

4.4 Quadratic Functions of the Form $y = a(x - p)^2 + q$ Part II

Example #1: For the quadratic function $y = \frac{1}{4}(x - 3)^2 + 1$

a) Identify:

- I. the direction of opening

up

- II. the vertex

(3, 1)

- III. the equation of the axis of symmetry

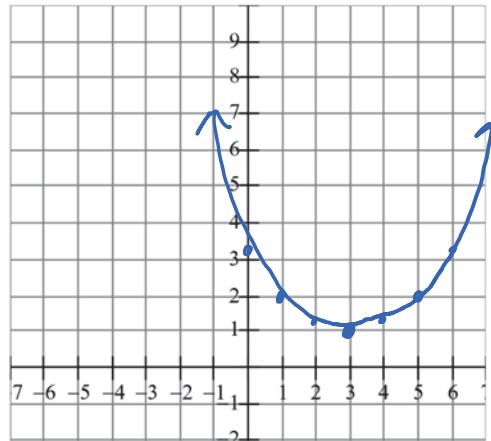
$x = 3$

- IV. the domain and range of the function

$x \in \mathbb{R}$

$y \geq 1$

b) Sketch the graph



Steps: 1, 3, 5

$$\times \frac{1}{4} : 0.25, 0.75, 1.25$$

Example #2: Determine a quadratic function that has a vertex at $(-1, -3)$ and passes through the point $(1, 5)$.

$\begin{matrix} x \\ y \end{matrix}$

$$y = a(x - p)^2 + q \rightarrow \text{sub in vertex } (-1, -3) \text{ and point } (1, 5)$$

$$5 = a(1 - (-1))^2 - 3$$

$$5 = a(2)^2 - 3$$

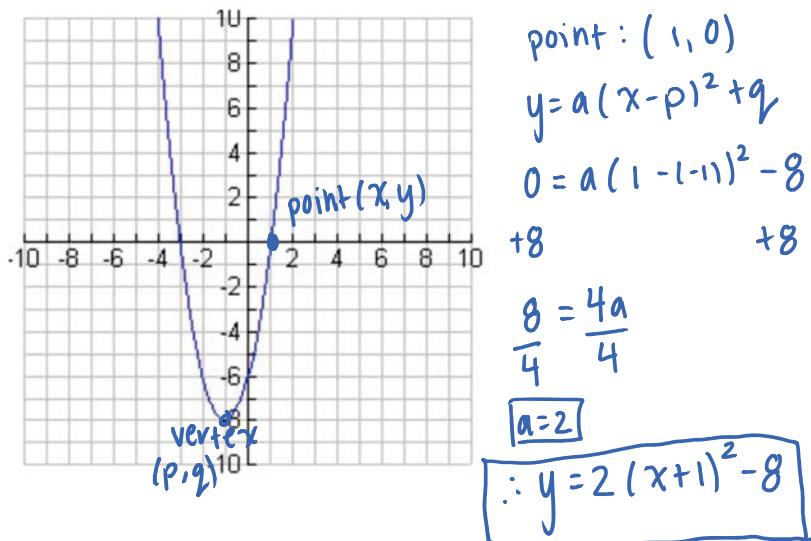
$$+3$$

$$\frac{8}{4} = \frac{4a}{4}$$

$$\boxed{a = 2}$$

$$\boxed{y = 2(x + 1)^2 - 3}$$

Example #3: The graph of a quadratic function is shown. What is the equation of the function?



4.5 Equivalent Forms of the Equation of a Quadratic Function

When the equation of a quadratic function is in general form $y = ax^2 + bx + c$, most characteristics of the graph cannot be identified. For this reason, we will convert from general form to standard form $y = a(x-p)^2 + q$ by completing the square.

Example #1: Determine the coordinates of the vertex of the parabola with equation $y = 3x^2 - 12x + 7$

*Always factor out leading coefficient on x^2 *

$$\begin{aligned}
& \frac{1}{2}(-4)^2 = 2 \\
& (-2)^2 = 4 \\
y &= 3(x^2 - 4x) + 7 \\
y &= 3(x^2 - 4x + 4 - 4) + 7 \\
y &= 3(x^2 - 4x + 4) - 12 + 7 \\
y &= 3(x-2)^2 - 5
\end{aligned}$$

↓ ↓
"p" "q"
(opposite)

$\therefore \text{vertex } (2, -5)$