More Review Questions for Chapter 1 Name KEY

1. Determine the general term of the following:

a) 6, 13, 20, 27, . . . (arithmetic)

$d=7$

$t_n = t_1 + (n-1)d$

$= 6 + (n-1)(7)$

$= 6 + 7n - 7$

$\boxed{t_n = 7n - 1}$

b) ½, -1, 2, -4, 8, . . . (geometric)

$r = -2$

$t_n = t_1 \cdot r^{n-1}$

$\boxed{t_n = \frac{1}{2}(-2)^{n-1}}$

2. Find the sum of the following series:

a) 5, 8, 11, 14, . . . , 65 (arithmetic)

Find "n"

$t_n = t_1 + (n-1)d$

$65 = 5 + (n-1)(3)$

$60 = (n-1)(3)$

$20 = n-1$

$21 = n$

$S_n = \frac{n(t_1 + t_n)}{2}$

$S_{21} = \frac{21(5+65)}{2}$

$\boxed{S_{21} = 735}$

b) 1, 3, 9, . . . , 59 049 (geometric)

Find "n"

$t_n = t_1 \cdot r^{n-1}$

$59049 = 1 \cdot 3^{n-1}$

Guess & check

$59049 = 3^{10}$

$\therefore n = 11$

$S_n = \frac{t_1(1-r^n)}{1-r}$

$S_{11} = \frac{1(1-3^{11})}{1-3}$

$\boxed{S_{11} = 88\,573}$

3. In an arithmetic series, $S_{13} = 806$, $S_{14} = 938$ and $t_1 = 2$, find the common difference.

t_{12} t_{13} t_{14}

$S_{13} = 806$

$S_{14} = 938$

$t_{14} = S_{14} - S_{13}$

$= 938 - 806$

$= 132$

$t_n = t_1 + (n-1)d$

$132 = 2 + (14-1)d$

$130 = 13d$

$\boxed{10 = d}$

4. In a geometric sequence, $t_1 = 3$ and $t_4 = 192$. Find t_6.

$\frac{3}{t_1}$ — — $\frac{192}{t_4}$

xr xr xr

$3 \cdot r^3 = 192$

$r^3 = 64$

$r = \sqrt[3]{64}$

$r = 4$

$t_n = t_1 \cdot r^{n-1}$

$t_6 = 3 \cdot (4)^{6-1}$

$\boxed{t_6 = 3072}$

5. Find the sum of the following series: $14 - 56 + 224 - \ldots - 3\,670\,016$ (geometric)

$r = \frac{-56}{14} = -4$

Find "n"

$t_n = t_1 \cdot r^{n-1}$

$-3670016 = 14 \cdot (-4)^{n-1}$

$-262144 = (-4)^{n-1}$

guess & check

$-262144 = (-4)^9 \quad \therefore n = 10$

$S_n = \frac{t_1(1 - r^n)}{1 - r}$

$S_{10} = \frac{14(1 - (-4)^{10})}{1 - (-4)}$

$\boxed{S_{10} = -2{,}936{,}010}$

6. In a geometric series, $S_3 = 105$, $S_4 = 425$, and $S_5 = 1705$. Find the common ratio.

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$t_2 \quad t_3 \quad t_4 \quad t_5$

$S_3 = 105$

$S_4 = 425$

$S_5 = 1705$

$t_5 = S_5 - S_4$

$= 1705 - 425$

$= 1280$

$t_4 = S_4 - S_3$

$= 425 - 105$

$= 320$

$r = \frac{t_5}{t_4}$

$r = \frac{1280}{320}$

$\boxed{r = 4}$

7. Use the infinite series formula to determine a fraction that is equal to 0.244444...

$0.2 + 0.04 + 0.004 + 0.0004 + \ldots$

infinite geometric series

$\frac{2}{10}$

$r = \frac{0.004}{0.04}$

$r = 0.1$

$S = \frac{t_1}{1 - r}$

$S = \frac{0.04}{1 - 0.1}$

$S = \frac{0.04}{0.9}$

$S = \frac{4}{100} \div \frac{9}{10}$

$S = \frac{4}{100} \times \frac{10}{9}$

$S = \frac{40}{900}$

$S = \frac{4}{90}$

$\therefore 0.2444\ldots = \frac{2}{10} + \frac{4}{90}$

$= \frac{18}{90} + \frac{4}{90}$

$= \frac{22}{90}$

$= \boxed{\frac{11}{45}}$