

Pre-Calculus II
7.3/7.4

Adding + Subtracting Rational Expressions



To add or subtract rational expressions we raise factors to higher terms to obtain the lowest common denominator

Examples: Simplify

1. a) $\frac{2}{3x} - \frac{5x}{4}$ LCD: $12x$

$$= \frac{2}{3x} \left(\frac{4}{4} \right) - \frac{5x}{4} \left(\frac{3x}{3x} \right)$$

$$= \frac{8}{12x} - \frac{15x^2}{12x}$$

$$= \frac{8 - 15x^2}{12x}$$

b) $\frac{2m}{n} + \frac{3n}{m^2} - \frac{2n-3}{5m}$ LCD: $5m^2n$

$$= \frac{2m}{n} \left(\frac{5m^2}{5m^2} \right) + \frac{3n}{m^2} \left(\frac{5n}{5n} \right) - \frac{(2n-3)(mn)}{5m(mn)}$$

$$= \frac{10m^3}{5m^2n} + \frac{15n^2}{5m^2n} - \frac{2mn^2 - 3mn}{5m^2n}$$

$$= \frac{10m^3 + 15n^2 - 2mn^2 + 3mn}{5m^2n}$$



Recall: Complex Fraction: $\frac{(2 + \frac{3}{4}) \cdot 12}{(5 - \frac{1}{3}) \cdot 12}$

To simplify, multiply num. + denom. by common denominator of the individual fractions.

$$= \frac{24 + \frac{36}{4}}{60 - \frac{12}{3}} = \frac{24 + 9}{60 - 4} = \frac{33}{56}$$

2. a) $\frac{(\frac{2}{3} - a) \times 12}{(4 + a) \times 12}$

$$= \frac{\frac{24}{3} - 12a}{\frac{12}{4} + 12a} = \frac{8 - 12a}{3 + 12a} = \frac{4(2 - 3a)}{3(1 + 4a)} = \frac{3b + 1}{2b - 1}$$

b) $\frac{(3 + \frac{1}{b}) \times b}{(2 - \frac{1}{b}) \times b}$



Frequently, we must factor the denominator of rational expressions so the lowest common denominator can be more readily determined



When the denominator is in factored form, don't forget to state restrictions (NPVs)

$$3. a) \frac{m}{m-4} + \frac{2m}{3m-12}$$

$$= \frac{m}{m-4} + \frac{2m}{3(m-4)} \quad \begin{array}{l} \text{LCD: } 3(m-4) \\ \text{NPV: } m \neq 4 \end{array}$$

$$= \frac{m}{m-4} \left(\frac{3}{3} \right) + \frac{2m}{3(m-4)}$$

$$= \frac{3m + 2m}{3(m-4)} = \frac{5m}{3(m-4)}$$

$$b) \frac{x}{x^2-3x+2} - \frac{x-5}{x^2+5x-14}$$

$$= \frac{x}{(x-2)(x-1)} - \frac{x-5}{(x+7)(x-2)} \quad \begin{array}{l} \text{LCD: } (x-1)(x-2)(x+7) \\ \text{NPVs: } x \neq 1, x \neq 2, x \neq -7 \end{array}$$

$$= \frac{x}{(x-2)(x-1)} \frac{(x+7)}{(x+7)} - \frac{(x-5)}{(x+7)(x-2)} \frac{(x-1)}{(x-1)}$$

$$= \frac{x^2+7x}{(x-2)(x-1)(x+7)} - \frac{x^2-x-5x+5}{(x-2)(x-1)(x+7)}$$

$$= \frac{x^2+7x - x^2 + 6x - 5}{(x-2)(x-1)(x+7)}$$

$$= \frac{13x-5}{(x-2)(x-1)(x+7)}$$



In Summary:

- ① FACTOR THE DENOMINATOR
- ② FORM LCD
- ③ STATE NPVs (restrictions)
- ④ COMBINE NUMERATORS
- ⑤ FACTOR/CANCEL

Practice

$$1. \frac{5m+25}{2m^2+13m+15} - \frac{10m-20}{m^2-4}$$

$$2(15) = 30$$

$$\begin{array}{c} \swarrow \quad \searrow \\ 10 \quad 3 \end{array}$$

$$= \frac{5(m+5)}{2m^2+10m+3m+15} - \frac{10(m-2)}{(m+2)(m-2)}$$

① factor denominator

$$= \frac{5(m+5)}{2m(m+5)+3(m+5)} - \frac{10(m-2)}{(m+2)(m-2)}$$

$$= \frac{5(m+5)}{(2m+3)(m+5)} - \frac{10(m-2)}{(m+2)(m-2)}$$

② form LCD:

$$(2m+3)(m+5)(m+2)(m-2)$$

*this is really messy!
cancel first to make your
life easier ;)

$$= \frac{5}{(2m+3)} \frac{(m+2)}{(m+2)} - \frac{10}{(m+2)} \frac{(2m+3)}{(2m+3)}$$

LCD is actually just:
 $(2m+3)(m+2)$

$$= \frac{5m+10 - 20m - 30}{(2m+3)(m+2)}$$

③ State NPVs: $m \neq -\frac{3}{2}, m \neq -2$
 $m \neq -5, m \neq 2$

$$= \frac{-15m - 20}{(2m+3)(m+2)}$$

④ combine numerators

$$= \frac{-5(3m+4)}{(2m+3)(m+2)}$$

⑤ Factor & simplify where possible

⑥ That's it! YAY! ;)