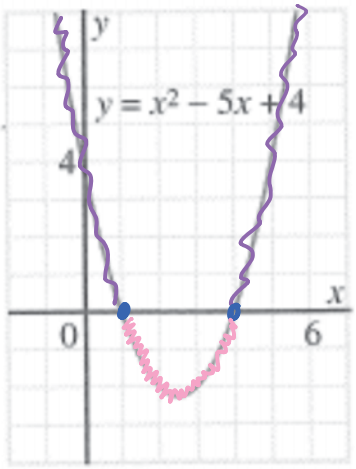
5.1 Solving Quadratic Inequalities in One Variable

When the equal sign in a quadratic equation is replaced with an inequality sign, a Quadratic Inequality in One Variable is formed.

< means **less than**	**>** means **greater than**
≤ means **less than or equal to**	**≥** means **greater than or equal to**

Example #1: Look at $y = x^2 - 5x + 4$



When is $x^2 - 5x + 4 = 0$? (roots)

$x = 1$ and $x = 4$

When is $x^2 - 5x + 4 > 0$?

$x < 1$ and $x > 4$

When is $x^2 - 5x + 4 < 0$?

$1 < x < 4$

Steps to Solving Quadratic Inequalities:

1. Move everything to one side of the inequality and factor it.
2. Using the zeros, sketch the graph. zeros = roots = x-intercepts
3. Write the solution to satisfy the inequality.
4. Test points from each region to check the solution.

must do this!!

Example #2: Solve the inequality $5x^2+13x-6>0$.

Step 1: Since everything is already on the left side of the inequality, we can go ahead and factor it.

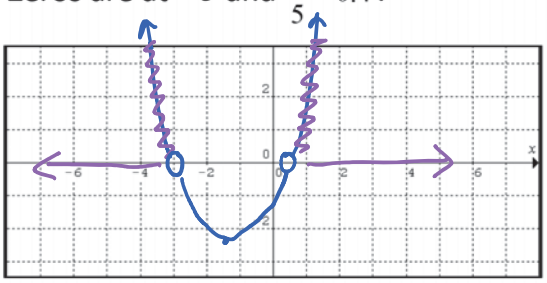
$$5x^2+13x-6>0$$
$$5x^2+15x-2x-6>0$$
$$5x(x+3)-2(x+3)>0$$
$$(x+3)(5x-2)>0$$

$x+3=0$ $\quad 5x-2=0$

$x=-3$ $\quad 5x=2$

$x=\frac{2}{5}=0.4$

Step 2: Zeros are at -3 and $\frac{2}{5}=0.4$.



Note: for $>$ or $<$ use an open circle ○

for $\geq$ or $\leq$ use a closed circle ●

Step 3: Since $5x^2+13x-6>0$, we are looking for the area where the graph is above zero. This happens to the left of -3 and to the right of 0.4.

We write this as:

$\boxed{x<-3}$ or $\boxed{x>0.4}$.

Step 4: To check, select one test point from each region.

- Less than -3 → try $x=-4$:

  $5(-4)^2+13(-4)-6=22>0$? YES

- Between -3 and 0.4 → try $x=0$:

  $5(0)^2+13(0)-6=-6>0$? NO

- Greater than 0.4 → try $x=1$:

  $5(1)^2+13(1)-6=12>0$ YES

Note: It is not necessary to sketch the graph; an alternate method is to choose test points from the regions (as in step 4) to write the solutions.

Example #3: Solve the inequality $-8x \le -3(x^2 - 1)$.

Step 1: Move everything to one side of the inequality and factor it.

$-8x \le -3(x^2-1)$

$+8x \quad +8x$

$0 \le -3(x^2-1) + 8x$

$0 \le -3x^2 + 3 + 8x$

$0 \ge 3x^2 - 8x - 3$

$0 \ge 3x^2 - 9x + x - 3$

$mn = 3(-3) = -9$

$-9 \quad +1$

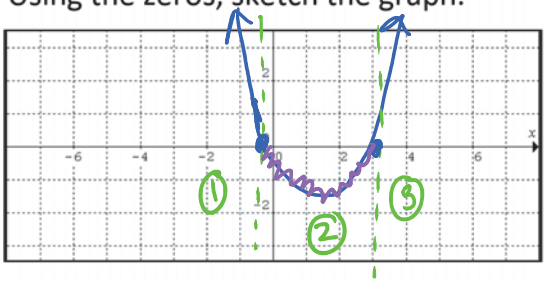
$0 \ge 3x(x-3) + (x-3)$

$0 \ge (3x+1)(x-3)$

$x = -\frac{1}{3} \quad x = 3$

*Recall: when ÷ by a negative, flip your inequality sign!

Step 2: Using the zeros, sketch the graph.



Step 3: Write the solution to satisfy the inequality. ≤ 0 means below x-axis

$-\frac{1}{3} \le x \le 3$

Step 4: To check, select one test point from each region.

$-8x \le -3(x^2-1)$

In ①: $x < -\frac{1}{3}$ test $x = -1$

$-8(-1) \le -3((-1)^2 - 1)$

$8 \le 0$ FALSE

In ②: $-\frac{1}{3} \le x \le 3$ test $x = 0$

$-8(0) \le -3(0^2 - 1)$

$0 \le 3$ TRUE

In ③: $x > 3$ test $x = 4$

$-8(4) \le -3(4^2 - 1)$

$-32 \le -45$ FALSE