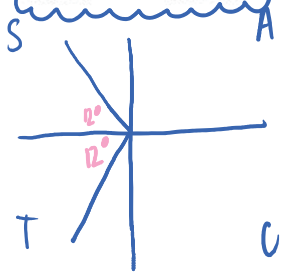
6.2 Angles in Standard Position in All Quadrants: Part 2

In the previous section, we found the trigonometric ratios of a given angle in standard position. We can reverse this process to use the ratios to find the standard position angles.

Example #1: Solve for θ to the nearest degree, given that $0 \leq \theta < 360°$ and $\cos\theta = -0.978$

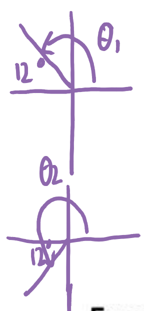


$\cos\theta = 0.978$

$\theta_r = \cos^{-1}(0.978)$

$\theta_r = 12°$

> **Steps**
> 5- Determine in which quadrants the angle could be based on the CAST rule.
> 6- Drop any negative sign. (This helps us find the reference angle).
> 7- Determine the reference angle by either using a calculator, or by looking at an exact value triangle.
> 8- Draw the reference angle in between the terminal arm and the *x*-axis.
> 9- Compute the standard position angle(s).



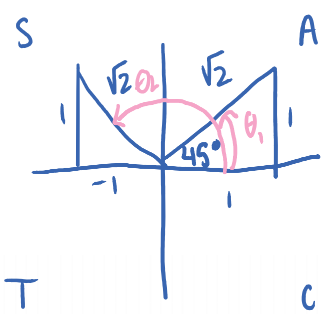
$\theta_1 = 180° - 12°$

$\boxed{\theta_1 = 168°}$

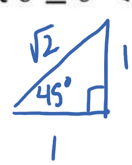
$\theta_2 = 180° + 12°$

$\boxed{\theta_2 = 192°}$

Example #2: Solve for θ exactly, given that $0 \leq \theta < 360°$ and $\sin\theta = \frac{1}{\sqrt{2}}$



Recall:



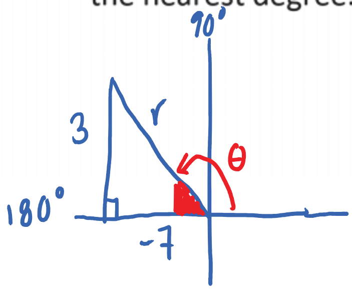
$\theta_r = 45°$

$\boxed{\theta_1 = 45°}$

$\theta_2 = 180° - 45°$

$\boxed{\theta_2 = 135°}$

Example #3: Given that $\tan\theta = \frac{-3}{7}$, and $90° \le \theta < 180°$, determine the exact values of the other two trigonometric ratios, and also determine the value of θ to the nearest degree.



$3^2 + (-7)^2 = r^2$

$9 + 49 = r^2$

$r = \sqrt{58}$

$\sin\theta = \frac{3}{\sqrt{58}} \rightarrow \theta_r = \sin^{-1}\left(\frac{3}{\sqrt{58}}\right)$

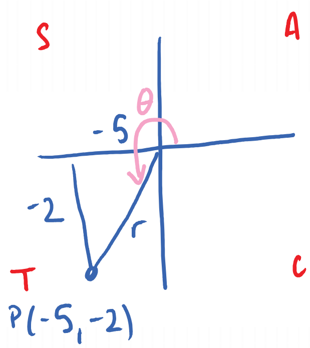
$\theta_r = 23.2°$

$\cos\theta = \frac{-7}{\sqrt{58}}$

$\theta = 180° - 23°$

$\boxed{\theta = 157°}$

Example #4: If the point $(-5, -2)$ is on the terminal arm of θ with $0° \le \theta < 360°$, then determine the exact values of all three basic trigonometric ratios, and also determine the value of θ to the nearest degree.



$r^2 = (-5)^2 + (-2)^2$

$r^2 = 25 + 4$

$r = \sqrt{29}$

$\sin\theta = \frac{-2}{\sqrt{29}}$

$\cos\theta = \frac{-5}{\sqrt{29}}$

$\tan\theta = \frac{-2}{-5} = \frac{2}{5}$

$\theta_r = \tan^{-1}\left(\frac{2}{5}\right)$

$\theta_r = 21.8°$

$\theta = 180° + 22°$

$\boxed{\theta = 202°}$