4.6 Analyzing Quadratic Functions of the Form $y = ax^2 + bx + c$

We will use the quadratic function $y = -2.5x^2 - 7.5x + 10$ to demonstrate how to find characteristics of the graph.

When the equation of a quadratic function is in general form, $y = ax^2 + bx + c$, the y-intercept is the constant term, ___C___ (where $x=0$)

If the equation is factorable, the x-intercepts can be found by factoring.

$y = -2.5x^2 - 7.5x + 10$

$y = -2.5(0)^2 - 7.5(0) + 10$

y-intercept = 10

$y = -2.5(x^2 + 3x - 4)$

$y = -2.5(x+4)(x-1)$

$x+4=0$ or $x-1=0$

$x = -4$, $x = 1$ — x-intercepts

When $y = ax^2 + bx + c$ can be written as ___$y = a(x-x_1)(x-x_2)$___, then the x-intercepts of the graph are x_1 and x_2

The *x*-intercepts are symmetrical about the axis of symmetry, so they can be used to calculate the constant term in the equation of the axis of symmetry.

The equation of the axis of symmetry is:

$$x = \frac{x_1 + x_2}{2}$$

$$x = \frac{-4+1}{2}$$

$x = \frac{-3}{2} = -1.5$ AXIS OF SYMMETRY

The constant term in the equation of the axis of symmetry is the ___x-coordinate___ of the vertex. So the *y*-coordinate of the vertex can then be determined by substituting in the equation of the quadratic function.

$y = -2.5x^2 - 7.5x + 10$

$y = -2.5(-1.5)^2 - 7.5(1.5) + 10$

$y = 15.625$

∴ Vertex is $(-1.5, 15.625)$

Note: You can use the discriminant to check if the equation factors (the discriminant will be a perfect square). If it does not factor, complete the square to determine the coordinates of the vertex of the graph.

Example #1: Sketch the graph of each quadratic function:

a. $y = \overset{a}{-2}x^2 + \overset{b}{10}x \overset{c}{-14}$

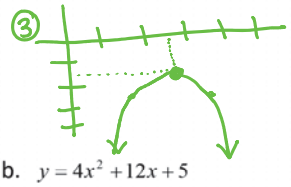
① Check to see if it will factor using the discriminant:

$b^2 - 4ac$

$= 10^2 - 4(-2)(-14)$

$= -12$ → not a perfect square so it doesn't factor. We must complete the square!

③



② $y = -2x^2 + 10x - 14$

$y = -2(x^2 - 5x) - 14$ ($\frac{1}{2}(-5) = \frac{-5}{2}$; $(\frac{-5}{2})^2 = \frac{25}{4}$)

$y = -2(x^2 - 5x + \frac{25}{4} - \frac{25}{4}) - 14$

$y = -2(x^2 - 5x + \frac{25}{4}) + \frac{50}{4} - 14$

$y = -2(x - \frac{5}{2})^2 + \frac{25}{2} - \frac{28}{2}$

$y = -2(x - \frac{5}{2})^2 - \frac{3}{2}$

$y = -2(x - 2.5)^2 - 1.5$

∴ vertex $(2.5, -1.5)$

b. $y = 4x^2 + 12x + 5$

① Check to see if factorable:

$b^2 - 4ac$

$= 12^2 - 4(4)(5)$

$= 64$ → perfect square so it is factorable!

$4(5) = 20$ (10, 2)

② $y = 4x^2 + 12x + 5$

$y = 4x^2 + 10x + 2x + 5$

$y = 2x(2x + 5) + (2x + 5)$

$y = (2x + 1)(2x + 5)$

$x = -\frac{1}{2}$ $\quad x = -\frac{5}{2}$

$x = -0.5 \quad x = -2.5$

x-intercepts

③ Axis of symmetry: $x = \frac{x_1 + x_2}{2}$

$x = \frac{-0.5 + -2.5}{2}$

$x = -1.5$

④ Find vertex when $x = -1.5$

$y = 4(-1.5)^2 + 12(-1.5) + 5$

$y = -4$

∴ vertex $(-1.5, -4)$

⑤