4.7 Modelling and Solving Problems with Quadratic Functions

In this section we will solve word problems by using the maximum or minimum points. We will need to use the information provided to write the function required to solve the problem.

Steps to Solve a Quadratic Word Problem:

1. Identify the variables.
2. Identify the quantity to be maximized or minimized, and write an algebraic expression for this quantity.
3. The expression must contain only one variable. If it contains more, use other information to write it in terms of one variable.
4. Identify whether the quadratic function has a maximum or minimum value. Then complete the square to determine this value and where it occurs.
5. Answer the question in the problem.

Example #1: Two numbers have a difference of 18. Does their product have a maximum or a minimum value? Determine this value and the two numbers.

Step 1 : let x be one number
y be the other number

Step 2: $x-y=18 \rightarrow y=x-18$
$P=xy$

Step 3: Solve by substitution
$P=xy$
$P=x(x-18)$
$P=x^2-18x$

Step 5: $P=x^2-18x$ → there will be a minimum
$P=x^2-18x+81-81$ $\frac{1}{2}(-18)=-9$ $(-9)^2=81$
$P=(x^2-18x+81)-81$
$P=(x-9)^2-81$
$\boxed{x=9}$ minimum value!

$y=x-18$
$y=9-18$
$\boxed{y=-9}$

Step 6: The two #s are 9 and -9 and the minimum value is -81.

Example #2: Every week, a take-out restaurant sells approximately 2000 chicken wraps for \$1.50 each. Through market research, the restaurant manager determines that for every \$0.10 increase in price, she will sell 100 fewer wraps.

a. What is the price of a wrap that will maximize the revenue?

Recall: Revenue = (# of items sold)(price per item)

Let x = # of increases in price

$$R = (2000 - 100x)(1.50 + 0.10x)$$
$$= 3000 + 200x - 150x - 10x^2$$
$$= -10x^2 + 50x + 3000$$
$$= -10(x^2 - 5x) + 3000$$
$$= -10\left(x^2 - 5x + \frac{25}{4} - \frac{25}{4}\right) + 3000$$

$\frac{1}{2}(-5) = -\frac{5}{2}$

$\left(-\frac{5}{2}\right)^2 = \frac{25}{4}$

$$= -10\left(x - \frac{5}{2}\right)^2 + \frac{250}{4} + 3000$$
$$= -10(x - 2.5)^2 + 3062.5$$

$$\underbrace{x = 2.5 \qquad y = 3062.5}_{\text{vertex!!}}$$

b. What is the maximum revenue?

Max Revenue = $\boxed{\$3062.50}$

when there are 2.5 increases in price.

$\rightarrow 2.5(10¢) = 25¢$ increase