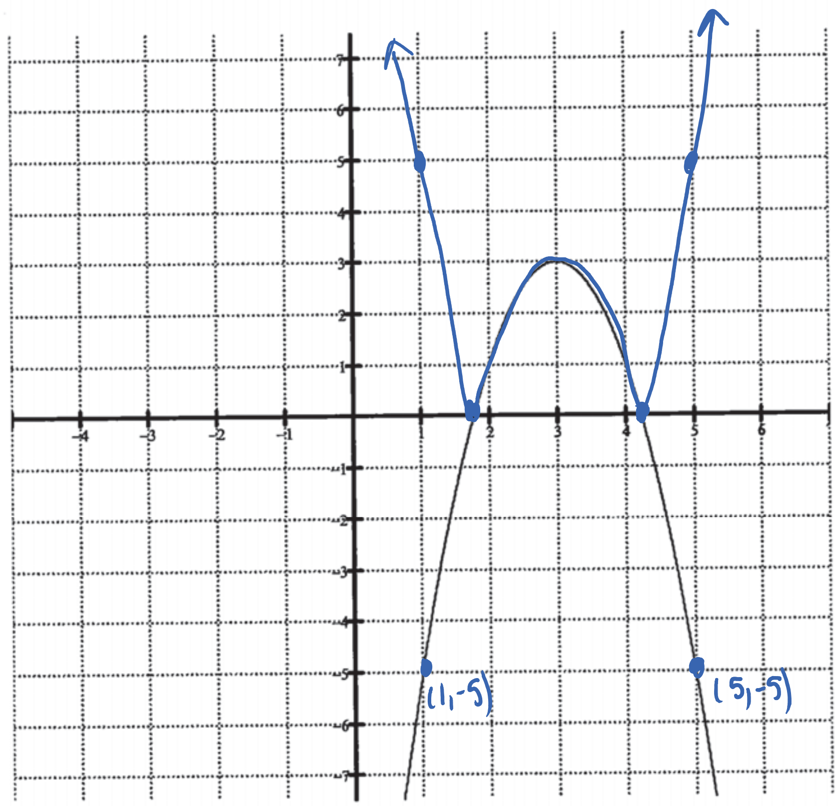
8.1 Absolute Value Functions: Part II

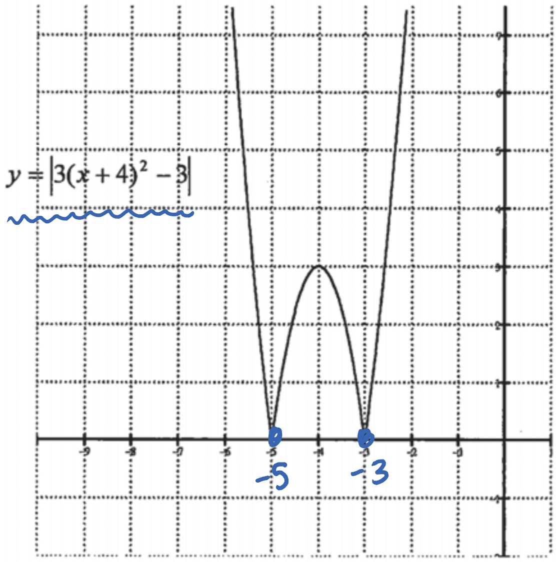
Example #1: Given the graph of $y = f(x)$. On the same set of axes, sketch the graph of $y = |f(x)|$.

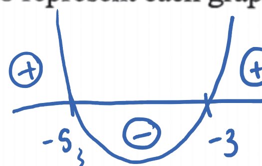
*Reflect all negative parts in the x-axis



Example #2: What piecewise function could you use to represent each graph of an absolute value function?

(a)



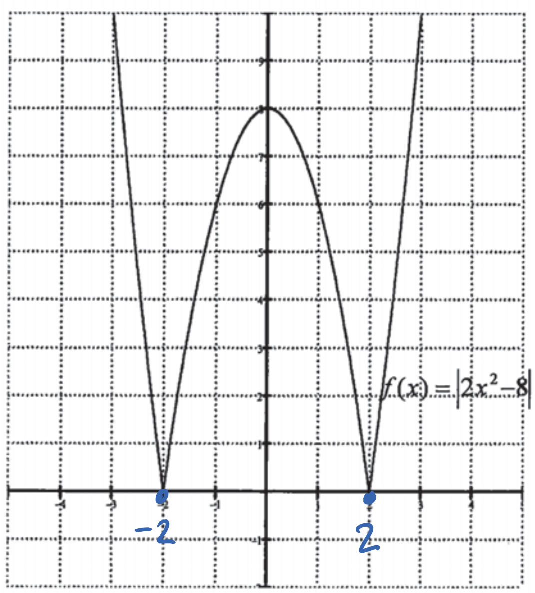


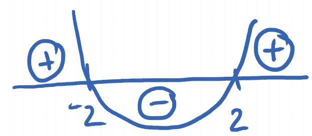
$x \leq -5$ | $-5 < x < -3$ | $x \geq -3$

↑ reflected this section

$$y = \begin{cases} 3(x+4)^2 - 3, \text{ if } x \leq -5 \text{ and } x \geq -3 \\ -[3(x+4)^2 - 3], \text{ if } -5 < x < -3 \end{cases}$$

(b)





$x \leq -2$ | $-2 < x < 2$ | $x \geq 2$

$$y = \begin{cases} 2x^2 - 8, \text{ if } x \leq -2 \text{ and } x \geq 2 \\ -(2x^2 - 8), \text{ if } -2 < x < 2 \end{cases}$$

Example #3: Consider the function $y=\left|-x^2+2x+8\right|$.

(a) Determine the x-intercept and the y-intercept.

x-int: $y=0$

$0=\left|-x^2+2x+8\right|$

$0=-x^2+2x+8$

$0=x^2-2x-8$

$0=(x-4)(x+2)$

$x=4,\ x=-2$

y-int: $x=0$

$y=\left|-(0)^2+2(0)+8\right|$

$y=|8|$

$y=8$

(b) Sketch the graph.

$y=\left|-x^2+2x+8\right|$

① Graph $-x^2+2x+8$

$\frac{1}{2}(-2)=-1$

$\hookrightarrow (-1)^2=1$

$y=-(x^2-2x)+8$

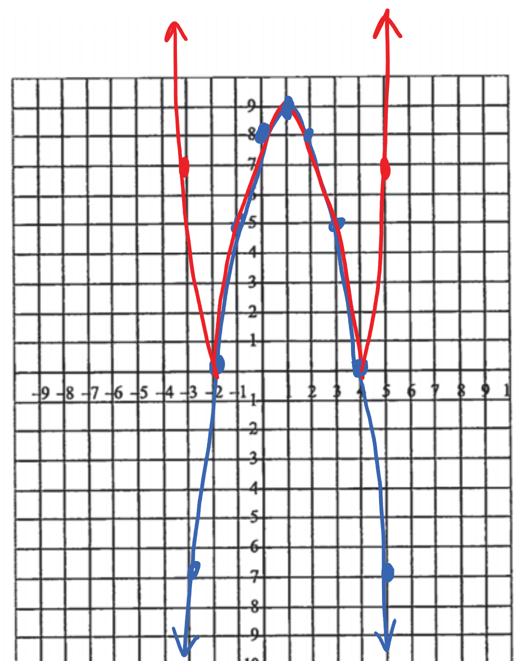
$y=-(x^2-2x+1-1)+8$

$y=-(x^2-2x+1)+1+8$

$y=-(x-1)^2+9$

$\therefore$ vertex is $(1,9)$

② Reflect the negative portion



Steps: 1, 3, 5

x-1: -1, -3, -5

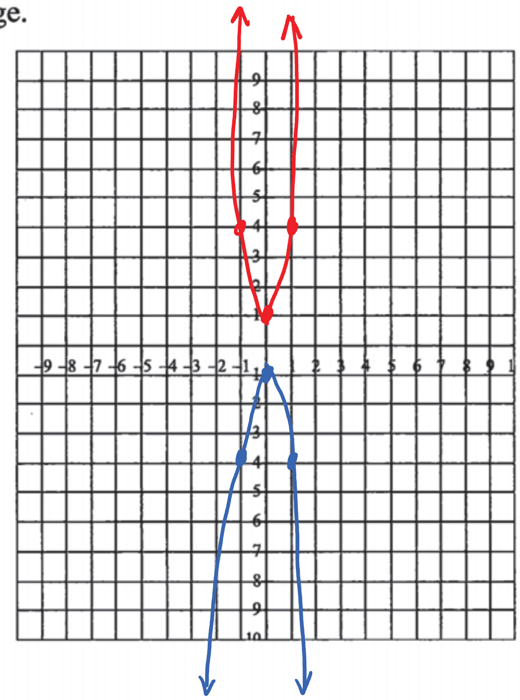
(c) State the domain and range.

D: $\{x \mid x \in \mathbb{R}\}$ R: $\{y \mid y \geq 0,\ y \in \mathbb{R}\}$

(d) Express as a piecewise function.

$$y=\begin{cases} -x^2+2x+8, & \text{if } -2 \leq x \leq 4 \\ -(-x^2+2x+8), & \text{if } x<-2 \text{ and } x>4 \end{cases}$$

Example #4: Consider the function $y = |-3x^2 - 1|$. Sketch its graph and determine its intercepts, domain, and range.



1 Graph $y = -3x^2 - 1$ ← y-int

steps: 1, 3, 5

x-3 : -3, -9, -15

2 Reflect all negatives in x-axis

3 x-int: $y = 0$

$0 = |-3x^2 - 1|$

$0 = -3x^2 - 1$

$1 = -3x^2$

$\frac{1}{-3} = x^2$

$\sqrt{\frac{1}{-3}} = x$

↳ never going to happen... no x-ints!

y-int: $x = 0$

$y = |-3(0)^2 - 1|$

$y = |0 - 1|$

$y = |-1|$

$y = 1$

y-int is $(0, 1)$

4 D: $\{x \mid x \in R\}$

R: $\{y \mid y \geq 1, y \in R\}$