Chapter 1 Sequences and Series Unit Review Name KEY

A sequence is an ordered list of objects or numbers.

- Each element of a sequence is called a term.
  - Ex. 2, 5, 8, 11, . . .

    $t_1, t_2, t_3, t_4 \ldots$

- An arithmetic sequence is a sequence in which the terms increase or decrease by a common value called the common difference (d)
- The general term of an arithmetic sequence is given by:

$t_n = t_1 + (n-1)d$

t_1 = first term

n = # of terms

d = common difference

t_n = general term or n^{th} term

Example: Given the sequence: 6, 12, 18, 24, . . .

+6 +6

a) Find the twentieth term

$t_n = t_1 + (n-1)d$

$t_{20} = 6 + (20-1)(6)$

$= 6 + 19(6)$

$\boxed{t_{20} = 120}$

$t_1 = 6$

$n = 20$

$d = 6$

$t_n = ?$

b) Find the general term

leave n as "n"

$t_n = t_1 + (n-1)d$

$t_n = 6 + (n-1)(6)$

$t_n = \cancel{6} + 6n - \cancel{6}$

$\boxed{t_n = 6n}$

- A geometric sequence is a sequence in which the terms increase or decrease by a common ratio (r)
- The general term of a geometric sequence is given by:

$t_n = t_1 \cdot r^{n-1}$

t_1 = first term
r = common ratio
n = # of terms
t_n = general

Example: Given the following sequence 4, 8, 16, . . .

×2

a) Find the twentieth term

$n = 20$
$t_1 = 4$
$r = \frac{8}{4} = 2$

$t_n = t_1 \cdot r^{n-1}$

$t_{20} = 4 \cdot (2)^{20-1}$

$= 4 \cdot (2)^{19}$

$\boxed{t_{20} = 2,097,152}$

b) Find the general term

leave n as "n"

$t_n = t_1 \cdot r^{n-1}$

$\boxed{t_n = 4(2)^{n-1}}$

A series is the ___sum___ of the terms of a sequence.

- An arithmetic series: "d"
  Ex: $2+5+8+11+\ldots$

  A geometric series: "r"
  Ex: $2+8+32+\ldots$

- The sum of the first *n* terms of an arithmetic series can be found:

$$S_n = \frac{n(2t_1+(n-1)d)}{2} \quad \text{or} \quad S_n = \frac{n(t_1+t_n)}{2}$$

if you don't know the last term

if you do know the last term

- The sum of the terms of a geometric series can be found using:

$$S_n = \frac{t_1(1-r^n)}{1-r}, \quad r \neq 1$$

Example:

a) Find the sum of the series: 39 — 36 — 33 — 30 — . . .

infinite series that goes on forever (divergent)

∴ no sum! ☺

b) Find the sum of the series: 4 + 9 + 14 + . . . + 59 (Arithmetic) → know the last term!

+5 +5

$t_1=4$
$t_n=59$
$n=?$
$d=5$

Find "n"

$t_n=t_1+(n-1)d$

$59=4+(n-1)(5)$

$-4 \quad -4$

$55=5n-5$

$+5 \quad +5$

$\frac{60}{5}=\frac{5n}{5}$

$\therefore n=12$

$S_n=\frac{n(t_1+t_n)}{2}$

$S_{12}=\frac{12(4+59)}{2}$

$\boxed{S_{12}=378}$

c) Find the sum of the series: 2 + 6 + 18 + 54 + . . . + 39366 (Geometric)

×3 ×3

$t_1=2$
$t_n=39366$
$r=3$
$n=?$

Find "n"

$t_n=t_1 \cdot r^{n-1}$

$\frac{39366}{2}=\frac{2(3)^{n-1}}{2}$

$19683=3^{n-1}$

Guess & check:

$19683=3^9$

$\therefore n=10$

$S_n=\frac{t_1(1-r^n)}{1-r}$

$S_{10}=\frac{2(1-3^{10})}{1-3}$

$\boxed{S_{10}=59\,048}$

An infinite geometric series is a geometric series with an infinite number of terms (goes on forever).

- A convergent infinite series approaches a finite value (has a sum)

  $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$ $r = \frac{\frac{1}{2}}{1}$ $r = \frac{1}{2}$

- A divergent series does not approach a finite value (has no sum)

  $5 + 25 + 125 + \ldots$ $r = \frac{25}{5}$ $r = 5$

- If $-1 < r < 1$ then an infinite series has a sum (convergent)

  $$S = \frac{t_1}{1-r}$$

Example: Find the sum of the following series, if possible:

a) 4 + 8 + 16 + 32 + . . .

$r = \frac{8}{4}$

$r = 2$

$\therefore$ no sum

DIVERGENT

b) 18 + 6 + 2 + . . .

$r = \frac{6}{18}$

$r = \frac{1}{3}$

$S = \frac{t_1}{1-r}$

$S = \frac{18}{1-\frac{1}{3}}$

$S = \frac{18}{\frac{3}{3}-\frac{1}{3}}$

$S = \frac{18}{\frac{2}{3}}$

$S = \frac{18}{1} \times \frac{3}{2}$

$\boxed{S = 27}$