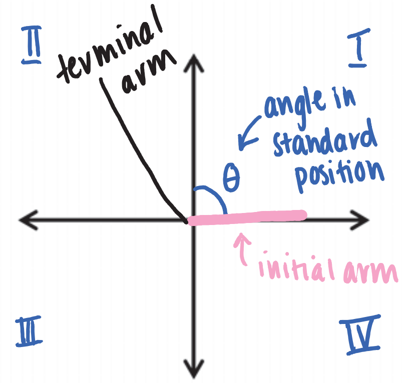
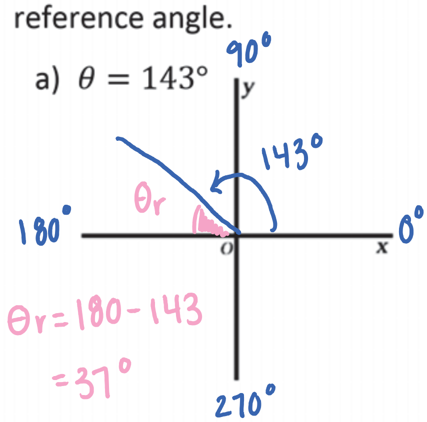
6.1 Angles in Standard Position in Quadrant I



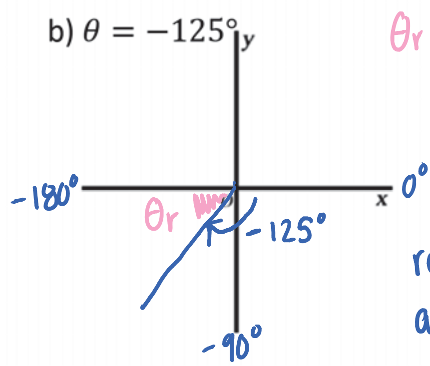
An angle is in __standard__ position when its vertex is at the origin and its initial ray (initial arm) is on the positive __x-axis__. The angle, θ, is the angle between the initial arm and the terminal arm. For a positive angle, the terminal arm moves in a __counter clockwise__ direction and for a negative angle, the terminal arm moves in a __clockwise__ direction. The smallest (acute) angle between the terminal arm and the __x-axis__ is known as the __reference__ angle.

Example #1: Sketch the following angles in standard position and determine their reference angle.

a) $\theta = 143°$



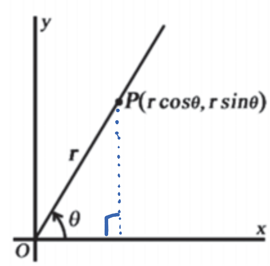
b) $\theta = -125°$



$\theta_r = 180 - 125$
$= 55°$

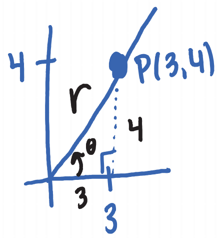
* NOTE: reference angle is always positive

If we define a point as lying on the terminal arm, we then call it a __terminal point__ and basic trigonometric operations can be used to define where the point is in space.



Example #2: The point P(3,4) is on the terminal arm of an angle in standard position.

a. Determine the distance r from the origin to P.



use Pythagoras:

$a^2+b^2=c^2$

$3^2+4^2=r^2$

$9+16=r^2$

$\sqrt{25}=\sqrt{r^2} \rightarrow \boxed{r=5}$

b. Determine the primary trigonometric ratios of θ

$\sin\theta = \frac{opp}{hyp} = \frac{4}{5}$ $\quad \cos\theta = \frac{adj}{hyp} = \frac{3}{5}$ $\quad \tan\theta = \frac{opp}{adj} = \frac{4}{3}$

use any ratio

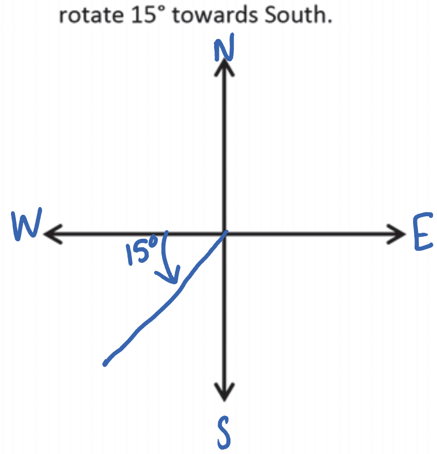
c. Determine the measure of θ to the nearest degree.

$\sin\theta = \frac{4}{5}$

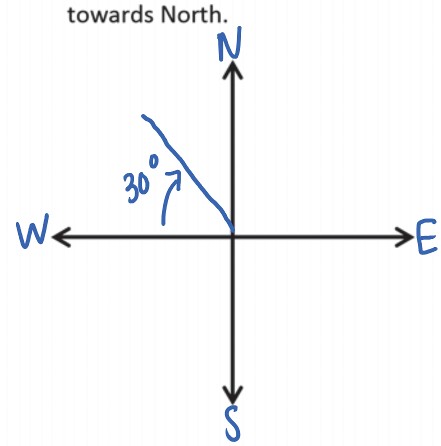
$\theta = \sin^{-1}\left(\frac{4}{5}\right)$ $\quad \boxed{\theta = 53^\circ}$

Trigonometry is essential to navigation. A direction can be described relating it to two of the compass points: North, South, East, West

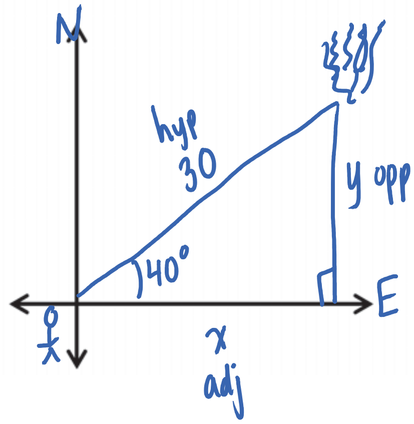
For example, a heading of W 15°S means from a direction due West, rotate 15° towards South.



A heading of W 30°N means from a direction due West, rotate 30° clock towards North.



Example #3: A forest ranger sees smoke rising from a point that lies in a direction E 40°N. She estimates that the distance from the ranger station is about 30 km. The firefighters at the ranger station have to travel east then north to get to the fire. To the nearest km, how far should the firefighters travel in each direction.



Distance east $= x$

$30 \cdot \cos 40° = \frac{x}{\cancel{30}} \cdot \cancel{30}$

$30\cos 40° = x$

$\boxed{x = 23\text{km}}$

Distance north $= y$

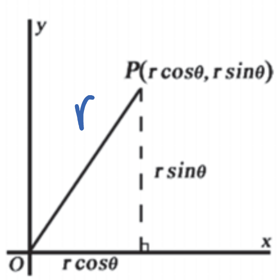
$\sin 40° = \frac{y}{30}$

$30\sin 40° = y$

$\boxed{y = 19\text{km}}$

$\therefore$ firefighters should travel 23km East and 19 km North

For an angle θ in standard position, the Pythagorean Theorem can be used to relate $\sin\theta$ and $\cos\theta$.



$a^2 + b^2 = c^2$

$(r\cos\theta)^2 + (r\sin\theta)^2 = r^2$

$\frac{\cancel{r^2}(\cos\theta)^2 + \cancel{r^2}(\sin\theta)^2}{r^2} = \frac{\cancel{r^2}}{\cancel{r^2}}$

$(\cos\theta)^2 + (\sin\theta)^2 = 1$

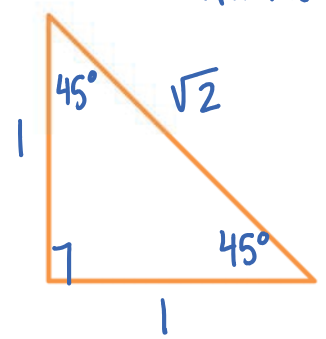
The Pythagorean Identity

*also written as $\cos^2\theta + \sin^2\theta = 1$

Special Right Triangles

$\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj}{hyp}$ $\tan\theta = \frac{opp}{adj}$

For angles $\underbrace{30^\circ, 45^\circ, 60^\circ}_{\text{reference angles}}$, you can determine the exact values of the trigonometric ratios.



$a^2 + b^2 = c^2$

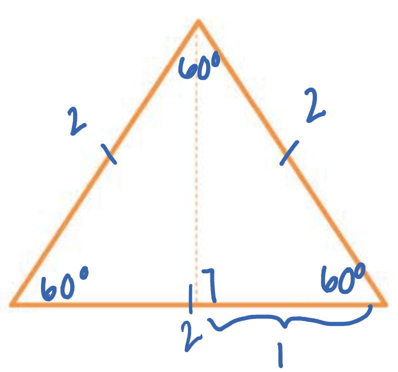
$1^2 + 1^2 = c^2$

$\sqrt{2} = c$

$\sin 45^\circ = \frac{1}{\sqrt{2}}$ $\cos 45^\circ = \frac{1}{\sqrt{2}}$

$\tan 45^\circ = \frac{1}{1} = 1$

Take an equilateral Δ with sides "2" and cut in half



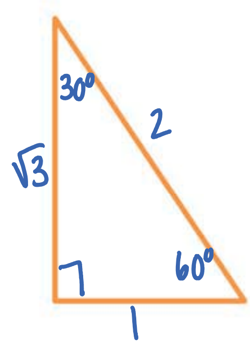
$\sin 60^\circ = \frac{\sqrt{3}}{2}$ $\cos 60^\circ = \frac{1}{2}$ $\tan 60^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$

$a^2 + b^2 = c^2$

$a^2 + 1^2 = 2^2$

$a^2 + 1 = 4$

$a = \sqrt{3}$



$\sin 30^\circ = \frac{1}{2}$ $\cos 30^\circ = \frac{\sqrt{3}}{2}$ $\tan 30^\circ = \frac{1}{\sqrt{3}}$

memorize these two special Δs