8.1 Absolute Value Functions: Part I

Recall: $|7| = 7$ $|-7| = 7$ $|-32| = 32$ $|53| = 53$

An absolute value function is a function that involves the absolute value of a variable. The absolute value of 'x' is defined as $y = |x|$ and can be written:

$$y = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

Since the function is defined by two different rules for each interval in the domain, this is an example of a piecewise function.

Example #1: Sketch the graph of $y = |x|$

x	y
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3

$y = |-3| = 3$

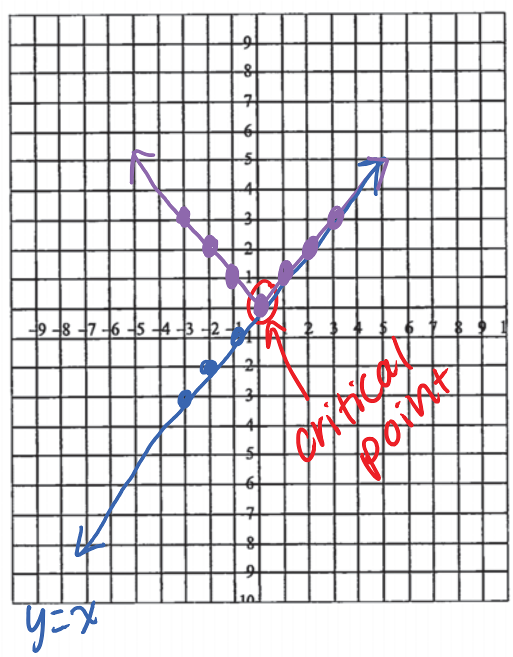
$y = |-2| = 2$

$y = |-1| = 1$

$y = |0| = 0$

$y = |1| = 1$

⋮



An invariant or critical point is a point that remains unchanged when a transformation is applied to it.

Example #2: Consider the function $y = |2x - 1|$.

(a) Determine the x-intercept and the y-intercept.

x-int: $y = 0$

$0 = |2x - 1|$

$0 = 2x - 1$

$1 = 2x$

$x = \frac{1}{2}$

$(\frac{1}{2}, 0)$

y-int: $x = 0$

$y = |2(0) - 1|$

$y = |-1| \rightarrow y = 1$

$(0, 1)$

(b) Sketch the graph.

Method 1: Table of Values

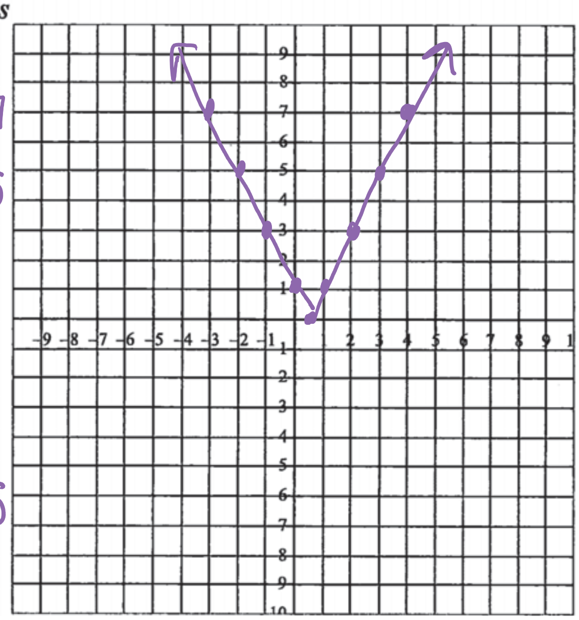
x	y
-3	7
-2	5
-1	3
0	1
1	1
2	3
3	5

$y = |2(-3) - 1| = |-7| = 7$

$y = |2(-2) - 1| = |-5| = 5$

$y = |2(2) - 1| = |4 - 1|$
$= |3| = 3$

$y = |2(3) - 1| = |5| = 5$



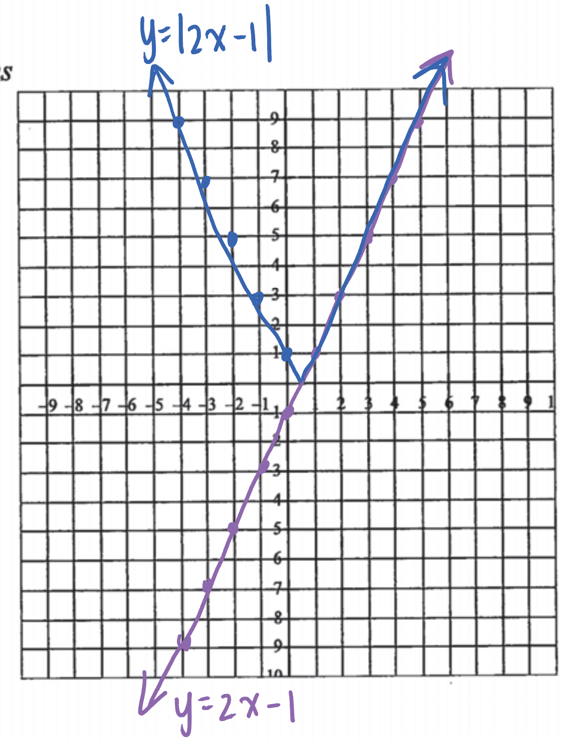
Method 2: Transformations

$y = |2x - 1|$

① Graph the line

$y = 2x - 1$

② Since the graph cannot have any negative values for y, reflect the graph over the x-axis for the neg. y-values.



(c) State the domain and range.

D: $\{x \mid x \in \mathbb{R}\}$ R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

(d) Express as a piecewise function.

$$y = \begin{cases} 2x-1, & \text{if } x \geq \frac{1}{2} \\ -(2x-1), & \text{if } x < \frac{1}{2} \end{cases}$$

⊖ original graph is negative $x < \frac{1}{2}$ | $\frac{1}{2}$ | ⊕ original graph is positive $x \geq \frac{1}{2}$

* critical point is where $y = 0$, so

$0 = |2x - 1|$

$0 = 2x - 1$

$1 = 2x$

$\frac{1}{2} = x$

Example #3: Consider the function $y = |-2x + 3|$.

(a) Determine the x-intercept and the y-intercept.

x-int: $y = 0$

$0 = |-2x + 3|$

$0 = -2x + 3$

$2x = 3$

$x = \frac{3}{2}$ $(\frac{3}{2}, 0)$

y-int: $x = 0$

$y = |-2(0) + 3|$

$y = |3|$

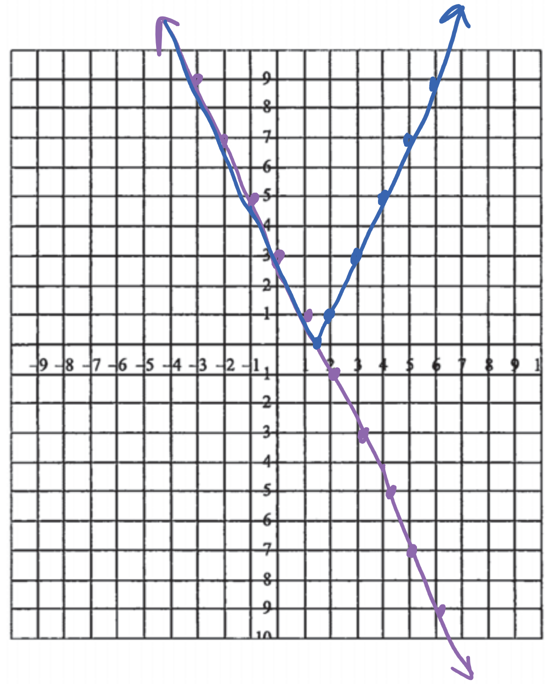
$y = 3$

$(0, 3)$

(b) Sketch the graph.

① Sketch $y = -2x+3$

② Reflect the negative portion



critical point: $0 = -2x+3$

$2x = 3$

$x = \frac{3}{2} = 1.5$

$x < 1.5 \quad x \geq 1.5$

⊕ 1.5 ⊖

this portion needs to flip

(c) State the domain and range.

D: $\{x \mid x \in \mathbb{R}\}$ R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

(d) Express as a piecewise function.

$$y = \begin{cases} -2x+3, & \text{if } x < 1.5 \\ -(-2x+3), & \text{if } x \geq 1.5 \end{cases}$$

Example #4: Write the piecewise function that represents the following graph.

$y = |3x-6|$

↳ x-int is (2,0) which is the critical point

$$y = \begin{cases} 3x-6, & \text{if } x \geq 2 \\ -(3x-6), & \text{if } x < 2 \end{cases}$$