ch1

ID: 1

Pre-Calculus 11 Name KEY

Sequences and Series

Date__________

Determine if the sequence is arithmetic. If it is, find the common difference and the term named in the problem.

1) $\underbrace{-35, -43}_{-8}, \underbrace{-51}_{-8}, \underbrace{-59}_{-8}, \ldots$
Find t_{28}

$n=28$
$t_1=-35$
$\boxed{d=-8}$

$t_n=t_1+(n-1)d$
$t_{28}=-35+(28-1)(-8)$
$\boxed{t_{28}=-251}$

2) $\underbrace{2, -198}_{-200}, \underbrace{-398}_{-200}, \underbrace{-598}_{-200}, \ldots$
Find t_{23}

$n=23$
$t_1=2$
$\boxed{d=-200}$

$t_n=t_1+(n-1)d$
$t_{23}=2+(23-1)(-200)$
$\boxed{t_{23}=-4398}$

Given two terms in an arithmetic sequence find the common difference and the term named in the problem.

3) $t_{11}=-75$ and $t_{37}=-257$
Find t_{40}

Find "d":
$37-11=26$

$\cancel{-75}+26d=-257$
$\cancel{+75} \quad\quad +75$
$26d=-182$
$\boxed{d=-7}$

Find "t_1":
$-75=t_1+(11-1)(-7)$
$t_1=-5$

$\therefore t_{40}=-5+(40-1)(-7)$
$\boxed{t_{40}=-278}$

4) $t_{10}=-38$ and $t_{37}=-173$
Find t_{40}

Find "d":
$37-10=27$
$\cancel{-38}+27d=-173$
$\cancel{+38} \quad\quad +38$
$27d=-135$
$\boxed{d=-5}$

Find "t_1":
$-38=t_1+(10-1)(-5)$
$t_1=7$

$\therefore t_{40}=7+(40-1)(-5)$
$\boxed{t_{40}=-188}$

Given a term in an arithmetic sequence and the common difference find the term named in the problem.

5) $t_{38}=-3661,\ d=-100$
Find t_{33}

Find "t_1":

$-3661=t_1+(38-1)(-100)$

$t_1=39$

$\therefore t_{33}=39+(33-1)(-100)$

$\boxed{t_{33}=-3161}$

6) $t_{10}=18,\ d=4$
Find t_{31}

Find "t_1":

$18=t_1+(10-1)(4)$

$t_1=-18$

$\therefore t_{31}=-18+(31-1)(4)$

$\boxed{t_{31}=102}$

Find the next three terms in each sequence.

7) 1, 2, 4, 8, 16, ...

$\times 2 \quad \times 2 \quad \times 2 \quad \times 2$

32, 64, 128

8) –2.5, –5, –10, –20, –40, ...

$\times 2 \quad \times 2 \quad \times 2 \quad \times 2$

–80, –160, –320

Find the tenth term in each sequence.

9) –3, 9, –27, 81, –243, ...

$t_{10} = ?$

$t_1 = -3$

$r = \frac{9}{-3} = -3$

$n = 10$

$t_{10} = (-3)\cdot(-3)^{10-1}$

$\boxed{t_{10} = 59\,049}$

10) –2, –6, –18, –54, –162, ...

$t_{10} = ?$

$t_1 = -2$

$r = \frac{-6}{-2} = 3$

$n = 10$

$t_{10} = (-2)\cdot(3)^{10-1}$

$\boxed{t_{10} = -39\,366}$

Determine if the sequence is geometric. If it is, find the common ratio, the 8th term, and the ~~explicit~~ general term formula.

11) –2, 8, –32, 128, ...

$\boxed{r = \frac{8}{-2} = -4}$

$t_1 = -2$

$n = 8$

$t_8 = (-2)\cdot(-4)^{8-1}$

$\boxed{t_8 = 32768}$

$\boxed{t_n = (-2)\cdot(-4)^{n-1}}$

12) –2, –6, –18, –54, ...

$\boxed{r = \frac{-6}{-2} = 3}$

$t_1 = -2$

$n = 8$

$t_8 = (-2)\cdot(3)^{8-1}$

$\boxed{t_8 = -4374}$

$\boxed{t_n = (-2)\cdot(3)^{n-1}}$

Find the missing term or terms in each geometric sequence.

13) ..., 2, 6, 18, 54, 162, 486, ...

$\times r \quad \times r \quad \times r \quad \times r \quad \times r$

$\frac{2\cdot r^5}{2} = \frac{486}{2}$

$r^5 = 243$

$r = \sqrt[5]{243}$

$\boxed{r = 3}$

14) ..., 1, 2, 4, 8, 16, 32, ...

$\times r \quad \times r \quad \times r \quad \times r \quad \times r$

$1\cdot r^5 = 32$

$r = \sqrt[5]{32}$

$\boxed{r = 2}$

Find the missing term or terms in each arithmetic sequence.

15) ..., 40, $\underline{10}$, $\underline{-20}$, $\underline{-50}$, –80, ...

+d +d +d +d

$40 + 4d = -80$

$4d = -120$

$\boxed{d = -30}$

16) ..., 3, $\underline{8}$, $\underline{13}$, $\underline{18}$, 23, ...

+d +d +d +d

$3 + 4d = 23$

$4d = 20$

$\boxed{d = 5}$

Evaluate each arithmetic series described.

17) $23 + 33 + 43 + 53 ...,\ n = 19$ *don't know last term

+10 +10

$t_1 = 23$

$n = 19$

$d = 10$

$S_n = \dfrac{n(2t_1 + (n-1)d)}{2}$

$S_{19} = \dfrac{19(2(23) + (19-1)(10))}{2}$

$\boxed{S_{19} = 2147}$

18) $(-16) + (-26) + (-36) + (-46) ...,\ n = 17$ *don't know last term

-10 -10 -10

$t_1 = -16$

$n = 17$

$d = -10$

$S_n = \dfrac{n(2t_1 + (n-1)d)}{2}$

$S_{17} = \dfrac{17(2(-16) + (17-1)(-10))}{2}$

$\boxed{S_{17} = -1632}$

Evaluate each geometric series described.

19) $3 - 6 + 12 - 24 ...,\ n = 8$

$t_1 = 3$

$n = 8$

$r = \frac{-6}{3} = -2$

$S_n = \dfrac{t_1(1 - r^n)}{1 - r}$

$S_8 = \dfrac{3(1 - (-2)^8)}{1 - (-2)}$

$\boxed{S_8 = -255}$

20) $3 - 6 + 12 - 24 ...,\ n = 7$

$t_1 = 3$

$n = 7$

$r = \frac{-6}{3} = -2$

$S_n = \dfrac{t_1(1 - r^n)}{1 - r}$

$S_7 = \dfrac{3(1 - (-2)^7)}{1 - (-2)}$

$\boxed{S_7 = 129}$

21) $2 + 10 + 50 + 250 ...,\ n = 9$

$t_1 = 2$

$n = 9$

$r = \frac{10}{2} = 5$

$S_9 = \dfrac{2(1 - 5^9)}{1 - 5}$

$\boxed{S_9 = 976{,}562}$

22) $2 + 10 + 50 + 250 ...,\ n = 8$

$t_1 = 2$

$n = 8$

$r = \frac{10}{2} = 5$

$S_8 = \dfrac{2(1 - 5^8)}{1 - 5}$

$\boxed{S_8 = 195312}$

Determine the number of terms *n* in each geometric series.

23) $t_1 = 2,\ r = 6,\ S_n = 18662$

$S_n = \dfrac{t_1(1 - r^n)}{1 - r}$

$18{,}662 = \dfrac{2(1 - 6^n)}{1 - 6}$

$-93{,}310 = 2(1 - 6^n)$

$-46{,}655 = 1 - 6^n$

$-46{,}656 = -6^n$

$46656 = 6^n$

Guess&Check

$46656 = 6^6$

$\boxed{\therefore n = 6}$

24) $t_1 = 1,\ r = 6,\ S_n = 259$

$259 = \dfrac{1(1 - 6^n)}{1 - 6}$

$-1295 = 1 - 6^n$

$-1296 = -6^n$

$1296 = 6^n$

Guess & Check

$1296 = 6^4$

$\boxed{\therefore n = 4}$

$\times 5 \quad \times 5 \quad \times 5$

25) $1+5+25+125\ldots,\ S_n=97656$

$97656=\frac{1(1-5^n)}{1-5}$

$-390624=1-5^n$

$-390625=-5^n$

$390625=5^n$

Guess and check

$\boxed{n=8}$

$\times 4 \quad \times 4 \quad \times 4$

26) $1+4+16+64\ldots,\ S_n=87381$

$87381=\frac{1(1-4^n)}{1-4}$

$-262143=1-4^n$

$-262144=-4^n$

$262144=4^n$

Guess and check

$\boxed{n=9}$

Determine if each geometric series converges or diverges.

27) $\frac{9}{5}+\frac{9}{10}+\frac{9}{20}+\frac{9}{40}\ldots$

$r=\frac{9}{10}\div\frac{9}{5}=\frac{\cancel{9}}{10}\times\frac{5}{\cancel{9}}=\frac{1}{2}$

$\therefore$ converges

28) $2-\frac{1}{2}+\frac{1}{8}-\frac{1}{32}\ldots$

$r=-\frac{1}{2}\div 2=-\frac{1}{2}\times\frac{1}{2}=-\frac{1}{4}$

$\therefore$ converges

Evaluate each infinite geometric series described.

29) $1.8-0.36+0.072-0.0144\ldots$

$r=\frac{-0.36}{1.8}=-0.2$

$S=\frac{t_1}{1-r}$

$=\frac{1.8}{1-(-0.2)}$

$=\frac{1.8}{1.2}$

$=\boxed{1.5}$

30) $1215+405+135+45\ldots$

$r=\frac{405}{1215}=\frac{1}{3}$

$S=\frac{1215}{1-\frac{1}{3}}$

$=\frac{1215}{\frac{3}{3}-\frac{1}{3}}$

$=\frac{1215}{\frac{2}{3}}$

$=1215\times\frac{3}{2}$

$=\boxed{\frac{3645}{2}\text{ or }1822.5}$

Determine the common ratio of the infinite geometric series.

31) $t_1=-3,\ S=-6$

$S=\frac{t_1}{1-r}$

$-6=\frac{-3}{1-r}$

$\frac{-6(1-r)}{-6}=\frac{-3}{-6}$

$1-r=\frac{1}{2}$

$-r=\frac{1}{2}-\frac{2}{2}$

$-r=-\frac{1}{2}$

$\boxed{r=\frac{1}{2}}$

32) $t_1=7.1,\ S=4.4375$

$S=\frac{t_1}{1-r}$

$4.4375=\frac{7.1}{1-r}$

$\frac{4.4375(1-r)}{4.4375}=\frac{7.1}{4.4375}$

$1-r=1.6$

$-r=0.6$

$\boxed{r=-0.6}$