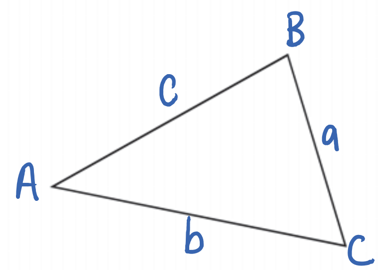
6.4 The Sine Law

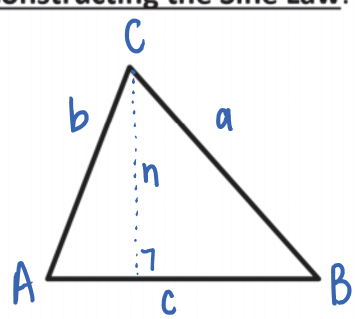
When we are asked to solve a triangle problem, we aren't always given a right triangle. Therefore, we can't always use our basic trigonometric ratios (ie. SOHCAHTOA).

Whenever you are given an oblique triangle,
label the lengths of the sides in terms of the vertices.

- The side of length __a__ is apposite angle __A__
- The side of length __b__ is opposite angle __B__
- The side of length __c__ is opposite angle __C__



Constructing the Sine Law: Let's compare $\angle A$ and $\angle B$:



$\sin A = \frac{n}{b}$ $\qquad$ $\sin B = \frac{n}{a}$

$b \sin A = n$ $\qquad$ $a \sin B = n$

$b \sin A = n = a \sin B$

$b \sin A = a \sin B$

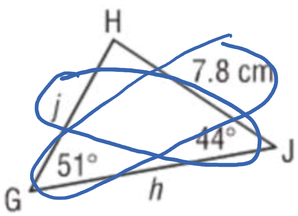
$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

THE SINE LAW

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{and} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

***In order to use the sine law, we must know an angle and the side opposite that angle. ***

Example #1: In ΔGHJ, determine the length of GH to the nearest tenth of a centimetre.



$$\frac{j}{\sin J} = \frac{g}{\sin G}$$

$$\frac{j}{\sin 44^\circ} = \frac{7.8}{\sin 51^\circ}$$

*cross multiply
OR
multiply both sides by sin44°

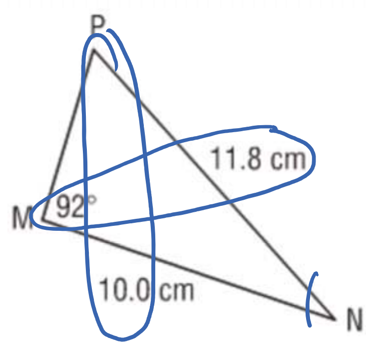
$$\cancel{\sin 44^\circ} \times \frac{j}{\cancel{\sin 44^\circ}} = \frac{7.8}{\sin 51^\circ} \times \sin 44^\circ$$

$$j = 6.97 \rightarrow \boxed{\therefore j = 7.0\text{cm}}$$

Example #2: In ΔMNP, determine the measure of angle N to the nearest degree.

Don't have enough info to find ∠N just yet...

→ Find ∠P, then ∠N



$$\frac{\sin P}{10} = \frac{\sin 92^\circ}{11.8}$$

$$\sin P = \frac{\sin 92^\circ}{11.8} \times 10$$

$$\sin P = 0.8469.....$$

$$\angle P = \sin^{-1}(\text{ans})$$

$$\boxed{\angle P = 58^\circ}$$

$$\angle N = 180^\circ - 92^\circ - 58^\circ$$

$$\boxed{\angle N = 30^\circ}$$