

Ch. 7 - Rational Expressions Review

1. Simplify the following. Identify the nonpermissible values.

a) $\frac{x-3}{x^2-9}$

NPV's: $x \neq -3, 3$

$= \frac{(x-3)}{(x/3)(x+3)}$

$= \boxed{\frac{1}{x+3}}$

b) $\frac{x^2+3x-4}{x^2+2x-8}$

NPV's: $x \neq -4, 2$

$= \frac{(x+4)(x-1)}{(x+4)(x-2)}$

$= \boxed{\frac{x-1}{x-2}}$

2. Simplify each expression.

a) $\frac{x^2-5x+4}{2x-8}$

$= \frac{(x-4)(x-1)}{2(x-4)}$

$= \boxed{\frac{x-1}{2}}$

b) $\frac{15m-5}{3-9m}$

$= \frac{5(3m-1)}{3(1-3m)} = \frac{-5(1-3m)}{3(1-3m)}$

$= \boxed{\frac{-5}{3}}$

* Recall: If you factor out "-1",

you can switch the order:

$\frac{1-3m}{-(-1+3m)} = -(3m-1)$

c) $\frac{a^2-3ab+2b^2}{2b-2a}$

$\begin{matrix} (2) = 2 \\ / \\ -1 - 2 \end{matrix}$

$= \frac{a^2-ab-2ab+2b^2}{2(b-a)}$

$= \frac{a(a-b)-2b(a-b)}{-2(a-b)}$

$= \boxed{\frac{a-2b}{2}}$

d) $\frac{4m^2+7mn-2n^2}{n-4m}$

$\begin{matrix} 4(-2) = -8 \\ / \\ 8 - 1 \end{matrix}$

$= \frac{4m^2+8mn-mn-2n^2}{n-4m}$

$= \frac{4m(m+2n)-n(m+2n)}{n-4m}$

$= \frac{(4m/n)(m+2n)}{-(4m/n)}$

$= -(m+2n)$

$= \boxed{-m-2n}$

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3. Simplify each expression.

$$\begin{aligned} \text{a)} \quad & \frac{x-2}{x^2+3x} \cdot \frac{2x^2+6x}{5x^2} \\ & = \frac{x-2}{x(x+3)} \cdot \frac{2x(x+3)}{5x^2} \\ & = \frac{x-2}{x} \cdot \frac{2}{5x} \\ & = \frac{2(x-2)}{5x^2} \end{aligned}$$

$$\begin{aligned} \text{b)} \quad & \frac{-2vw}{6v^2} \div \frac{-15w^2}{18v^2w} \\ & = \frac{-2vw}{6v^2} \cdot \frac{18v^2w}{-15w^2} \\ & = \frac{-36vw}{-90} = \frac{2v}{5} \end{aligned}$$

$$\begin{aligned} \text{c)} \quad & \frac{x^2+7x+12}{x^2-3x-10} \div \frac{x-4}{x+2} \\ & = \frac{(x+4)(x+3)}{(x-5)(x+2)} \cdot \frac{(x+2)}{(x-4)} \\ & = \frac{(x+4)(x+3)}{(x-5)(x-4)} \end{aligned}$$

$$\begin{aligned} \text{d)} \quad & \frac{h^2-4}{h^2-h-6} \times \frac{h-3}{h+2} \\ & = \frac{(h-2)(h+2)}{(h-3)(h+2)} \cdot \frac{(h-3)}{(h+2)} \\ & = \frac{h-2}{h+2} \end{aligned}$$

$$\begin{aligned} \text{e)} \quad & \frac{m^2n-5mn^2}{m-3n} \div \frac{-m^2+5mn}{m^2-6mn+9n^2} \\ & = \frac{m \cancel{n} (m-5n)}{m-3n} \cdot \frac{(m-3n)(m-3n)}{-m(m-5n)} \\ & = \frac{-n(m-3n)}{m-3n} \end{aligned}$$

$$\begin{aligned} \text{f)} \quad & \frac{a^2+ab-6b^2}{a^2+4ab+3b^2} \cdot \frac{-a^2-ab}{a^2-3ab+2b^2} \\ & = \frac{(a+3b)(a-2b)}{(a+3b)(a+b)} \cdot \frac{-a(a+b)}{(a-2b)(a-b)} \\ & = \frac{-a}{a-b} \end{aligned}$$

g) $\frac{x^2-x-2}{3x^2-11x+6} \div \frac{2x^2-x-6}{2x^2-3x-9}$ \leftarrow State ALL the nonpermissible values for x

NPV'S: $x \neq 3, 2, \frac{2}{3}, -\frac{3}{2}$

$$\begin{aligned} & = \frac{(x-2)(x+1)}{3x^2-9x-2x+6} \cdot \frac{2x^2-6x+3x-9}{2x^2-4x+3x-6} \\ & = \frac{(x-2)(x+1)}{3x(x-3)-2(x-3)} \cdot \frac{2x(x-3)+3(x-3)}{2x(x-2)+3(x-2)} \\ & = \frac{(x-2)(x+1)}{(3x-2)(x-3)} \cdot \frac{(2x+3)(x-3)}{(2x+3)(x-2)} \\ & = \frac{x+1}{3x-2} \end{aligned}$$

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4. Simplify

$$\begin{aligned}
 \text{a.) } & \frac{x}{x+5} - \frac{x-5}{x+5} \\
 & = \frac{x - (x-5)}{x+5} \\
 & = \frac{x-x+5}{x+5} \\
 & = \frac{5}{x+5}
 \end{aligned}$$

$$\begin{aligned}
 \text{b.) } & \frac{4}{2a} - \frac{1}{2a} \quad \text{LCD} = 2a \\
 & = \frac{4}{2a} - \frac{1}{2a} \\
 & = \frac{4-1}{2a} \\
 & = \frac{3}{2a}
 \end{aligned}$$

$$\begin{aligned}
 \text{c.) } & \frac{4t-1}{5t^2} - \frac{3t+2}{5t} \quad \text{LCD} = 5t^2 \\
 & = \frac{4t-1}{5t^2} - \frac{(3t+2)t}{5t^2} \\
 & = \frac{4t-1-3t^2-2t}{5t^2} \\
 & = \frac{-3t^2+18t-5}{5t^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{d.) } & \frac{x}{x-3} + \frac{4}{x-3} \quad \text{LCD} = x-3 \\
 & = \frac{x+4}{x-3} \\
 & = \frac{x^2-3x+4}{x-3}
 \end{aligned}$$

$$\begin{aligned}
 \text{e.) } & \frac{3}{a-3} - \frac{1}{a^2-9} \quad \text{LCD} = a^2-9 = (a+3)(a-3) \\
 & \frac{3}{a-3} - \frac{1}{(a+3)(a-3)} \\
 & = \frac{3(a+3)-1}{(a+3)(a-3)} \\
 & = \frac{3a+9-1}{(a+3)(a-3)} \\
 & = \frac{3a+8}{(a+3)(a-3)}
 \end{aligned}$$

$$\begin{aligned}
 \text{f.) } & \frac{5}{6m-18} + \frac{2}{15-5m} \quad \text{LCD} = 30(m-3) \\
 & = \frac{5}{6(m-3)} + \frac{2}{5(3-m)} \\
 & = \frac{5}{6(m-3)} + \frac{-2}{5(m-3)} \\
 & = \frac{25 + (-12)}{30(m-3)} \\
 & = \frac{13}{30(m-3)}
 \end{aligned}$$

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5. Simplify

LCD = (x-3)(x-2)(x+5)

a) $\frac{x}{x^2-3x-4} + \frac{x+1}{x^2+2x+1}$

b) $\frac{x+2}{x^2-5x+6} - \frac{2x}{x^2+3x-10}$

LCD = (x-4)(x+1) = $\frac{x}{(x-4)(x+1)} + \frac{\cancel{x}+1}{(\cancel{x}+1)(x+1)} \cdot \frac{(x-4)}{(x-4)}$

= $\frac{x+2}{(x-3)(x-2)} - \frac{2x}{(x+5)(x-2)} \cdot \frac{(x-3)}{(x-3)}$

= $\frac{x+x-4}{(x-4)(x+1)}$

= $\frac{x^2+2x+5x+10 - (2x^2-6x)}{(x-3)(x-2)(x+5)}$

= $\frac{2x-4}{(x-4)(x+1)}$

= $\frac{-x^2+13x+10}{(x-3)(x-2)(x+5)}$

c) $\frac{4}{2x^2-5x-3} + \frac{3}{2x^2+5x+2}$

2(2) = 4

2(-3) = -6
-6 + 4 = -2

4 1

$\frac{4}{2x^2-6x+x-3} + \frac{3}{2x^2+4x+x+2}$

LCD = (2x+1)(x-3)(x+2)

= $\frac{4}{(2x(x-3)+(x-3))} + \frac{3}{2x(x+2)+(x+2)}$

= $\left(\frac{4}{(2x+1)(x-3)}\right)_{x+2} + \left(\frac{3}{(2x+1)(x+2)}\right)_{x-3}$

= $\frac{4x+8+3x-9}{(2x+1)(x-3)(x+2)}$

= $\frac{7x-1}{(2x+1)(x-3)(x+2)}$

d) $\frac{3}{x+2} + \frac{2x-2}{x^2-x-6} \div \frac{x-1}{x-3}$

= $\frac{3}{x+2} + \frac{2(\cancel{x}-1)}{(\cancel{x}-3)(x+2)} \cdot \frac{(x-3)}{(\cancel{x}-1)}$

= $\frac{3}{x+2} + \frac{2}{x+2}$

= $\frac{5}{x+2}$

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6. Simplify

$$a.) \left(1 - \frac{16}{m^2}\right) \cdot m^2$$

$$\left(\frac{1-4}{m}\right) \cdot m^2$$

$$= \frac{m^2 - 16}{m^2 - 4m}$$

$$= \frac{(m-4)(m+4)}{m(m-4)}$$

$$= \boxed{\frac{m+4}{m}}$$

$$b.) \left(\frac{4+x}{x}\right) \cdot 4x$$

$$\left(\frac{x+x}{4}\right) \cdot 4x$$

$$= \frac{16 + 4x^2}{x^2 + 4x^2}$$

$$= \boxed{\frac{4(4+x^2)}{5x^2}}$$

$$\frac{\frac{4+x^2}{x}}{\frac{x+4x}{4}}$$

$$c.) \frac{3}{4x^3} - \left(\frac{1}{2x}\right) \frac{2x^2}{2x^2}$$

LCD = $4x^3$

$$2x^2 \left(\frac{3}{4x^3}\right) + \frac{5}{4x^3}$$

$$= \frac{3 - 2x^2}{4x^3}$$

$$\frac{6x^2 + 5}{4x^3}$$

$$= \boxed{\frac{3 - 2x^2}{6x^2 + 5}}$$

$$d.) m - \frac{3m+2}{m+4}$$

*common denominator!

$$m - 2 + \frac{5}{m+4}$$

$$= m \left(\frac{m+4}{m+4}\right) - \frac{3m+2}{m+4}$$

$$\frac{m-2}{m+4} + \frac{5}{m+4}$$

$$= \frac{m^2 + 4m - 3m + 2}{m+4} \div \frac{m^2 + 4m - 2m - 8 + 5}{m+4}$$

$$= \frac{(m+2)(m-1)}{(m+3)(m-1)}$$

$$= \boxed{\frac{m+2}{m+3}}$$

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7. Solve. Don't forget to identify nonpermissible values.

a) $\frac{1}{x+4} = \frac{2}{x-6}$ NPV'S: $x \neq -4, 6$

$$\left(\frac{1}{x+4}\right)\left(\frac{x-6}{x-6}\right) = \left(\frac{2}{x-6}\right)\left(\frac{x+4}{x+4}\right)$$

$$\begin{array}{r} x-6 = 2x+8 \\ -x-8 \quad -x-8 \end{array}$$

$$\boxed{-14 = x}$$

b) $\frac{2}{5x-10} = \frac{3}{x-2}$ NPV'S: $x \neq 2$

$$\frac{2}{5(x-2)} = \frac{3}{(x-2)}\left(\frac{5}{5}\right)$$

$$\frac{2}{5(x-2)} = \frac{15}{5(x-2)}$$

$$2 = 15$$

↳ except it doesn't \therefore $\boxed{\text{no solution}}$

c) $\frac{x^2}{x^2-9} = \frac{x}{x+3} + \frac{x}{3-x}$

NPV'S: $x \neq -3, 3$

$$\frac{x^2}{(x+3)(x-3)} = \frac{x}{(x+3)}\left(\frac{x-3}{x-3}\right) - \frac{x}{(x-3)}\left(\frac{x+3}{x+3}\right)$$

$$\frac{x^2}{(x+3)(x-3)} = \frac{x^2-3x - x^2-3x}{(x+3)(x-3)}$$

$$x^2 = -6x$$

$$x^2 + 6x = 0 \rightarrow \boxed{x=0}$$

$$x(x+6) = 0 \rightarrow \boxed{x=-6}$$

d) $\frac{x}{2x-1} + \frac{5}{x-6} = 0$

NPV'S: $x \neq \frac{1}{2}, 6$

$$\frac{x}{(2x-1)}\left(\frac{x-6}{x-6}\right) + \frac{5}{(x-6)}\left(\frac{2x-1}{2x-1}\right) = 0$$

$$x^2 - 6x + 10x - 5 = 0$$

$$x^2 + 4x - 5 = 0$$

$$(x+5)(x-1) = 0$$

$$\begin{array}{cc} \downarrow & \downarrow \\ \boxed{x=-5} & \boxed{x=1} \end{array}$$

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e.) $x - \frac{6}{x+1} = -2$

NPV's:
 $x \neq -1$

$$x \left(\frac{x+1}{x+1} \right) - \frac{6}{x+1} = -2 \left(\frac{x+1}{x+1} \right)$$

$$x^2 + x - 6 = -2x - 2$$

+2x +2 +2x +2

$$x^2 + 3x - 4 = 0$$

$$(x+4)(x-1) = 0$$

$$\boxed{x = -4} \quad \boxed{x = 1}$$

f.) $x - \frac{2}{x+7} = -\frac{2}{x+7}$

NPV's:
 $x \neq -7$

$$x \left(\frac{x+7}{x+7} \right) - \frac{2}{x+7} = -\frac{2}{x+7}$$

$$x^2 + 7x - 2 = -2$$

+2 +2

$$x^2 + 7x = 0$$

$$x(x+7) = 0$$

$$\boxed{x = 0}$$

$$\cancel{x = -7}$$

Reject - NPV!

g.) $\frac{m+3}{m-2} + \frac{m+2}{m+3} = \frac{2m+1}{m-2} - \frac{1}{3}$ LCD = $3(m-2)(m+3)$

NPV's: $m \neq 2, -3$

$$\left(\frac{m+3}{m-2} \right) \left(\frac{3}{3} \right) \left(\frac{m+3}{m+3} \right) + \left(\frac{m+2}{m+3} \right) \left(\frac{3}{3} \right) \left(\frac{m-2}{m-2} \right) = \left(\frac{2m+1}{m-2} \right) \left(\frac{3}{3} \right) \left(\frac{m+3}{m+3} \right) - \left(\frac{1}{3} \right) \left(\frac{m-2}{m-2} \right) \left(\frac{m+3}{m+3} \right)$$

$$\frac{(3m+9)(m+3) + (3m+6)(m-2)}{3(m-2)(m+3)} = \frac{(6m+3)(m+3) - (m^2+3m-2m-6)}{3(m-2)(m+3)}$$

$$3m^2 + 9m + 9m + 27 + 3m^2 - 6m + 6m - 12 = 6m^2 + 18m + 3m + 9 - m^2 - m + 6$$

$$6m^2 + 18m + 15 = 5m^2 + 20m + 15$$

$$-5m^2 - 20m - 15 \quad -5m^2 - 20m - 15$$

$$m^2 - 2m = 0$$

$$m(m-2) = 0$$

$$\downarrow \quad \downarrow$$

$$\boxed{m = 0} \quad \cancel{m = 2} \text{ - reject, NPV!}$$

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8. Give an example of a rational expression that has the following non permissible values: $x \neq 4, -3$.

$$x \neq 4 \quad x \neq -3$$

Nonpermissible values are where the denominator = 0.

$$x-4 \neq 0 \quad x+3 \neq 0$$

$$(x-4)(x+3)$$

Example: $\frac{1}{(x-4)(x+3)}$ \leftarrow anything can be in the numerator.

9. Given that both rational expressions are defined, what is the value of k ?

a) $\frac{x^2 - 2x - 8}{x^2 + 8x + 12} = \frac{x-4}{x+k}$

$$\frac{(x-4)(\cancel{x+2})}{(x+6)(\cancel{x+2})} = \frac{x-4}{x+k}$$

$$\frac{x-4}{x+6} = \frac{x-4}{x+k}$$

$$\boxed{\therefore k=6}$$

b) $\frac{x(x+k)}{x^2+2x-3} = \frac{x}{x-1}$

$$\frac{x(x+k)}{(x+3)(x-1)} = \frac{x}{x-1}$$

$$\frac{x}{(x-1)} \underbrace{\frac{(x+k)}{(x+3)}}_{\text{must be equal to "1" so that LHS=RHS}} = \frac{x}{x-1}$$

$$\boxed{\therefore k=3}$$