

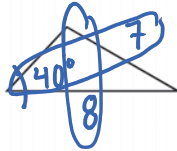
### 6.5 The Cosine Law

When can we use the basic trigonometric ratios? (ie. SOHCAHTOA)

when we have a right (90°) triangle

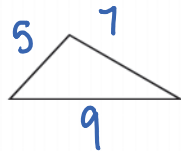
When can we use the Sine Law? when we have an angle and its opposite side + another angle or side (pair and a half)

Example:

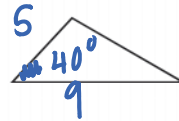


When can we use the Cosine Law? when we have all 3 sides or 2 sides and an angle between them

Example:



or



#### The Cosine Law

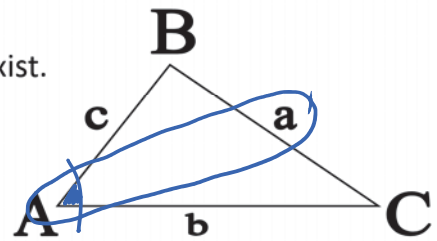
In any triangle ABC, the following relationships exist.

$$\underline{a^2 = b^2 + c^2 - 2bc \cos A}$$

$$\underline{b^2 = a^2 + c^2 - 2ac \cos B}$$

$$\underline{c^2 = a^2 + b^2 - 2ab \cos C}$$

Use the above formulas when you are given 2 sides and the angle contained by them and you want to find the 3<sup>rd</sup> side.



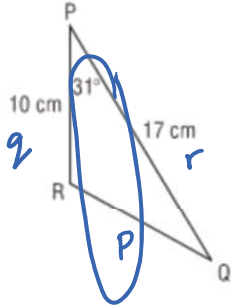
When you are given 3 sides and you want to find an angle, solve for cos A:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 - b^2 - c^2 = -2bc \cos A$$

$$\boxed{\frac{a^2 - b^2 - c^2}{-2bc} = \cos A}$$

**Example #1:** In  $\triangle PQR$ , determine the length of QR to the nearest tenth of a centimetre.



$$p^2 = q^2 + r^2 - 2qr \cos P$$

$$p^2 = 10^2 + 17^2 - 2(10)(17)\cos 31^\circ$$

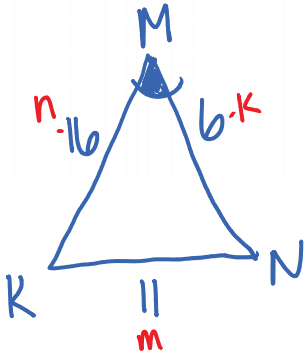
$$p^2 = 97.56 \dots$$

$$p = \sqrt{\text{ans}}$$

$$p = 9.877 \dots$$

$$p = 9.9 \text{ cm}$$

**Example #2:** In  $\triangle KMN$ ,  $KM = 16$  cm,  $KN = 11$  cm, and  $MN = 6$  cm; determine the measure of angle M to the nearest degree.



$$m^2 = n^2 + k^2 - 2nk \cos M$$

$$\frac{m^2 - n^2 - k^2}{-2nk} = \cos M$$

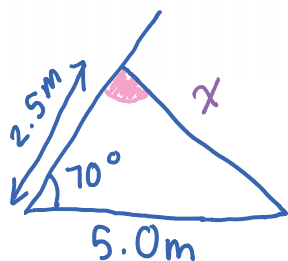
$$\frac{11^2 - 16^2 - 6^2}{-2(16)(6)} = \cos M$$

$$0.890625 = \cos M$$

$$\angle M = \cos^{-1}(\text{ans})$$

$$\angle M = 27^\circ$$

**Example #3:** A retaining wall is leaning at an angle of  $70^\circ$  to the horizontal. A rigid support is to be placed 5.0 m from the base of the wall and it will be attached to the wall 2.5 m from its base. Determine the length of the support to the nearest tenth of a metre and the measures of the angle between the support and the wall to the nearest degree.



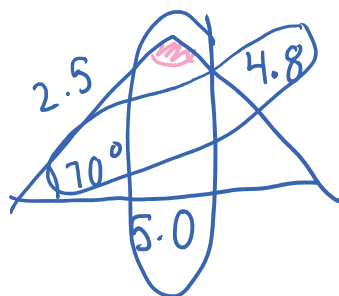
\* we have 2 sides and an angle between them, so use the cosine law to find  $x$

$$x^2 = 5^2 + 2.5^2 - 2(5)(2.5)\cos 70^\circ$$

$$x^2 = 22.699$$

$$x = \sqrt{\text{ans}}$$

$$\boxed{x = 4.8\text{m}}$$



$$\frac{\sin \theta}{5.0} = \frac{\sin 70^\circ}{4.8}$$

$$\sin \theta = \frac{\sin 70^\circ}{4.8} \times 5$$

$$\sin \theta = 0.978 \dots$$

$$\theta = \sin^{-1}(\text{ans})$$

$$\boxed{\theta = 78^\circ}$$