Pre-Calculus 11

Chapter 1 Review **Name** KEY

Match each formula with its description.

1. C $S_n = \frac{n(t_1 + t_n)}{2}$

2. E $S = \frac{t_1}{1-r}$

3. B $t_n = t_1 \cdot r^{n-1}$

4. A $S_n = \frac{t_1(1-r^n)}{1-r}$

5. D $t_n = t_1 + (n-1)d$

6. F $S_n = \frac{n(2t_1 + (n-1)d)}{2}$

A. The sum of a geometric series

B. The nth term of a geometric sequence

C. The sum of an arithmetic series when you know the last term

D. The nth term of an arithmetic sequence

E. The sum of a convergent infinite geometric series

F. The sum of an arithmetic series when you don't know the last term

Fill in the blanks.

The common difference in an arithmetic sequence can be found by: subtracting a term by the previous term (i.e. $t_2 - t_1$).

An arithmetic series is the sum of the terms of an arithmetic sequence.

To find the general term of a sequence we substitute n for the variable n.

The common ratio can be found by: dividing a term by the previous term (i.e. $\frac{t_2}{t_1}$)

How can you tell whether an infinite geometric series has a finite sum? Is it a convergent or divergent series?

It has a finite sum if the series converges.

$-1 < r < 1$

7. State the general term for the following:

a) 1, 4, 7, … Arithmetic

$t_1 = 1$

$d = 3$

$n = n$

always! for general term

$t_n = 1 + (n-1)3$

$t_n = 1 + 3n - 3$

$\boxed{t_n = 3n - 2}$

b) 2, 6, 18, … Geometric

$t_1 = 2$

$r = 3$

$n = n$

$\boxed{t_n = 2(3)^{n-1}}$

8. Determine the first term and common difference of the arithmetic sequence with $t_5 = 16$ and $t_8 = 25$. Find t_{12}.

$16 + 3d = 25$

$3d = 9$

$\boxed{d = 3}$

$t_5 = 16$
$n = 5$
$d = 3$

$t_n = t_1 + (n-1)d$
$16 = t_1 + (5-1)(3)$
$16 = t_1 + 12$
$\boxed{t_1 = 4}$

$t_{12} = 4 + (12-1)(3)$
$t_{12} = 4 + (11)(3)$
$t_{12} = 4 + 33$
$\boxed{t_{12} = 37}$

9. Determine the number of terms in the arithmetic sequence: 3, 5, 7, …., 129

$n = ?$
$t_1 = 3$
$d = 2$
$t_n = 129$

$t_n = t_1 + (n-1)d$
$129 = 3 + (n-1)(2)$
$-3 \quad -3$
$126 = 2n - 2$
$+2 \quad +2$
$128 = 2n$

$\frac{128}{2} = n$

$\boxed{n = 64}$

$\therefore$ 64 terms

10. Determine the value of x in the arithmetic sequence: $2 + x$, $10 - x$, 9

$t_1 \quad t_2 \quad t_3$

$d = t_3 - t_2$ and $d = t_2 - t_1$
$= 9 - (10 - x)$ $\quad = (10 - x) - (2 + x)$

$9 - (10 - x) = (10 - x) - (2 + x)$
$9 - 10 + x = 10 - x - 2 - x$
$-1 + x = 8 - 2x$
$+2x \quad +2x$
$-1 + 3x = 8$
$+1 \quad +1$
$3x = 9 \rightarrow \boxed{x = 3}$

11. Find the sum of the first ten terms for each series.

a) $2 + 8 + 14 + \ldots$

$n = 10$
$t_1 = 2$
$d = 6$

$S_n = \frac{n(2t_1 + (n-1)d)}{2}$

$S_{10} = \frac{10(2(2) + (10-1)(6))}{2}$

$= 5(4 + (9)(6))$

$= \boxed{290}$

b) $t_1 = 4$ and $t_5 = 24$

$n = 10$
$t_1 = 4$
$d = ?$

"Find "d":

$4 + 4d = 24$
$-4 \quad -4$
$4d = 20$

$\boxed{d = 5}$

$S_{10} = \frac{10(2(4) + (10-1)(5))}{2}$

$= 5(8 + (9)(5))$

$= \boxed{265}$

12. Find the sum of the following arithmetic series: $2 + 7 + 12 + \ldots + 92$

$t_1 = 2$
$n = ?$
$t_n = 92$
$d = 5$

$t_n = t_1 + (n-1)d$
$92 = 2 + (n-1)5$
$-2 \quad -2$
$90 = 5n - 5$
$+5 \quad +5$
$95 = 5n \rightarrow \boxed{n = 19}$

$S_{19} = \frac{19(2(2) + (19-1)(5))}{2}$

OR

$S_{19} = \frac{19(2 + 92)}{2}$

$= \boxed{893}$

13. In an arithmetic series, $S_{15} = 1305$, $S_{16} = 1488$ and $t_1 = 3$, find the common difference.

$\ldots + \underset{t_{14}}{\_\_} + \underset{t_{15}}{\_\_} + \underset{t_{16}}{\_\_}$

$S_{15} = 1305$

$S_{16} = 1488$

$t_{16} = S_{16} - S_{15}$
$= 1488 - 1305$
$= 183$

$t_n = t_1 + (n-1)d$
$183 = 3 + (16-1)d$
$-3 \quad -3$
$\frac{180}{15} = \frac{15d}{15}$

$\boxed{d = 12}$

14. In a geometric sequence $t_1 = 2$ and $t_5 = 162$. Find t_8.

$t_n = t_1 \cdot r^{n-1}$

$\underset{t_1}{2}$ __ __ __ $\underset{t_5}{162}$

$\times r \quad \times r \quad \times r \quad \times r$

$\frac{2r^4}{2} = \frac{162}{2}$

$r^4 = 81$

$r = \sqrt[4]{81} \rightarrow \boxed{r=3}$

$r = 3$

$t_1 = 2$

$n = 8$

$t_8 = 2(3)^{8-1}$

$= 2(3)^7$

$= 2(2187)$

$= \boxed{4374}$

15. Find the sum of the first ten terms of the following: 24 – 12 + 6 – ...

$n = 10$

$r = \frac{-12}{24} = -\frac{1}{2}$

$t_1 = 24$

$S_n = \frac{t_1(1-r^n)}{1-r}$

$S_{10} = \frac{24(1-(-\frac{1}{2})^{10})}{1-(-\frac{1}{2})}$

$= \frac{24(1-\frac{1}{1024})}{1+\frac{1}{2}}$

$= \frac{24\left(\frac{1024}{1024} - \frac{1}{1024}\right)}{\frac{2}{2}+\frac{1}{2}}$

$= \frac{24\left(\frac{1023}{1024}\right)}{\frac{3}{2}}$

$= \frac{24552}{1024} \div \frac{3}{2}$

$= \frac{24552}{1024} \times \frac{2}{3}$

$= \frac{49104}{3072}$

$\boxed{= \frac{1023}{64} \text{ or } 15.98}$

16. Find the sum of the following geometric series: 17 – 51 + 153 – .... – 334611

$t_1 = 17$

$t_n = -334611$

$r = \frac{-51}{17} = -3$

$n = ?$

Find "n":

$t_n = t_1 \cdot r^{n-1}$

$\frac{-334611}{17} = \frac{17 \cdot (-3)^{n-1}}{17}$

$-19683 = (-3)^{n-1}$ *guess and check*

$-19683 = (-3)^9 \rightarrow n = 10$

$S_n = \frac{t_1(1-r^n)}{1-r}$

$S_{10} = \frac{17(1-(-3)^{10})}{1-(-3)}$

$= \boxed{-250954}$

17. State whether each infinite geometric series is convergent or divergent. Find the sum if it exists.

a) – 64 + 16 – 4 + 1 - ...

$r = \frac{16}{-64} = \frac{-1}{4}$ ∴ convergent

$S_\infty = \frac{t_1}{1-r}$

$= \frac{-64}{1-(\frac{-1}{4})}$

$= \frac{-64}{1+\frac{1}{4}}$

$= \frac{-64}{\frac{5}{4}}$

$= -64 \times \frac{4}{5}$

$= \boxed{-51.2}$

b) $\frac{5}{12} - \frac{5}{6} + \frac{5}{3} - \frac{10}{3} + \cdots$

$r = -\frac{5}{6} \div \frac{5}{12}$

$= -\frac{\cancel{5}}{6} \times \frac{12}{\cancel{5}}$

$= \frac{-12}{6} = -2$

∴ Divergent, no sum ($r < -1$)

18. In a geometric series, $S_3 = 9$, $S_4 = -15$, and $S_5 = 33$. Find the common ratio.

$\underbrace{\underbrace{\underbrace{t_1 + t_2 + t_3}_{S_3=9} + t_4}_{S_4=-15} + t_5}_{S_5=33}$

$t_5 = S_5 - S_4$

$= 33 - (-15)$

$= 33 + 15$

$= 48$

$t_4 = S_4 - S_3$

$= -15 - 9$

$= -24$

$r = \frac{48}{-24}$

$\boxed{r = -2}$

19. In a geometric series, $S_6 = 6825$, $r = 4$, find the first term.

$S_6 = 6825$
$r = 4$
$n = 6$
$t_1 = ?$

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

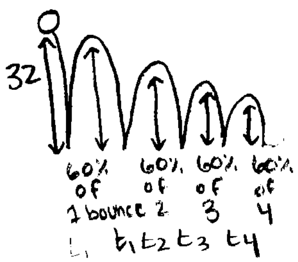
$$6825 = \frac{t_1(1-4^6)}{1-4}$$

$$6825 = \frac{t_1(-4095)}{-3}$$

$$\frac{6825}{1365} = \frac{t_1(1365)}{1365}$$

$$\boxed{t_1 = 5}$$

20. A rubber ball is dropped from a height of 32 m. Each time it bounces, it rebounds to 60% of its previous height. Determine the total vertical distance the ball has travelled after it has hit the ground for the fifth time.



Consider: $32 + 2(32)(0.6) + 2(32)(0.6)^2 + 2(32)(0.6)^3 + 2(32)(0.6$

$= 32 + 64(0.6) + 64(0.6)^2 + 64(0.6)^3 + 64(0.6)^4$

$= 115.5584$

OR// $S_5 = \frac{t_1(1-r^n)}{1-r}$

$= \frac{64(1-0.6^5)}{1-0.6}$

$= 147.5584$

Total Vertical Distance

$= 147.5584 - 32$

$= 115.5584$

21. Amy deposited \$500 in a long-term savings account on her 10th birthday. She did not make any more deposits or withdrawal. The account earned 8% per year. How much money did she have in the account on her 25th birthday?

$500,\ 500(1.08),\ 500(1.08)^2, \ldots$

$n = 25 - 10$
$= 15$
$t_1 = 500$
$r = 1.08$

$t_n = t_1 \cdot r^{n-1}$

$t_{15} = 500(1.08)^{15-1}$

$= 1468.597 \Rightarrow \boxed{\$1468.60}$

22. Determine a fraction that is equal to 0.12222 ...

$$0.1222\ldots = \underset{\frac{1}{10}}{0.1} + \underbrace{0.02 + 0.002 + 0.0002 + 0.00002 + \ldots}_{\text{infinite geometric series}}$$

$t_1 = 0.02$
$r = \frac{0.002}{0.02}$
$r = 0.1$

$$S_\infty = \frac{t_1}{1-r} = \frac{0.02}{1-0.1} = \frac{0.02}{0.9} = \frac{2}{90}$$

$\therefore 0.1222\ldots$

$= \frac{9}{9} \times \frac{1}{10} + \frac{2}{90}$

$= \frac{9}{90} + \frac{2}{90}$

$= \boxed{\frac{11}{90}}$