Pre-Calculus 11

Chapter 2 Review **Name** KEY

1. For which values of the variable, x, are the following radicals defined:

a) $\sqrt{4x^2}$

$x \in \mathbb{R}$

↳ "x" is squared and anything squared gives a positive answer

b) $\sqrt[5]{32x}$

$x \in \mathbb{R}$

↳ can take the 5th root of a positive or negative

c) $\sqrt[4]{-16x}$

$x \leq 0$

(neg 16) × (neg "x") = positive

↳ can only take the 4th root of a positive.

2. For which values of the variable, x, are the following radicals defined:

a) $\sqrt{8x}$

$x \geq 0$

↳ can only take the square root of a positive

b) $\sqrt{-8x^5}$

$x \leq 0$

↳ $(-8)(-"x") =$ positive; can only take the square root of a positive

c) $\sqrt[3]{16x}$

$x \in \mathbb{R}$

↳ can take the cube root of a positive or negative

3. Express the following mixed radicals as entire radicals: *(Note that brackets have been used to avoid confusion on whether the small number is an exponent or an index to the radical.)*

a) $3\sqrt{5}$

$= \sqrt{3^2}\sqrt{5}$

$= \sqrt{9}\sqrt{5}$

$= \sqrt{45}$

b) $2(\sqrt[4]{3})$

$= \sqrt[4]{2^4}\sqrt[4]{3}$

$= \sqrt[4]{16 \cdot 3}$

$= \sqrt[4]{48}$

c) $3x(\sqrt[3]{2})$

$= \sqrt[3]{(3x)^3}\sqrt[3]{2}$

$= \sqrt[3]{27x^3 \cdot 2}$

$= \sqrt[3]{54x^3}$

d) $4m^2n\sqrt{3}$

$= \sqrt{(4m^2n)^2}\sqrt{3}$

$= \sqrt{16 \cdot 3 \cdot m^4 \cdot n^2}$

$= \sqrt{48m^4n^2}$

NOTE: must keep the neg. out in front so each line is still equal to the previous line (i.e. negative)

e) $-2(\sqrt[4]{6xy^3})$

$= -\sqrt[4]{(2)^4}\sqrt[4]{6xy^3}$

$= -\sqrt[4]{16 \cdot 6xy^3}$

$= -\sqrt[4]{96xy^3}$

f) $2m^4(\sqrt[5]{3m^2})$

$= \sqrt[5]{(2m^4)^5}\sqrt[5]{3m^2}$

$= \sqrt[5]{32m^{20} \cdot 3m^2}$

$= \sqrt[5]{96m^{22}}$

4. Express the following entire radicals as mixed radicals

a) $\sqrt{162}$

$= \sqrt{9 \cdot 18}$

$= \sqrt{9}\sqrt{9 \cdot 2}$

$= 3\sqrt{9}\sqrt{2}$

$= (3)(3)\sqrt{2}$

$= 9\sqrt{2}$

b) $\sqrt[3]{72}$

$= \sqrt[3]{8}\sqrt[3]{9}$

$= 2\sqrt[3]{9}$

c) $\sqrt{\frac{32}{50}}$

$= \frac{\sqrt{16 \cdot 2}}{\sqrt{25 \cdot 2}}$

$= \frac{4\sqrt{2}}{5\sqrt{2}}$

$= \frac{4}{5}$

d) $\sqrt[3]{16x^3y}$

$= \sqrt[3]{2 \cdot 8 \cdot (x x x) y}$

$= 2x\sqrt[3]{2y}$

e) $\sqrt{2m^5n^4}$

$= \sqrt{2\,(mm)(mm)m(nn)(nn)}$

$= m \cdot m \cdot n \cdot n\sqrt{2m}$

$= m^2n^2\sqrt{2m}$

f) $\sqrt[3]{\frac{375n^6}{24m^3}}$

$= \frac{\sqrt[3]{125 \cdot 3}\ \sqrt[3]{(nnn)(nnn)}}{\sqrt[3]{8 \cdot 3}\ \sqrt[3]{(mmm)}}$

$= \frac{5n^2\sqrt[3]{3}}{2m\sqrt[3]{3}}$

$= \frac{5n^2}{2m}$

5. Simplify the following. Answer in simplest form.

a) $\sqrt{7} - \sqrt{28} + 3\sqrt{63}$

$= \sqrt{7} - \sqrt{4 \cdot 7} + 3\sqrt{9 \cdot 7}$

$= \sqrt{7} - 2\sqrt{7} + 9\sqrt{7}$

$= 8\sqrt{7}$

b) $3\sqrt{175} - 6\sqrt{32} + \sqrt{98}$

$= 3\sqrt{25 \cdot 7} - 6\sqrt{16 \cdot 2} + \sqrt{49 \cdot 2}$

$= 15\sqrt{7} - 24\sqrt{2} + 7\sqrt{2}$

$= 15\sqrt{7} - 17\sqrt{2}$

c) $\sqrt[4]{48} - \frac{2}{3}\sqrt[4]{243}$

$= \sqrt[4]{16 \cdot 3} - \frac{2}{3}\sqrt[4]{81 \cdot 3}$

$= 2\sqrt[4]{3} - 2\sqrt[4]{3}$

$= 0$

d) $2(\sqrt[3]{16}) + \sqrt[3]{375} - \sqrt[3]{54} + 3(\sqrt[3]{24})$

$= 2\sqrt[3]{8 \cdot 2} + \sqrt[3]{125 \cdot 3} - \sqrt[3]{27 \cdot 2} + 3\sqrt[3]{8 \cdot 3}$

$= 4\sqrt[3]{2} + 5\sqrt[3]{3} - 3\sqrt[3]{2} + 6\sqrt[3]{3}$

$= \sqrt[3]{2} + 11\sqrt[3]{3}$

e) $3\sqrt{3x^3} - 3\sqrt{12x^3}$

$= 3\sqrt{3(xx)x} - 3\sqrt{4 \cdot 3(xx)x}$

$= 3x\sqrt{3x} - 6x\sqrt{3x}$

$= -3x\sqrt{3x}$

f) $\sqrt{32a^2b^3} - ab\sqrt{98b}$

$= \sqrt{16 \cdot 2 \cdot (aa)(bb)b} - ab\sqrt{49 \cdot 2b}$

$= 4ab\sqrt{2b} - 7ab\sqrt{2b}$

$= -3ab\sqrt{2b}$

g) $\frac{\sqrt{64n^3}}{2} - \sqrt{9n^3} + \frac{1}{5}\sqrt{25n^5}$

$= \frac{8\sqrt{(nn)n}}{2} - 3\sqrt{(nn)n} + \sqrt{(nn)(nn)n}$

$= 4n\sqrt{n} - 3n\sqrt{n} + n^2\sqrt{n}$

$= n\sqrt{n} + n^2\sqrt{n}$

6. Simplify the following. Answer in simplest form.

a) $(6\sqrt{3})(5\sqrt{2})$

$= 30\sqrt{6}$

b) $(4\sqrt{18a^2})(\sqrt{3a^2})$

$= 4\sqrt{54a^4}$

$= 4\sqrt{9 \cdot 6\,(aa)(aa)}$

$= 12a^2\sqrt{6}$

c) $4\sqrt{5}(2\sqrt{80} - 3\sqrt{45})$

$= 8\sqrt{400} - 12\sqrt{225}$

$= 8(20) - 12(15)$

$= 160 - 180$

$= -20$

d) $2\sqrt{5}(\sqrt{6} + \sqrt{2})$

$= 2\sqrt{30} + 2\sqrt{10}$

e) $(5 - 4\sqrt{3})(-2 + \sqrt{3})$

$= -10 + 5\sqrt{3} + 8\sqrt{3} - 4(3)$

$= -22 + 13\sqrt{3}$

f) $(8(\sqrt[3]{4c^2}) - c)(\sqrt[3]{2c} + 5c)$

$= (8\sqrt[3]{4c^2} - c)(\sqrt[3]{2c} + 5c)$

$= 8\sqrt[3]{8c^3} + 40c\sqrt[3]{4c^2} - c\sqrt[3]{2c} - 5c^2$

$= 16c + 40c\sqrt[3]{4c^2} - c\sqrt[3]{2c} - 5c^2$

g) $(-2 - 3\sqrt{6})^2$

$= (-2 - 3\sqrt{6})(-2 - 3\sqrt{6})$

$= 4 + 6\sqrt{6} + 6\sqrt{6} + 9(6)$

$= 58 + 12\sqrt{6}$

h) $(\sqrt{2} - 3\sqrt{5m})^2$

$= (\sqrt{2} - 3\sqrt{5m})(\sqrt{2} - 3\sqrt{5m})$

$= 2 - 3\sqrt{10m} - 3\sqrt{10m} + 9(5m)$

$= 2 - 6\sqrt{10m} + 45m$

7. Simplify the following. Answer in simplest form. *(Note that this means you must rationalize the denominator if need be...)*

a) $\frac{-5\sqrt{80}}{\sqrt{5}}$

$= \frac{-5\sqrt{16} \cdot \sqrt{5}}{\sqrt{5}}$

$= -5(4)$

$= -20$

b) $\frac{4}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{4\sqrt{5}}{5}$

c) $\frac{72\sqrt{45}}{-24\sqrt{20}}$

$= -\frac{3\sqrt{9 \cdot 5}}{1\sqrt{4 \cdot 5}}$

$= \frac{-3(3)\sqrt{5}}{2\sqrt{5}}$

$= \frac{-9}{2}$

d) $\frac{3\sqrt{6}}{2\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}}$

$= \frac{3\sqrt{90}}{2(15)}$

$= \frac{3\sqrt{9\cdot 10}}{30}$

$= \frac{3\sqrt{10}}{10}$

e) $\frac{-3\sqrt{3a}}{4\sqrt{8a}}$

$= \frac{-3\sqrt{3a}}{4\sqrt{4\cdot 2a}}$

$= \frac{-3\sqrt{3a}}{8\sqrt{2a}} \cdot \frac{\sqrt{2a}}{\sqrt{2a}}$

$= \frac{-3\sqrt{6a^2}}{8(2a)}$

$= \frac{-3\sqrt{6}\sqrt{a^2}}{16a}$

$= \frac{-3\sqrt{6}}{16}$

f) $\frac{\sqrt{15xy}}{\sqrt{10xy}} \cdot \frac{\sqrt{10xy}}{\sqrt{10xy}}$

$= \frac{\sqrt{150x^2y^2}}{10xy}$

$= \frac{\sqrt{25\cdot 6\cdot(x\cdot x)\cdot(y\cdot y)}}{10xy}$

$= \frac{5xy\sqrt{6}}{10xy}$

$= \frac{\sqrt{6}}{2}$

g) $\frac{(2\sqrt{5})}{(7+\sqrt{3})} \cdot \frac{(7-\sqrt{3})}{(7-\sqrt{3})}$

$= \frac{14\sqrt{5} - 2\sqrt{15}}{49 - 7\sqrt{3} + 7\sqrt{3} - 3}$

$= \frac{14\sqrt{5} - 2\sqrt{15}}{46}$

$= \frac{7\sqrt{5} - \sqrt{15}}{23}$

h) $\frac{(2+\sqrt{3})}{(5\sqrt{2}-\sqrt{5})} \cdot \frac{(5\sqrt{2}+\sqrt{5})}{(5\sqrt{2}+\sqrt{5})}$

$= \frac{10\sqrt{2} + 2\sqrt{5} + 5\sqrt{6} + \sqrt{15}}{25(2) + 5\sqrt{10} - 5\sqrt{10} - 5}$

$= \frac{10\sqrt{2} + 2\sqrt{5} + 5\sqrt{6} + \sqrt{15}}{45}$

8. Solve the following. State the restrictions on the variables.

a) $\sqrt{m-1} + 7 = 13$

$-7 \quad -7$

$(\sqrt{m-1})^2 = (6)^2$

$m - 1 = 36$

$+1 \quad +1$

$m = 37$

Restrictions:

$m - 1 \geq 0$

$\boxed{m \geq 1}$

b) $8 + \sqrt{5a-5} = -3$

$-8 \quad -8$

$(\sqrt{5a-5})^2 = (-11)^2$

$5a - 5 = 121$

$+5 \quad +5$

$\frac{5a}{5} = \frac{126}{5}$

$a = 25.2$

Restrictions:

$5a - 5 \geq 0$

$5a \geq 5$

$\boxed{a \geq 1}$

c) $(\sqrt{3n})^2 = (\sqrt{4n-1})^2$

$3n = 4n - 1$

$+1 \quad +1$

$3n + 1 = 4n$

$-3n \quad -3n$

$1 = n$

Restrictions:

$3n \geq 0 \quad 4n - 1 \geq 0$

$n \geq 0 \quad 4n \geq 1$

$n \geq \frac{1}{4}$

$\boxed{n \geq \frac{1}{4}}$

d) $\left(\sqrt{\frac{x}{10}}\right)^2 = \left(\sqrt{3x - 58}\right)^2$

$10 \cdot \frac{x}{10} = (3x - 58) \cdot 10$

$x = 30x - 580$

$-30x \quad -30x$

$\frac{-29x}{-29} = \frac{-580}{-29}$

$x = 20$

Restrictions:

$\frac{x}{10} \geq 0 \quad 3x - 58 \geq 0$

$x \geq 0 \quad 3x \geq 58$

$x \geq 19.3$

$\boxed{x \geq 19.3}$