Pre-Calculus 11

1. Graph each equation and then answer the given questions.

$\rightarrow 2 \uparrow 1$

(a) $y = (x - 2)^2 + 1$
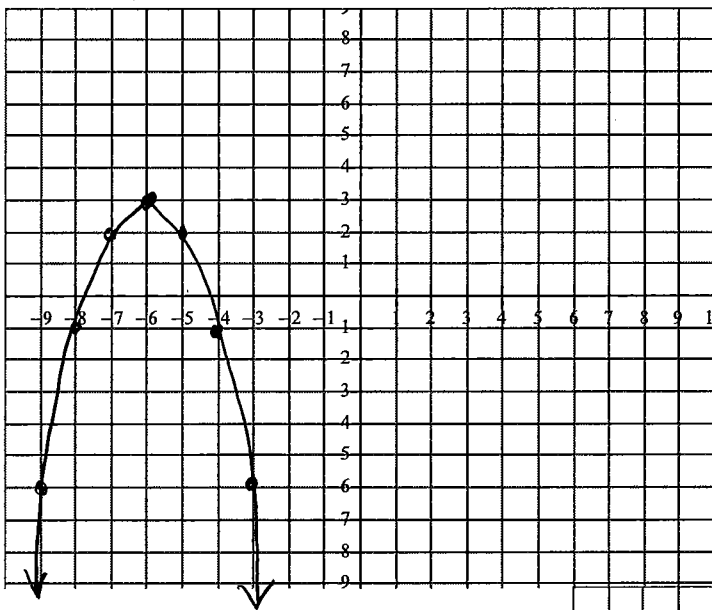
Vertex: $(2, 1)$

Axis of Symmetry: $x = 2$

Direction of Opening: UP

Domain: $x \in \mathbb{R}$

Range: $y \geq 1$

$\leftarrow 6 \uparrow 3$

(b) $y = -(x + 6)^2 + 3$

Vertex: $(-6, 3)$

Axis of Symmetry: $x = -6$

Direction of opening: down

Domain: $x \in \mathbb{R}$

Range: $y \leq 3$

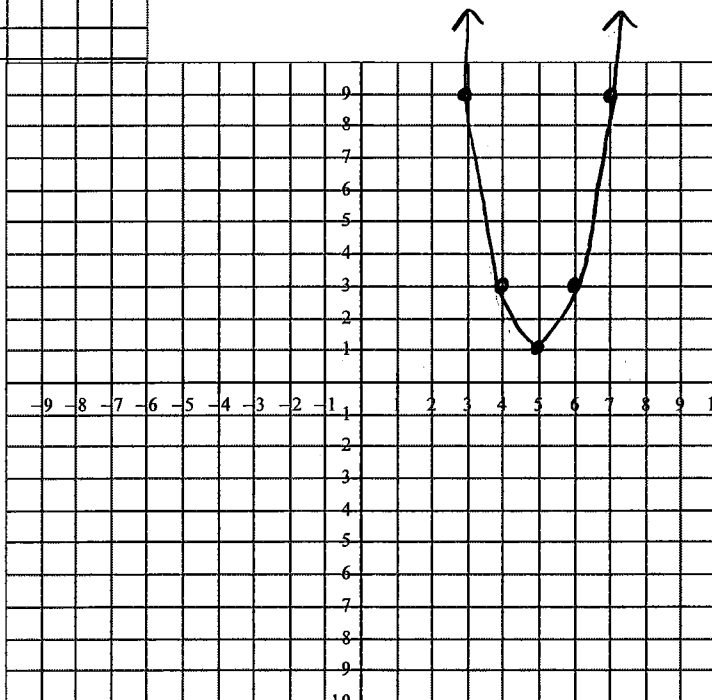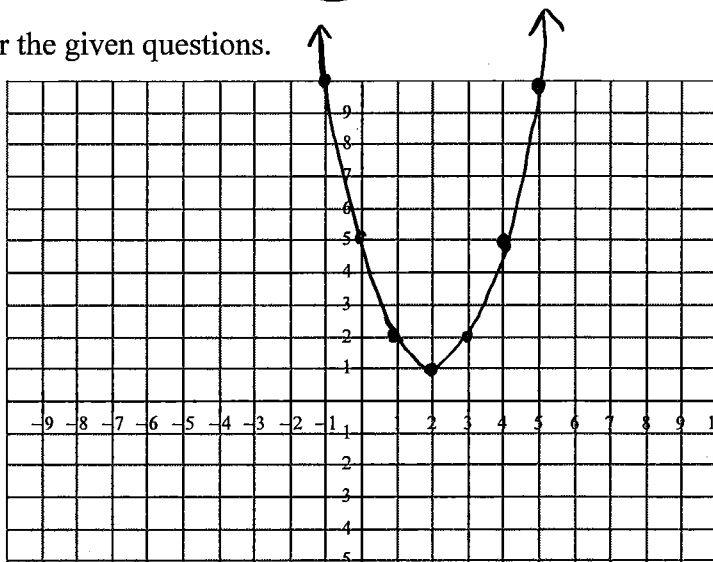$\rightarrow 5 \uparrow 1$

(c) $y = 2(x - 5)^2 + 1$

Steps: 1, 3, 5
x2: 2, 6, 10

Vertex: $(5, 1)$

Axis of symmetry: $x = 5$

Direction of Opening: UP

Domain: $x \in \mathbb{R}$

Range: $y \geq 1$

Steps: 1, 3, 5
2, -1.5,
$x - \frac{1}{2} : -\frac{1}{2},$

$\leftarrow 3 \quad \downarrow 4$
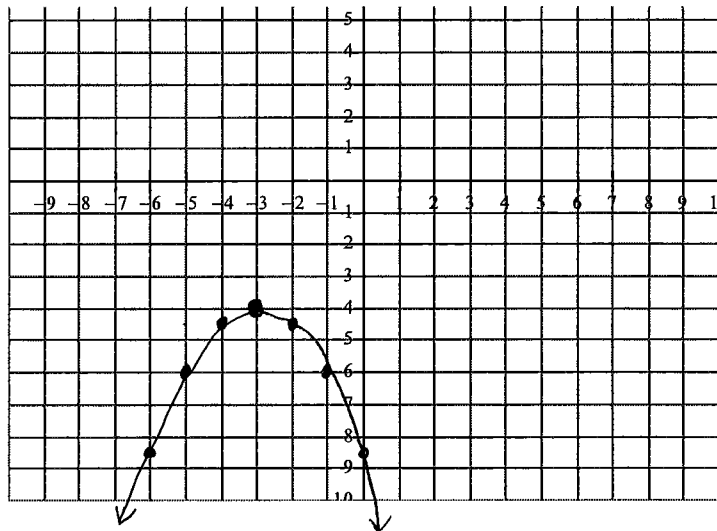
(d) $y = -\frac{1}{2}(x+3)^2 - 4$

Vertex: $(-3, -4)$

Axis of symmetry: $x = -3$

Direction of opening: down

Domain: $x \in \mathbb{R}$

Range: $y \leq -4$

2. Write the equation of the quadratic given the following.

(a) Vertex $(0, 3)$ passing through the point $(-1, 2)$

$\overset{P \quad q}{y = a(x-p)^2 + q}$

Find "a":
$2 = a(-1-0)^2 + 3$
$-3 \qquad\qquad -3$
$-1 = a(-1)^2$

$\boxed{-1 = a}$

$\overset{x \quad y}{y = -(x-0)^2 + 3}$

$\boxed{y = -(x)^2 + 3}$

(b) Vertex $(-5, -2)$ passing through the point $(-3, 0)$

$\overset{P \quad q}{y = a(x-p)^2 + q}$

$0 = a(-3-(-5))^2 + (-2)$

$0 = a(2)^2 - 2$
$+2 \qquad\qquad +2$

$\dfrac{2}{4} = \dfrac{4a}{4} \quad \Rightarrow \boxed{a = \frac{1}{2}}$

$\boxed{y = \frac{1}{2}(x+5)^2 - 2}$

(c) Passing through $A(-4, -6)$ with x −intercepts -3 and -1.

$\overset{x \quad y}{y = a(x-x_1)(x-x_2)} \qquad \overset{x_1 \quad x_2}{}$

$-6 = a(-4-(-3))(-4-(-1))$

$-6 = a(-1)(-3)$

$\dfrac{-6}{3} = \dfrac{3a}{3}$

$\boxed{-2 = a}$

$\boxed{y = -2(x+3)(x+1)}$

(d) State the equation of a quadratic in vertex form if the axis of symmetry is x = -2 and the range is $y \geq -1$ and passing through the point (1, 2).

$$y = a(x - (-2))^2 - 1$$

$$2 = a(1 + 2)^2 - 1$$

$$3 = 9a$$

$$\frac{3}{9} = \frac{9a}{9}$$

$$\boxed{\frac{1}{3} = a}$$

$$\boxed{y = \frac{1}{3}(x+2)^2 - 1}$$

(e) Passes through $A(1, -10)$ and x−intercepts of 0 and 6.

$$y = a(x - x_1)(x - x_2)$$

$$-10 = a(1 - 0)(1 - 6)$$

$$-10 = a(1)(-5)$$

$$\frac{-10}{-5} = \frac{-5a}{-5}$$

$$\boxed{2 = a}$$

$$y = 2(x - 0)(x - 6)$$

$$\boxed{y = 2x(x - 6)}$$

or

$$\boxed{y = 2x^2 - 12x}$$

(f) x-intercepts of −2 and 3 and range $y \geq -6.25$

Axis of symmetry: $\frac{-2+3}{2} = \frac{1}{2} = 0.5$

vertex: $(\frac{1}{2}, -6.25)$

Passes through: (3, 0)

$$0 = a(3 - 0.5)^2 - 6.25$$

$$6.25 = a(2.5)^2$$

$$\frac{6.25}{6.25} = \frac{6.25a}{6.25}$$

$$\boxed{a = 1}$$

$$\boxed{y = (x - 0.5)^2 - 6.25}$$

3. Express each equation in standard form.

(a) $y = \underline{x^2 - 6x} + 8$

$\frac{1}{2}(-6) = -3$

$\hookrightarrow (-3)^2 = 9$

$$y = (\underline{x^2 - 6x + 9}, -9) + 8$$
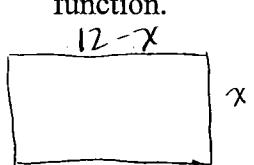
$$y = (x - 3)(x - 3) - 1$$

$$\boxed{y = (x - 3)^2 - 1}$$

(b) $y = 2x^2 + 4x + 7$

$\frac{1}{2}(2) = 1$

$y = 2(x^2 + 2x) + 7$

$\hookrightarrow (1)^2 = 1$

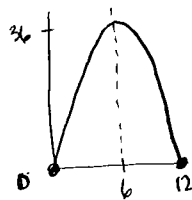$y = 2(x^2 + 2x + 1 - 1) + 7$

$\boxed{y = 2(x+1)^2 + 5}$

(c) $y = -5x^2 - 20x - 30$

$\frac{1}{2}(4) = 2$

$y = -5(x^2 + 4x) - 30$

$\hookrightarrow (2)^2 = 4$

$y = -5(x^2 + 4x + 4 - 4) - 30$

$\boxed{y = -5(x+2)^2 - 10}$

4. Brooklyn has 24m of fencing. She wants to build a rectangular enclosure for her dog with the maximum possible area.

(a) Write a function to represent the rectangular area of the enclosure. Sketch the function.

$\text{Area} = x(12 - x)$

Method ①: $A = x(12 - x)$

x-intercepts: $x = 0$ $x = 12$

Axis of symmetry: $\frac{0 + 12}{2} = 6$

when $x = 6$, vertex: $A = 6(12 - 6)$

$A = 36m^2$

vertex: $(6, 36)$

method ②: $A = 12x - x^2$

$A = -x^2 + 12x$

$A = -(x^2 - 12x)$

$A = -(x^2 - 12x + 36 - 36)$

$A = -(x - 6)^2 + 36$

$\frac{1}{2}(12) = 6$

$\hookrightarrow (6)^2 = 36$

(b) What are the dimensions of the enclosure that achieve Brooklyn's goals? What area will be available to her dog?

$x = 6$ Dimensions: $\boxed{6m \text{ by } 6m}$

$\boxed{\text{Area} = 36m^2}$

5. A student club is planning a fundraising car wash. Last year they charged $10 per vehicle and washed 120 vehicles. They would like to earn more money this year. For every $1 increase in price, they know they will wash 5 fewer vehicles.

(a) Write a quadratic function to model this situation using v as the number of vehicles and R as the revenue.

$$R = (10 + x)(120 - 5x)$$
$$R = 1200 - 50x + 120x - 5x^2$$
$$R = -5x^2 + 70x + 1200$$

$$\rightarrow R = -5(x^2 - 14x) + 1200 \qquad \frac{1}{2}(-14) = -7$$
$$R = -5(x^2 - 14x + 49 - 49) + 1200 \qquad (-7)^2 = 49$$
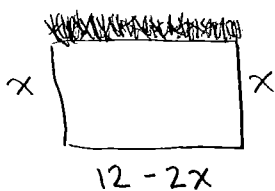$$R = -5(x^2 - 14x + 49) + 245 + 1200$$
$$R = -5(x - 7)^2 + 1445$$

(b) Determine the best price to charge for the car wash and the revenue expected at that price.

$$x = 7$$

∴ 7 $1 increases

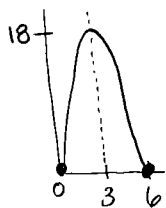⟿ Best Price = 10 + 7 = $\boxed{\$17}$ ⟹ max Revenue : $\boxed{\$1445}$

6. Rylee has 12m of edging material to place along the three sides of the garden to separate it from her lawn. What dimensions will give the maximum area for the garden?

$$A = x(12 - 2x)$$

12 - 2x

METHOD ①: $A = x(12 - 2x)$

x-intercepts: $x = 0$, $x = 6$

Axis of symmetry $= \dfrac{0 + 6}{2} = 3$

Vertex when $x = 3$:
$$A = 3(12 - 2(3))$$
$$= 3(6)$$
$$= 18$$

∴ max Area = $18 m^2$
Dimensions: 3m × 6m

METHOD ②: $A = 12x - 2x^2$
$$A = -2x^2 + 12x$$
$$A = -2(x^2 - 6x)$$
$$A = -2(x^2 - 6x + 9 - 9)$$
$$A = -2(x - 3)^2 + 18$$

$\uparrow$ $\uparrow$
$x = 3$ max = 18

$\frac{1}{2}(6) = 3$
$(3)^2 = 9$

Dimensions: 3m × 6m
max Area = $18 m^2$