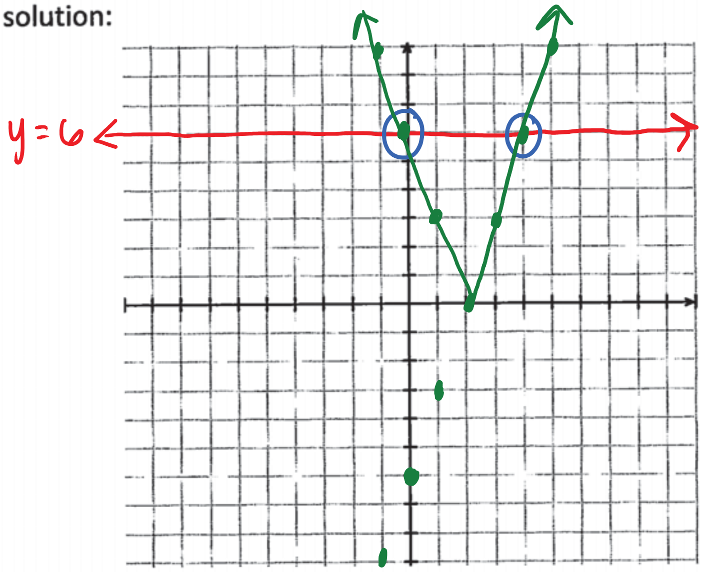
8.2 Solving Absolute Value Equations

We can solve absolute value equations by graphing, using the strategies learned in the previous lesson or we can solve using algebra.

Example #1: *Solving* $y=|f(x)|$ *Graphically for Linear* $f(x)$

Solve by graphing, then verify the solution:

$|3x-6|=6$



① Graph $y=|3x-6|$ and $y=6$

② Look for where the graphs intersect

$x=0$, $x=4$

check:

$|3(0)-6|=6$
$|-6|=6$
$6=6$ ✓

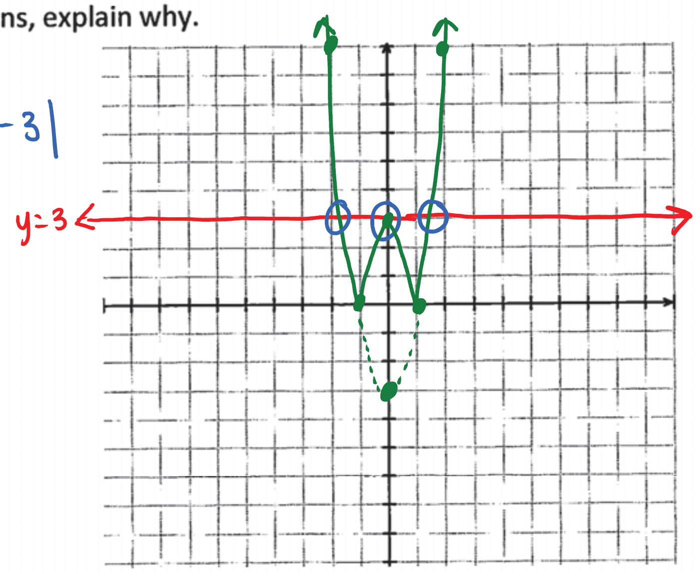
$|3(4)-6|=6$
$|12-6|=6$
$6=6$ ✓

Example #2: *Solving* $y=|f(x)|$ *Graphically for when* $f(x)$ *is Quadratic*

Solve each equation by graphing. Where necessary, give the solutions to the nearest tenth. If there is no solutions, explain why.

a. $3=|3x^2-3|$

① Graph $y=3$ and $y=|3x^2-3|$



Steps: 1, 3, 5

x3: 3, 9, 15

$x=0$ $\quad x \doteq -1.7$ $\quad x \doteq 1.7$

b. $|2x^2+4x-1|=-2$

$y=|2x^2+4x-1|$ $\quad$ $y=-2$

$y=|2(x^2+2x)-1|$

$\frac{1}{2}(2)=1$

$\hookrightarrow 1^2=1$

$y=|2(x^2+2x+1-1)-1|$

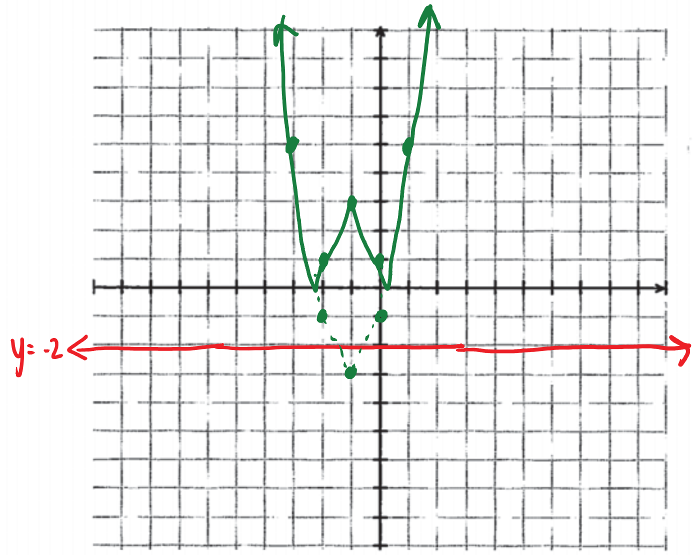
$y=|2(x^2+2x+1)-2-1|$

$y=|2(x+1)^2-3|$

$\leftarrow 1 \downarrow 3$

Steps: 1, 3, 5

x2: 2, 6, 10



Solutions: there are no solutions since the graphs do not intersect. This happens when $|f(x)|<0$

An absolute value equation of the form $|ax^2+bx+c|=d$ can have 0, 1, 2, 3, or 4 solutions. The number of solutions depends on the absolute value function graphed and the value of d.

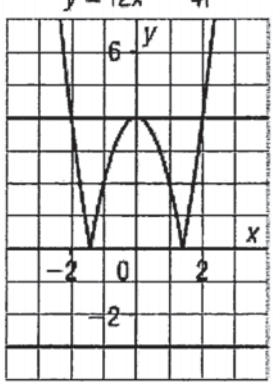
$y=|2x^2-4|$

3 solutions

4 solutions

2 solutions

0 solutions



$|2x^2-4|=4$

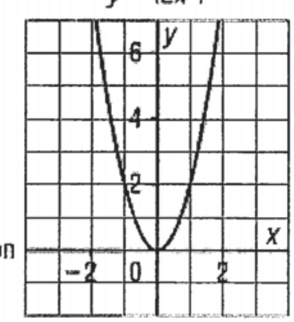
$|2x^2-4|=2$

$|2x^2-4|=0$

$|2x^2-4|=-3$

$y=|2x^2|$

1 solution



$|2x^2|=0$

One of the problems with solving problems by graphing is that the solutions are often approximate. To determine exact solutions we can use algebra.

Example #3: *Solving* $y=|f(x)|$ *Algebraically for Linear* $f(x)$

Solve algebraically: $x+8=|4x+6|$

case ①: the expression in the absolute value is positive or zero

$x+8=4x+6$

$-x-6 \quad -x-6$

$2=3x$

$\boxed{x=\frac{2}{3}}$

check:

$\frac{2}{3}+8=|4(\frac{2}{3})+6|$

$\frac{2}{3}+\frac{24}{3}=|\frac{8}{3}+\frac{18}{3}| \rightarrow \frac{26}{3}=\frac{26}{3}$ ✓

case ②: the expression in the absolute value is negative

$x+8=-(4x+6)$

$x+8=-4x-6$

$+4x-8 \quad +4x \quad -8$

$5x=-14$

$\boxed{x=\frac{-14}{5}}$

check:

$\frac{-14}{5}+8=|4(\frac{-14}{5})+6|$

$\frac{-14}{5}+\frac{40}{5}=|\frac{-56}{5}+\frac{30}{5}| \rightarrow \frac{26}{5}=|\frac{-26}{5}|$ ✓

Example #4: *Solving* $y=|f(x)|$ *Algebraically for when* $f(x)$ *is Quadratic*

Solve algebraically $|x^2-6x+5|=5$

case ①

$x^2-6x+5=5$

$-5 \quad -5$

$x^2-6x=0$

$x(x-6)=0$

$x=0, x=6$

check:

$x=0$

$|0^2-6(0)+5|=5$

$|5|=5$ ✓

$x=6$

$|6^2-6(6)+5|=5$

$|5|=5$ ✓

case ②

$-(x^2-6x+5)=5$

$-x^2+6x-5=5$

$-5 \quad -5$

$-x^2+6x-10=0$

$x^2-6x+10=0$ *not factorable

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $a=1 \quad b=-6 \quad c=10$

$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(10)}}{2(1)}$

$\cancel{x=\frac{6\pm\sqrt{36-40}}{2}}$ ← negative ∴ no solutions