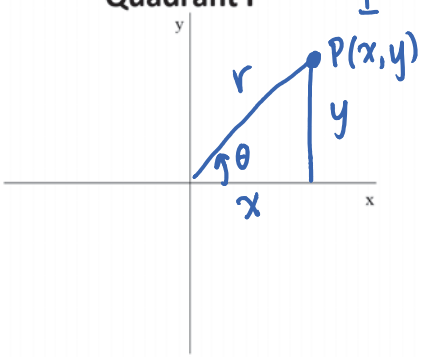
6.2 Angles in Standard Position in All Quadrants: Part 1

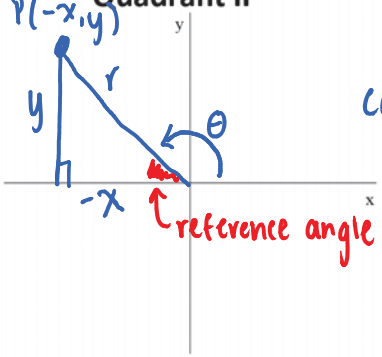
In the previous section, we found that the trigonometric ratios of an angle in standard position in Quadrant I were related to the coordinates of a point on the terminal arm of the angle. These relationships can be extended to other Quadrants as well.

Quadrant I (I)



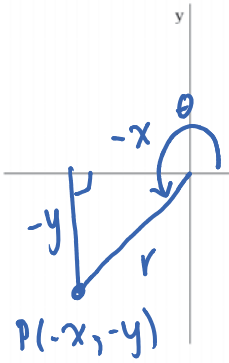
$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

Quadrant II



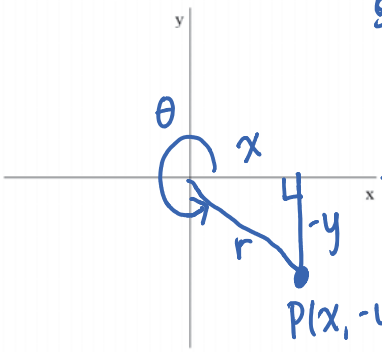
$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{-x}{r}$ (neg) $\tan\theta = \frac{y}{-x}$ (neg)

Quadrant III



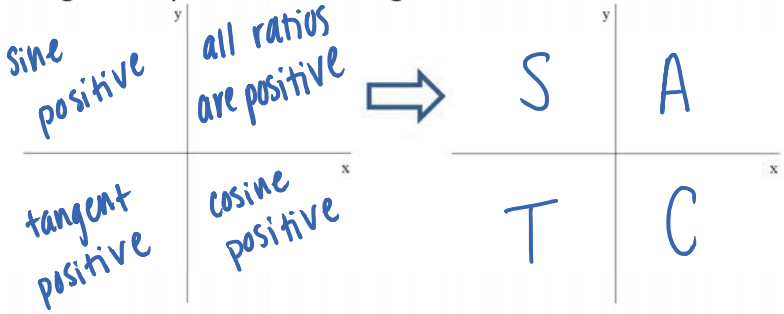
$\sin\theta = \frac{-y}{r}$ (neg) $\cos\theta = \frac{-x}{r}$ (neg) $\tan\theta = \frac{-y}{-x} = \frac{y}{x}$

Quadrant IV



$\sin\theta = \frac{-y}{r}$ (neg) $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{-y}{x}$ (neg)

We can generalize which trigonometric ratios are positive in which quadrants by just looking at the positives and negatives from above.



We use the term CAST to help us remember which trigonometric ratios are positive in which quadrants.

Example #1: The point $P(-3,\ 2)$is on the terminal arm of the angle θ. Draw θ in standard position, determine the exact value of the primary trigonometric ratios, and then determine the value of θ to the nearest degree.

S A T C

P(-3,2), r, θ, 2, -3

$2^2+(-3)^2=r^2$

$4+9=r^2$

$\sqrt{13}=r$

$\sin\theta=\frac{2}{\sqrt{13}}$

$\cos\theta=\frac{-3}{\sqrt{13}}$

$\tan\theta=\frac{2}{-3}$

$\theta_r=\sin^{-1}\left(\frac{2}{\sqrt{13}}\right)$

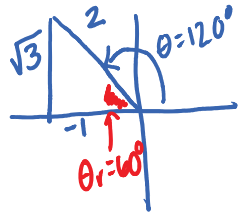
$\theta_r=34^\circ$

$\theta=180^\circ-34^\circ$

$\theta=146^\circ$

Example #2: Determine the exact values for all the trigonometric ratios for the standard position angle $\theta=120^\circ$.

$\theta_r=180^\circ-120^\circ=60^\circ$



$\sin\theta=\frac{\sqrt{3}}{2}$

$\cos\theta=\frac{-1}{2}$

$\tan\theta=\frac{\sqrt{3}}{-1}=-\sqrt{3}$

Steps
1- Draw the angle in standard position.
2- Determine the reference angle.
3- Draw the appropriate exact value triangle making sure to label sides as either positive or negative.
4- Pick off the desired trigonometric ratios.

We can use the above definitions for the each of the three trigonometric ratios to determine what the values are for angles that are on the cardinal directions (ie: 0°, 90°, 180°, 270°, 360°)

θ	Point on the terminal arm ($r=1$)	$\cos\theta=\frac{x}{r}$	$\sin\theta=\frac{y}{r}$	$\tan\theta=\frac{y}{x}$
0°	x y $(1,0)$	$\frac{1}{1}=1$	$\frac{0}{1}=0$	$\frac{0}{1}=0$
90°	x y $(0,1)$	$\frac{0}{1}=0$	$\frac{1}{1}=1$	$\frac{1}{0}=$ DNE
180°	$(-1,0)$	$\frac{-1}{1}=-1$	$\frac{0}{1}=0$	$\frac{0}{-1}=0$
270°	$(0,-1)$	$\frac{0}{1}=0$	$\frac{-1}{1}=-1$	$\frac{-1}{0}=$ DNE
360°	$(1,0)$	$\frac{1}{1}=1$	$\frac{0}{1}=0$	$\frac{0}{1}=0$