} \\
 &= 2\sqrt{9 \cdot 6} \\
 &= 6\sqrt{6}
 \end{aligned}$$

$$\begin{aligned}
 2) \sqrt{20} \\
 &= \sqrt{5 \cdot 4} \\
 &= 2\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 3) \sqrt{54} \\
 &= \sqrt{9 \cdot 6} \\
 &= 3\sqrt{6}
 \end{aligned}$$

$$\begin{aligned}
 4) \sqrt{100} \\
 &= 10
 \end{aligned}$$

$$\begin{aligned}
 5) -4\sqrt{64} \\
 &= -4 \cdot 8 \\
 &= -32
 \end{aligned}$$

$$\begin{aligned}
 6) -6\sqrt{320} \\
 &= -6\sqrt{64 \cdot 5} \\
 &= -48\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 7) \sqrt{96r^4} \\
 &= \sqrt{16 \cdot 6 \cdot (rr) \cdot (rr)} \\
 &= 4r^2\sqrt{6}
 \end{aligned}$$

$$\begin{aligned}
 8) \sqrt{18x^4} \\
 &= \sqrt{9 \cdot 2 \cdot (xx)(xx)} \\
 &= 3x^2\sqrt{2}
 \end{aligned}$$

$$9) \sqrt{12xy^3z}$$

$$= \sqrt{4 \cdot 3 \cdot x \cdot (yy) \cdot y \cdot z}$$

$$= 2y\sqrt{3xyz}$$

$$10) \sqrt{63mp^4q^2}$$

$$= \sqrt{9 \cdot 7 \cdot m \cdot (pp)(pp) \cdot (qq)}$$

$$= 3p^2q\sqrt{7m}$$

$$11) -5\sqrt{54p^2q^2r^3}$$

$$= -5\sqrt{9 \cdot 6 \cdot (pp) \cdot (qq) \cdot (rr) \cdot r}$$

$$= -15pqr\sqrt{6r}$$

$$12) 8\sqrt{12x^2y^4z^2}$$

$$= 8\sqrt{4 \cdot 3 \cdot (xx) \cdot (yy) \cdot (yy) \cdot (zz)}$$

$$= 16xy^2z\sqrt{3}$$

$$13) 5\sqrt{10} \cdot \sqrt{6}$$

$$= 5\sqrt{60}$$

$$= 5\sqrt{4 \cdot 15}$$

$$= 10\sqrt{15}$$

$$14) \sqrt{2} \cdot 5\sqrt{5}$$

$$= 5\sqrt{10}$$

$$15) -4\sqrt{3(\sqrt{5} + \sqrt{3})}$$

$$= -4\sqrt{15} - 4(3)$$

$$= -4\sqrt{15} - 12$$

$$16) \sqrt{15(\sqrt{10} + 5\sqrt{3})}$$

$$= \sqrt{150} + 5\sqrt{45}$$

$$= \sqrt{25 \cdot 6} + 5\sqrt{9 \cdot 5}$$

$$= 5\sqrt{6} + 15\sqrt{5}$$

$$17) \sqrt{10x(2 - 4\sqrt{5})}$$

$$= 2\sqrt{10x} - 4\sqrt{50x}$$

$$= 2\sqrt{10x} - 4\sqrt{25 \cdot 2 \cdot x}$$

$$= 2\sqrt{10x} - 20\sqrt{2x}$$

$$18) \sqrt{15n(-4\sqrt{5} + 3)}$$

$$= -4\sqrt{75n} + 3\sqrt{15n}$$

$$= -4\sqrt{25 \cdot 3 \cdot n} + 3\sqrt{15n}$$

$$= -20\sqrt{3n} + 3\sqrt{15n}$$

$$\begin{aligned}
 19) & \quad (-4+4\sqrt{2})(3+\sqrt{2}) \\
 & = -12 - 4\sqrt{2} + 12\sqrt{2} + 4(2) \\
 & = -4 + 8\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 20) & \quad (1+\sqrt{5})(5+\sqrt{5}) \\
 & = \sqrt{5} + 5 + 5\sqrt{5} + 5 \\
 & = 10 + 6\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 21) & \quad (\sqrt{2}+4)(\sqrt{2x}-5) \\
 & = \sqrt{4x} - 5\sqrt{2} + 4\sqrt{2x} - 20 \\
 & = 2\sqrt{x} - 5\sqrt{2} + 4\sqrt{2x} - 20
 \end{aligned}$$

$$\begin{aligned}
 22) & \quad (5\sqrt{2n}-2)(\sqrt{2n}-1) \\
 & = 5(2n) - 5\sqrt{2n} - 2\sqrt{2n} + 2 \\
 & = 10n - 7\sqrt{2n} + 2
 \end{aligned}$$

$$\begin{aligned}
 23) & \quad \frac{\sqrt{20}}{2\sqrt{25}} \\
 & = \frac{\sqrt{5 \cdot 4}}{2(5)} \\
 & = \frac{2\sqrt{5}}{10} \\
 & = \frac{\sqrt{5}}{5}
 \end{aligned}$$

$$\begin{aligned}
 24) & \quad \frac{\sqrt{20}}{\sqrt{16}} \\
 & = \frac{\sqrt{5 \cdot 4}}{4} \\
 & = \frac{2\sqrt{5}}{4} \\
 & = \frac{\sqrt{5}}{2}
 \end{aligned}$$

$$\begin{aligned}
 25) & \quad \frac{2-\sqrt{2}}{\sqrt{4}} \\
 & = \frac{2-\sqrt{2}}{2}
 \end{aligned}$$

$$\begin{aligned}
 26) & \quad \frac{2+\sqrt{2}}{4\sqrt{4}} \\
 & = \frac{2+\sqrt{2}}{4(2)} \\
 & = \frac{2+\sqrt{2}}{8}
 \end{aligned}$$

Rationalize the denominator (#27-30)

$$27) \frac{4+3\sqrt{2}}{5\sqrt{8}}$$

$$= \frac{4+3\sqrt{2}}{5\sqrt{4 \cdot 2}}$$

$$= \frac{(4+3\sqrt{2}) \cdot \sqrt{2}}{(10\sqrt{2}) \cdot \sqrt{2}}$$

$$= \frac{4\sqrt{2} + 3(2)}{10(2)}$$

$$= \frac{\cancel{4\sqrt{2}} + 6}{\cancel{20}}$$

$$= \frac{2\sqrt{2} + 3}{10}$$

$$28) \frac{(5-4\sqrt{3}) \cdot \sqrt{14}}{(3\sqrt{14}) \cdot \sqrt{14}}$$

$$= \frac{5\sqrt{14} - 4\sqrt{42}}{3(14)}$$

$$= \frac{5\sqrt{14} - 4\sqrt{42}}{42}$$

$$29) \frac{2 \cdot (4+5\sqrt{2})}{(4-5\sqrt{2}) \cdot (4+5\sqrt{2})}$$

$$= \frac{8 + 10\sqrt{2}}{16 - 25(2)}$$

$$= \frac{8 + 10\sqrt{2}}{16 - 50}$$

$$= \frac{8 + 10\sqrt{2}}{-34}$$

$$= \frac{4 + 5\sqrt{2}}{-17}$$

or

$$= \frac{-4 - 5\sqrt{2}}{17}$$

$$30) \frac{2 \cdot (5+\sqrt{5})}{(5-\sqrt{5}) \cdot (5+\sqrt{5})}$$

$$= \frac{10 + 2\sqrt{5}}{25 - 5}$$

$$= \frac{10 + 2\sqrt{5}}{20}$$

$$= \frac{5 + \sqrt{5}}{10}$$

$$31) \frac{(\sqrt{2}-5) \cdot (5-\sqrt{2})}{(5+\sqrt{2}) \cdot (5-\sqrt{2})}$$

$$= \frac{5\sqrt{2} - 2 - 25 + 5\sqrt{2}}{25 - 2}$$

$$= \frac{10\sqrt{2} - 27}{23}$$

$$32) \frac{(-1+3\sqrt{2}) \cdot (3-\sqrt{2})}{(3+\sqrt{2}) \cdot (3-\sqrt{2})}$$

$$= \frac{-3 + \sqrt{2} + 9\sqrt{2} - 3(2)}{9 - 2}$$

$$= \frac{10\sqrt{2} - 9}{7}$$

$$33) -2\sqrt{3} - \sqrt{3}$$

$$= -3\sqrt{3}$$

$$34) -3\sqrt{6} + 2\sqrt{6}$$

$$= -\sqrt{6}$$

$$\begin{aligned}
 35) \quad & 2\sqrt{6} + 2\sqrt{54} \\
 & = 2\sqrt{6} + 2\sqrt{9 \cdot 6} \\
 & = 2\sqrt{6} + 6\sqrt{6} \\
 & = 8\sqrt{6}
 \end{aligned}$$

$$\begin{aligned}
 37) \quad & -\sqrt{54} + 3\sqrt{45} + 3\sqrt{24} \\
 & = -\sqrt{9 \cdot 6} + 3\sqrt{9 \cdot 5} + 3\sqrt{4 \cdot 6} \\
 & = -3\sqrt{6} + 9\sqrt{5} + 6\sqrt{6} \\
 & = 3\sqrt{6} + 9\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 39) \quad & 3\sqrt{18} + 3\sqrt{27} - \sqrt{2} - 2\sqrt{45} \\
 & = 3\sqrt{9 \cdot 2} + 3\sqrt{9 \cdot 3} - \sqrt{2} - 2\sqrt{9 \cdot 5} \\
 & = 9\sqrt{2} + 9\sqrt{3} - \sqrt{2} - 6\sqrt{5} \\
 & = 8\sqrt{2} + 9\sqrt{3} - 6\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 36) \quad & -2\sqrt{8} - \sqrt{2} \\
 & = -2\sqrt{4 \cdot 2} - \sqrt{2} \\
 & = -4\sqrt{2} - \sqrt{2} \\
 & = -5\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 38) \quad & -2\sqrt{27} - 2\sqrt{45} - \sqrt{3} \\
 & = -2\sqrt{9 \cdot 3} - 2\sqrt{9 \cdot 5} - \sqrt{3} \\
 & = -6\sqrt{3} - 6\sqrt{5} - \sqrt{3} \\
 & = -7\sqrt{3} - 6\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 40) \quad & 2\sqrt{6} - \sqrt{45} + 2\sqrt{2} - 3\sqrt{24} \\
 & = 2\sqrt{6} - \sqrt{9 \cdot 5} + 2\sqrt{2} - 3\sqrt{4 \cdot 6} \\
 & = 2\sqrt{6} - 3\sqrt{5} + 2\sqrt{2} - 6\sqrt{6} \\
 & = -4\sqrt{6} - 3\sqrt{5} + 2\sqrt{2}
 \end{aligned}$$

Solve each equation. Remember to check for extraneous solutions.

$$41) \quad 3 = (\sqrt{x})^2$$

$$\boxed{9 = x}$$

check

$$3 = \sqrt{9}$$

$$3 = 3 \checkmark$$

$$42) \quad (\sqrt{v})^2 = 7^2$$

$$\boxed{v = 49}$$

check

$$\sqrt{49} = 7$$

$$7 = 7 \checkmark$$

$$43) (\sqrt{11x+4})^2 = 9^2$$

$$11x+4 = 81$$

$$11x = 77$$

$$x = 7$$

check
 $\sqrt{11(7)+4} = 9$

$$\sqrt{81} = 9$$

$$9 = 9$$

$$44) (\sqrt{5a+1})^2 = 6^2$$

$$5a+1 = 36$$

$$5a = 35$$

$$a = 7$$

check
 $\sqrt{5(7)+1} = 6$

$$\sqrt{36} = 6$$

$$6 = 6$$

$$45) 15 = 6 + \sqrt{p}$$

$$9^2 = (\sqrt{p})^2$$

$$81 = p$$

check
 $15 = 6 + \sqrt{81}$

$$15 = 6 + 9$$

$$15 = 15$$

$$46) 2^2 = (\sqrt{k})^2$$

$$4 = k$$

check

$$2 = \sqrt{4}$$

$$2 = 2$$

$$47) 1^2 = (\sqrt{13-2n})^2$$

$$1 = 13 - 2n$$

$$-13 = -13$$

$$\frac{-12}{-2} = \frac{-2n}{-2}$$

$$n = 6$$

check

$$1 = \sqrt{13-2(6)}$$

$$1 = \sqrt{1}$$

$$1 = 1$$

$$48) \frac{-2\sqrt{x+6}}{-2} = \frac{-10}{-2}$$

$$(\sqrt{x+6})^2 = 5^2$$

$$x+6 = 25$$

$$-6 = -6$$

$$x = 19$$

check

$$-2\sqrt{19+6} = -10$$

$$-2\sqrt{25} = -10$$

$$-2(5) = -10$$

$$-10 = -10$$

$$49) (\sqrt{1-m})^2 = (\sqrt{-2-2m})^2$$

$$1-m = -2-2m$$

$$+2m = -1 + 2m$$

$$m = -3$$

check

$$\sqrt{1+3} = \sqrt{-2-2(-3)}$$

$$\sqrt{4} = \sqrt{-2+6}$$

$$\sqrt{4} = \sqrt{4}$$

$$2 = 2$$

$$50) (\sqrt{r+1})^2 = (\sqrt{13-2r})^2$$

$$r+1 = 13-2r$$

$$+2r = 12 - 2r$$

$$\frac{3r}{3} = \frac{12}{3}$$

$$r = 4$$

check

$$\sqrt{4+1} = \sqrt{13-2(4)}$$

$$\sqrt{5} = \sqrt{13-8}$$

$$\sqrt{5} = \sqrt{5}$$