

8.3 Graphing Reciprocals of Linear Functions

Recall that the reciprocal of a number x , $x \neq 0$, is the number $\frac{1}{x}$. Similarly, the reciprocal of a function $f(x)$ is $\frac{1}{f(x)}$ where $f(x) \neq 0$.

Consider the function: $y = x + 1$ \Rightarrow Its reciprocal function is: $y = \frac{1}{x + 1}$

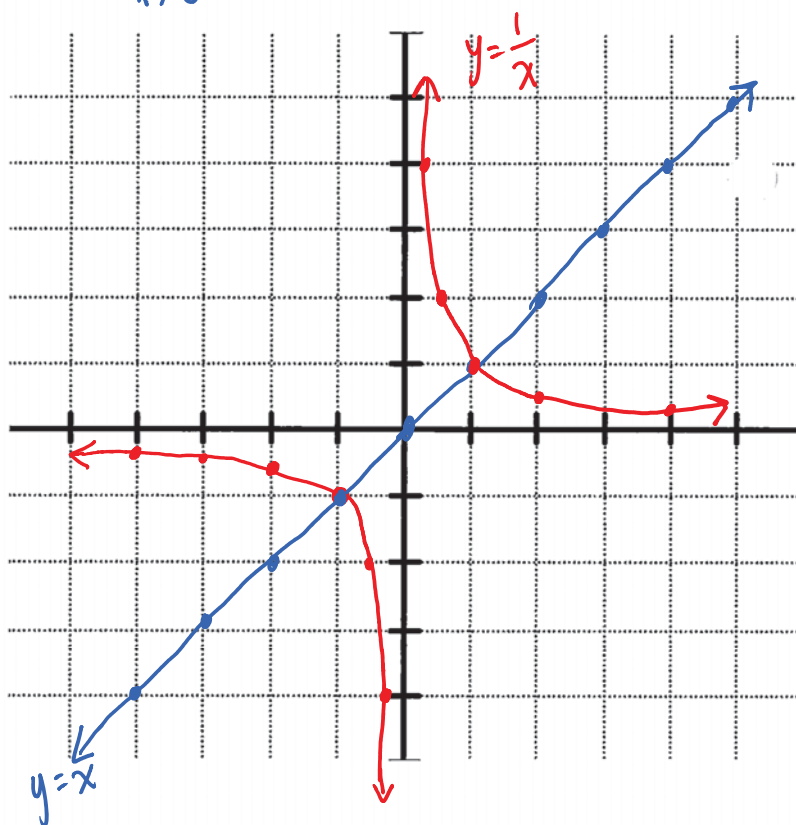
Example #1: Sketch the graphs of $y = x$ and its reciprocal.

What is the equation of the reciprocal? $y = \frac{1}{x}$

What is the non-permissible value? $x \neq 0$

$$\frac{1}{\frac{1}{2}} = 1 \div \frac{1}{2} = 1 \times \frac{2}{1} = 2$$

x	$y=x$	$y=\frac{1}{x}$
-4	-4	$-\frac{1}{4}$
-2	-2	$-\frac{1}{2}$
-1	-1	-1
$-\frac{1}{2}$	$-\frac{1}{2}$	-2
$-\frac{1}{4}$	$-\frac{1}{4}$	-4
0	0	DNE
$\frac{1}{4}$	$\frac{1}{4}$	4
$\frac{1}{2}$	$\frac{1}{2}$	2
1	1	1
2	2	$\frac{1}{2}$
4	4	$\frac{1}{4}$



An asymptote is a line that a graph approaches more and more closely the further the graph is followed. You can have horizontal and vertical asymptotes.

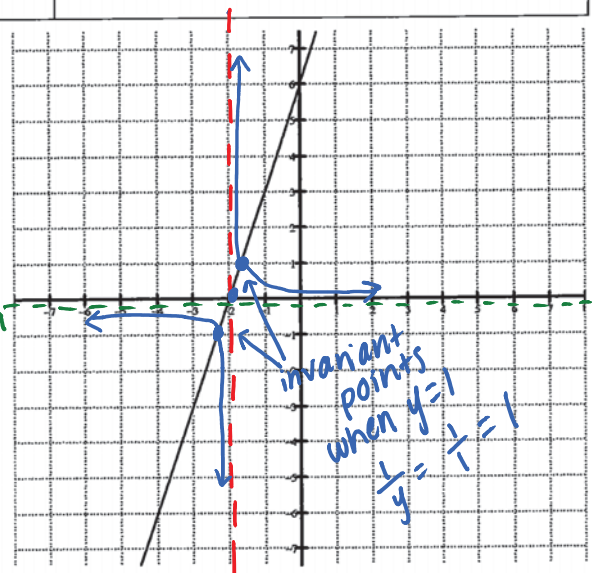
Properties of Reciprocal Functions

Original Function $f(x)$	Reciprocal Function $\frac{1}{f(x)}$
equals 1 equals -1 equals 0 positive negative approaches 0 approaches ∞	equals 1 (invariant point) equals -1 (invariant point) has a vertical asymptote positive negative approaches ∞ approaches 0

Example #2: Given the following graph sketch the reciprocal.

$y = \frac{1}{f(x)}$ will never = 0
 $\frac{1}{\text{anything}} \neq 0$

horizontal asymptote $y=0$



vertical asymptote $x = -2$

Example #3: Sketch a graph of

$$y = \frac{1}{-2x-4}$$

① Graph $y = -2x - 4$

② Asymptotes

Hor: $y = 0$

Ver: $-2x - 4 = 0$

$$-2x = 4$$

$$x = -2$$

③ Invariant points where $y = 1$ and $y = -1$

④ Pick some points & graph the reciprocal

What are the domain and range of $y = \frac{1}{2x-4}$?

D: $\{x \mid x \neq -2, x \in \mathbb{R}\}$ → where the vertical asymptote is

R: $\{y \mid y \neq 0, y \in \mathbb{R}\}$ → where the horizontal asymptote is

Example #4: Use the graph of $y = \frac{1}{f(x)}$ to graph the

linear function $y = f(x)$.

- vertical asymptote at $x = -0.5$
 ↳ that is where the line crosses the x-axis
- look for points where $y = \pm 1$

