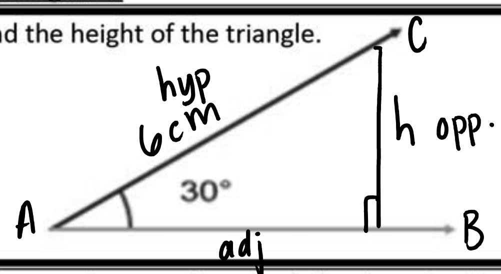
6.4b The Ambiguous Case of the Sine Law

Investigation

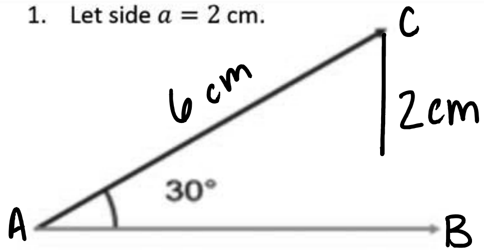
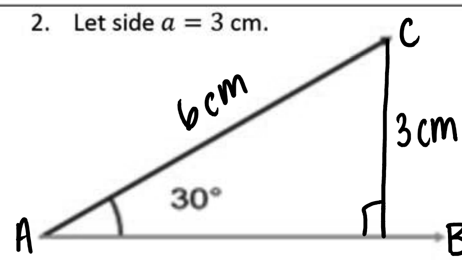
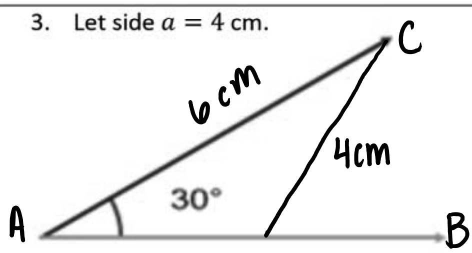
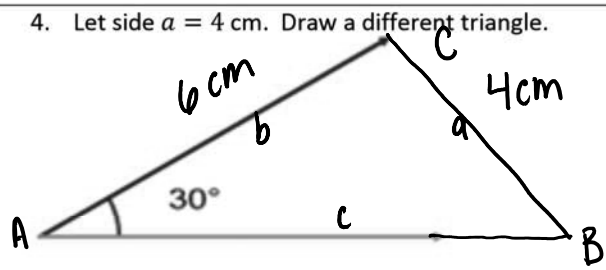
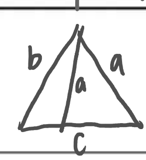
Find the height of the triangle.



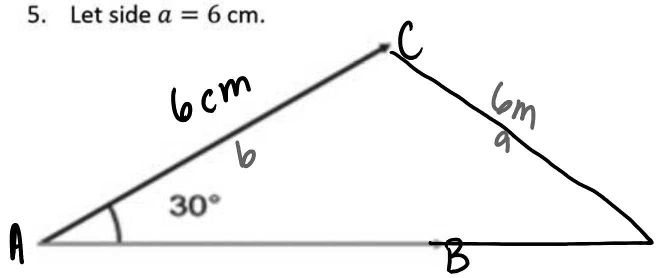
$\sin A = \frac{opp}{hyp}$ $\quad h = 6\sin 30°$

$\sin 30° = \frac{h}{6}$ $\quad h = 3$

Using a ruler, complete the following triangles. In each triangle, you are given $\angle A = 30°$, and side $b = 6$cm. You are not given the length of side c or the measures of the other two angles.

Triangles	Need to Know
1. Let side $a = 2$ cm.  	If $\angle A$ is acute and $a < h$, there is no triangle
2. Let side $a = 3$ cm.  	If $\angle A$ is acute and $a = h$, there is one right triangle
3. Let side $a = 4$ cm.  	If $\angle A$ is acute and $h < a < b$, there are two possible Δs
4. Let side $a = 4$ cm. Draw a different triangle.  	If $\angle A$ is acute and $h < a < b$, there are two possible Δs 

5. Let side $a = 6$ cm.	If $\angle A$ is acute and $a \geq b$, there is one triangle

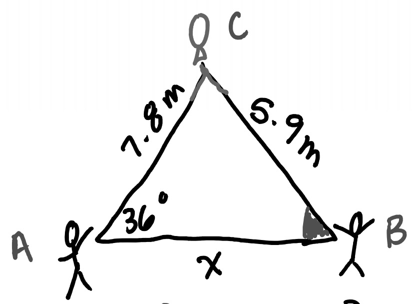


The Ambiguous Case of the Sine Law

When you are given two side lengths and the measure of an angle that is opposite one of these sides, you may be able to construct and solve zero, one, or two triangles.

Example #1:

Daniel and Zane are amusing themselves by playing with a helium filled balloon out in the back field. Daniel's rope is 7.8 m long and makes an angle of 36.0° with the ground. Zane's rope is 5.9 m long. Assuming that Daniel and Zane form a triangle in a vertical plane with the balloon, what is the distance between Daniel and Zane, to the nearest tenth of a metre?



step①: find ∠B

$$\frac{\sin B}{7.8} = \frac{\sin 36}{5.9}$$

$\angle B = \sin^{-1}(0.777\ldots)$

$\angle B = 51°$

step②: find ∠C

$\angle C = 180 - 36 - 51 = 93°$

Step③: find side c ("x")

$$\frac{c}{\sin 93°} = \frac{5.9}{\sin 36°}$$

$\boxed{c = 10.0\text{ m}}$

Step④: consider ∠B being obtuse

$\angle B = 180° - 51°$

$= 129°$

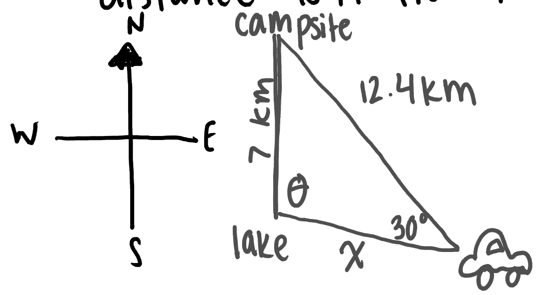
$\therefore \angle C = 180 - 36 - 129$

$\angle C = 15°$

Step⑤: find side c

$$\frac{c}{\sin 15°} = \frac{5.9}{\sin 36°} \rightarrow \boxed{c = 2.6\text{m}}$$

Try this! Leanne & Kerry are hiking in the mountains. They left their car and walked NW for 12.4 km to the campsite. They then turned due south and walked another 7.0 km to a lake. The weather took a turn for the worse, so they wanted to head back to the car. The angle from the path to the campsite and path to lake is 30°. What distance is it from the lake to the car?



Step ①: find ∠ of lake (θ)

$$\frac{\sin\theta}{12.4} = \frac{\sin 30^\circ}{7}$$

$$\theta = 62^\circ$$

* note: the angle in the Δ is obtuse, so take the supplement:

$$\theta = 180 - 62 = 118^\circ$$

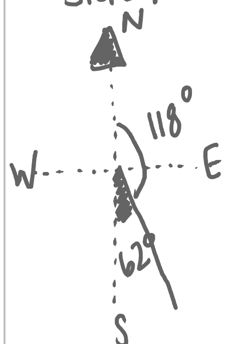
$\therefore \angle$ of campsite $= 180 - 118 - 30 = 32^\circ$

Step ② find "x":

$$\frac{x}{\sin 32^\circ} = \frac{7}{\sin 30^\circ}$$

$$\boxed{x = 7.4\text{ km}}$$

Side note:



S62°E to the car