

More Review Questions for Chapter 1

Name KEY

1. Determine the general term of the following:

a) 6, 13, 20, 27, ... (arithmetic)

$d=7$

$$t_n = t_1 + (n-1)d$$

$$= 6 + (n-1)(7)$$

$$= 6 + 7n - 7$$

$$t_n = 7n - 1$$

b) $\frac{1}{2}, -1, 2, -4, 8, \dots$ (geometric)

$r = -2$

$$t_n = t_1 \cdot r^{n-1}$$

$$t_n = \frac{1}{2}(-2)^{n-1}$$

2. Find the sum of the following series:

a) 5, 8, 11, 14, ..., 65 (arithmetic)

Find "n"

$$t_n = t_1 + (n-1)d$$

$$65 = 5 + (n-1)(3)$$

$$60 = (n-1)(3)$$

$$20 = n-1$$

$$21 = n$$

$$S_n = \frac{n(t_1 + t_n)}{2}$$

$$S_{21} = \frac{21(5 + 65)}{2}$$

$$S_{21} = 735$$

b) 1, 3, 9, ..., 59 049 (geometric)

Find "n"

$$t_n = t_1 \cdot r^{n-1}$$

$$59049 = 1 \cdot 3^{n-1}$$

Guess & check

$$59049 = 3^{10}$$

$\therefore n = 11$

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

$$S_{11} = \frac{1(1-3^{11})}{1-3}$$

$$S_{11} = 88\,573$$

3. In an arithmetic series, $S_{13} = 806$, $S_{14} = 938$ and $t_1 = 2$, find the common difference.

$$\begin{array}{ccc} \overline{\quad} & \overline{\quad} & \overline{\quad} \\ t_{12} & t_{13} & t_{14} \\ \underbrace{\hspace{1.5cm}} & & \\ S_{13} = 806 & & \\ \underbrace{\hspace{1.5cm}} & & \\ S_{14} = 938 & & \end{array}$$

$$t_{14} = S_{14} - S_{13}$$

$$= 938 - 806$$

$$= 132$$

$$t_n = t_1 + (n-1)d$$

$$132 = 2 + (14-1)d$$

$$130 = 13d$$

$$10 = d$$

4. In a geometric sequence, $t_1 = 3$ and $t_4 = 192$. Find t_6 .

$$\begin{array}{ccc} \overline{\quad} & \overline{\quad} & \overline{\quad} \\ t_1 & & t_4 \\ \uparrow & \uparrow & \uparrow \\ \times r & \times r & \times r \end{array}$$

$$3 \cdot r^3 = 192$$

$$r^3 = 64$$

$$r = \sqrt[3]{64}$$

$$r = 4$$

$$t_n = t_1 \cdot r^{n-1}$$

$$t_6 = 3 \cdot (4)^{6-1}$$

$$t_6 = 3072$$

5. Find the sum of the following series: $14 - 56 + 224 - \dots - 3\,670\,016$ (geometric)

$$r = \frac{-56}{14} = -4$$

Find "n"

$$t_n = t_1 \cdot r^{n-1}$$

$$-3\,670\,016 = 14 \cdot (-4)^{n-1}$$

$$-262\,144 = (-4)^{n-1}$$

guess & check

$$-262\,144 = (-4)^9 \quad \therefore n = 10$$

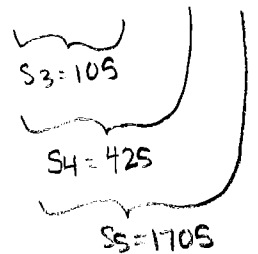
$$S_n = \frac{t_1(1-r^n)}{1-r}$$

$$S_{10} = \frac{14(1-(-4)^{10})}{1-(-4)}$$

$$S_{10} = -2,936,010$$

6. In a geometric series, $S_3 = 105$, $S_4 = 425$, and $S_5 = 1705$. Find the common ratio.

$$\frac{\quad}{t_2} + \frac{\quad}{t_3} + \frac{\quad}{t_4} + \frac{\quad}{t_5}$$



$$t_5 = S_5 - S_4 = 1705 - 425 = 1280$$

$$r = \frac{t_5}{t_4}$$

$$t_4 = S_4 - S_3 = 425 - 105 = 320$$

$$r = \frac{1280}{320}$$

$$r = 4$$

7. Use the infinite series formula to determine a fraction that is equal to $0.24444\dots$

$$0.2 + 0.04 + 0.004 + 0.0004 + \dots$$

infinite geometric series

\downarrow
 $\frac{2}{10}$

$$r = \frac{0.004}{0.04}$$

$$r = 0.1$$

$$S = \frac{t_1}{1-r}$$

$$S = \frac{0.04}{1-0.1}$$

$$S = \frac{0.04}{0.9}$$

$$S = \frac{4}{100} \div \frac{9}{10}$$

$$S = \frac{4}{100} \times \frac{10}{9}$$

$$S = \frac{40}{900}$$

$$S = \frac{4}{90}$$

$$\begin{aligned} \therefore 0.2444\dots &= \frac{2}{10} + \frac{4}{90} \\ &= \frac{18}{90} + \frac{4}{90} \\ &= \frac{22}{90} \\ &= \frac{11}{45} \end{aligned}$$