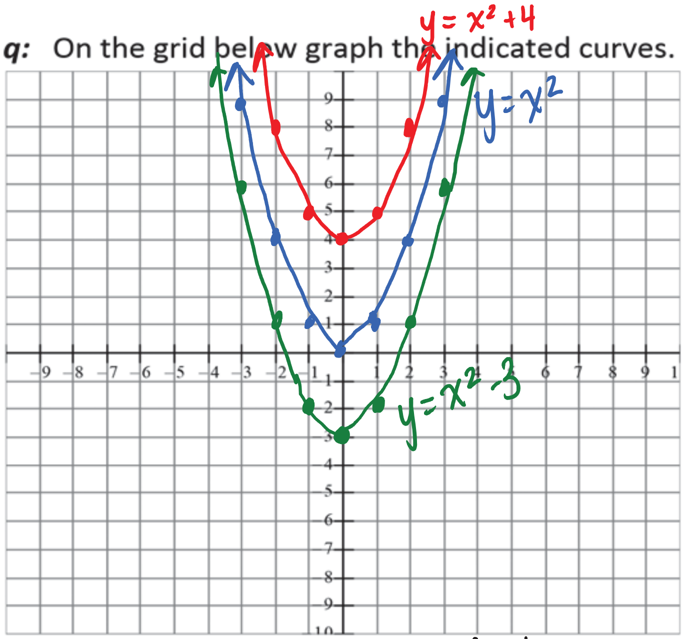
4.4 Quadratic Functions of the Form $y = a(x-p)^2 + q$ Part I

Investigating $y = x^2 + q$: On the grid below graph the indicated curves.

$y = x^2$ → Steps 1, 3, 5

$y = x^2 + 4$

$y = x^2 - 3$



What do you notice about the graphs? The shape of the graphs are the same, the graphs are just moved up or down.

In general the graph of $y = x^2 + q$ is congruent to the graph of $y = x^2$.

- If $q > 0$ the graph is translated q units up
- If $q < 0$ the graph is translated q units down

Example #1: Sketch the graph of $y = x^2 - 7$ on the grid below and answer the following questions.

Vertex: (0, −7)

Max or Min: min.

Axis of Symmetry: $x = 0$

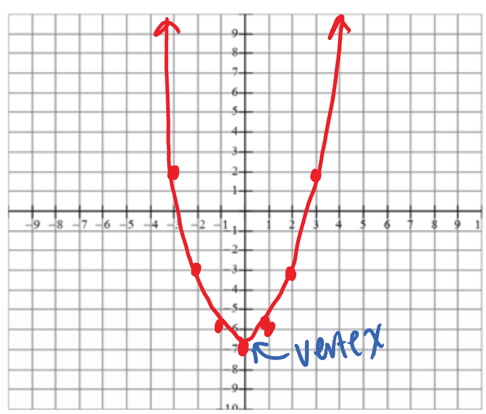
Domain: $x \in \mathbb{R}$

Range: $y \geq -7$

y-int → set $x = 0$

$y = 0^2 - 7$

$y = -7$



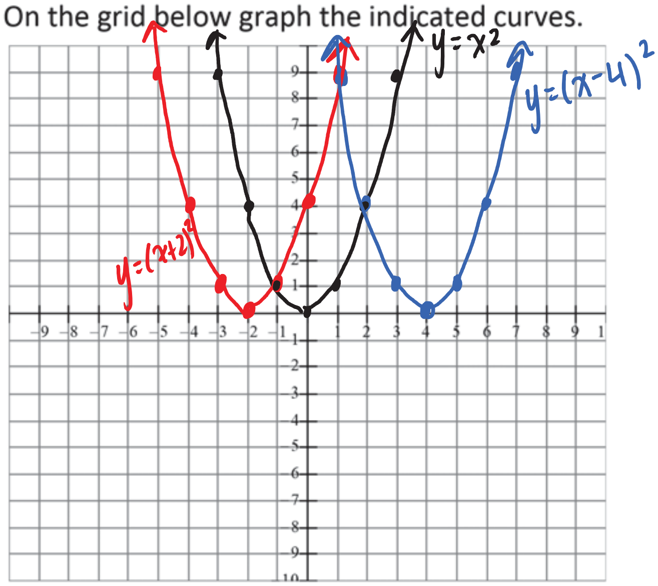
steps: 1, 3, 5 "Y steps"

Investigating $y = (x - p)^2$ On the grid below graph the indicated curves.

$y = x^2$

$y = (x + 2)^2$

$y = (x - 4)^2$

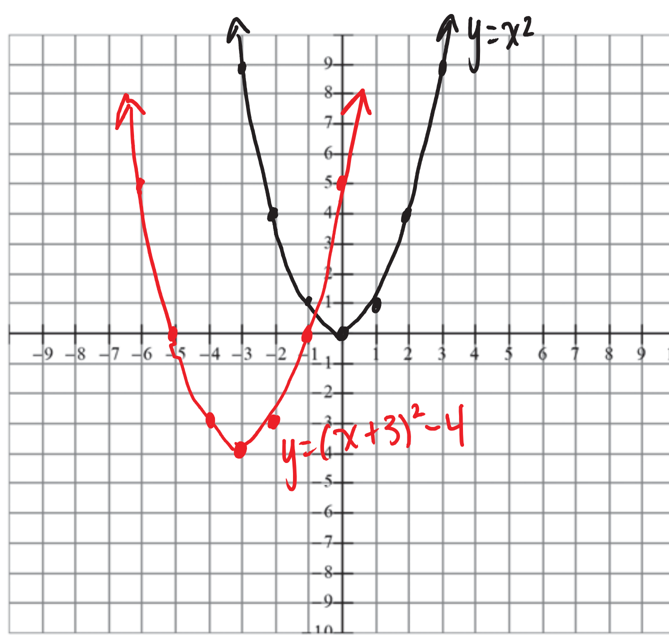


What do you notice about the graphs? The graphs are the same shape, but the graphs are moved left/right.

In general the graph of $y = (x - p)^2$ is congruent to the graph of $y = x^2$.

- If p > 0 the graph is translated p units right "Do the opposite"
- If p < 0 the graph is translated p units left

Example #2: Sketch the graph of the equation $y = (x + 3)^2 - 4$ by translating the graph of $y = x^2$.



Vertex: (-3,-4) ← 3 ↓ 4

Axis of Symmetry: x = -3

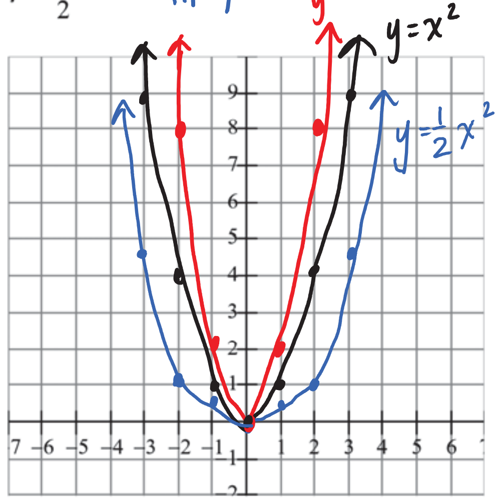
Domain: x∈R

Range: y ≥ -4

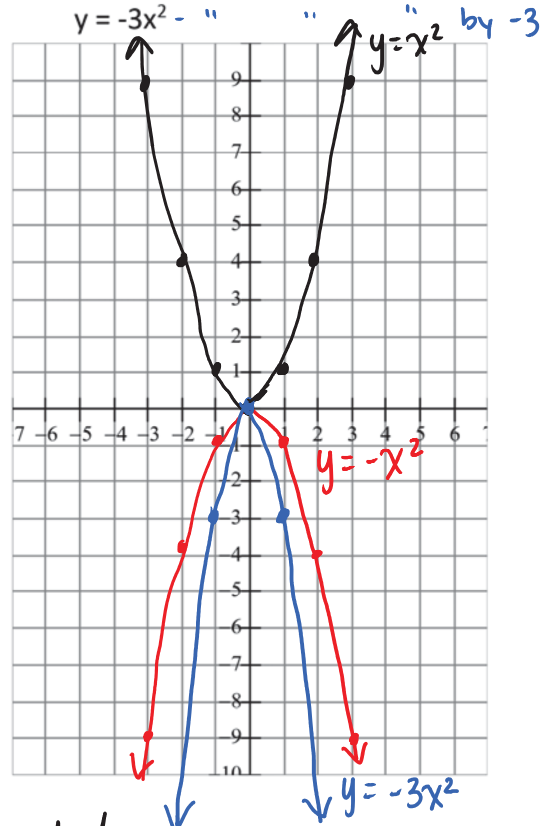
Steps: 1, 3, 5

Investigating $y = ax^2$ Graph the following equations on the axes provided.

$y = x^2$
$y = 2x^2$ -multiply original values by 2
$y = \frac{1}{2}x^2$ -multiply " $y=2x^2$ " by $\frac{1}{2}$



$y = x^2$
$y = -x^2$ - multiply original values by -1
$y = -3x^2$ - " " " by -3



When *a* is positive, the graph opens up ⇑ .

When *a* is negative, the graph opens down ⇓ .

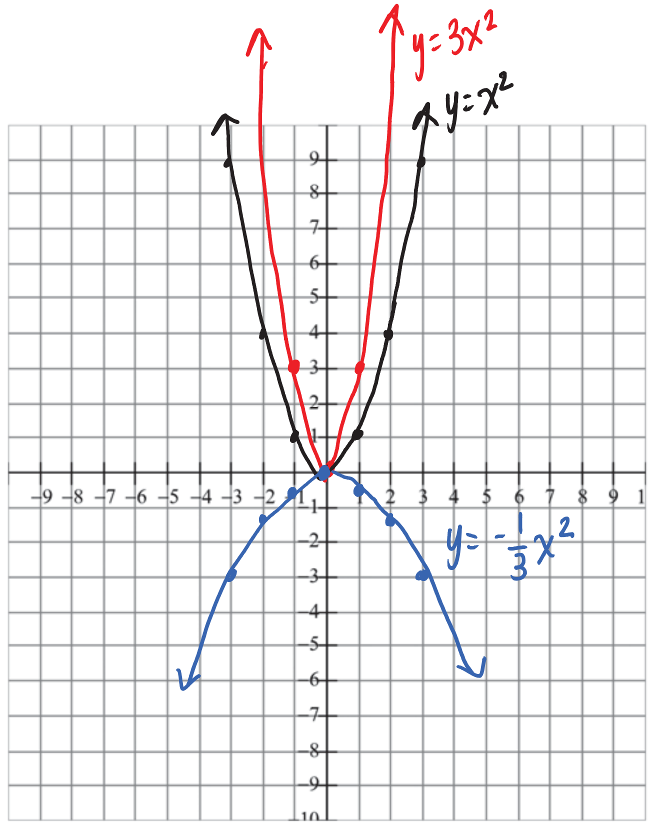
When $a > 1$ or $a < -1$, the graph is expanded vertically.

When $-1 < a < 1$ $(a \neq 0)$ the graph is compressed vertically.

In general, for the function $y = x^2$ the graph of $y = ax^2$, where a is any real number, is obtained by multiply the "y" value of the original $y=x^2$ graph by "a" .

Example #3: Sketch the following graphs.

$y = 3x^2$ and $y = -\frac{1}{3}x^2$



Summary

A quadratic function can be expressed in standard form as follows:

"vertex form"

$$y = a(x - p)^2 + q$$

a negative flips the parabola

moves the parabola left or right ("think opposite")

stretches or compresses the parabola

moves parabola up or down

- The coordinates of the vertex are (p, q)
- The equation of the axis of symmetry is x = p
- The parabola is congruent to $y = ax^2$.
- The parabola opens up if a is positive and opens down if a is negative.
- The constant p moves the graph left or right from the origin.
- The constant q moves the graph up or down from the origin.

Example #4:

Graph the function $y = 2(x-4)^2 - 3$.

a p q

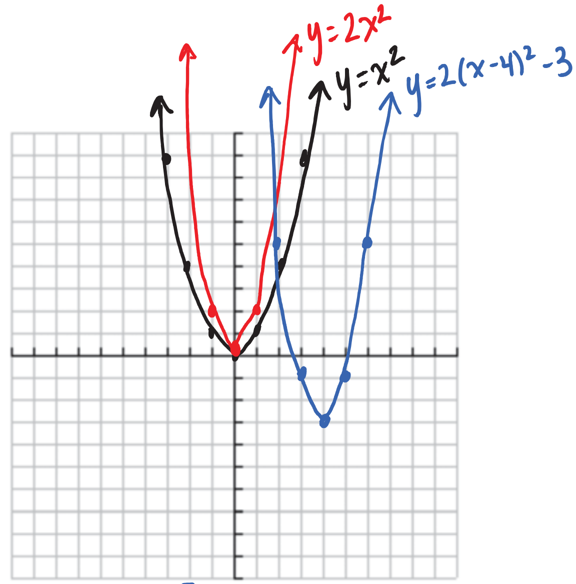
→ ↓3

① Start with $y = x^2$

② Graph $y = 2x^2$

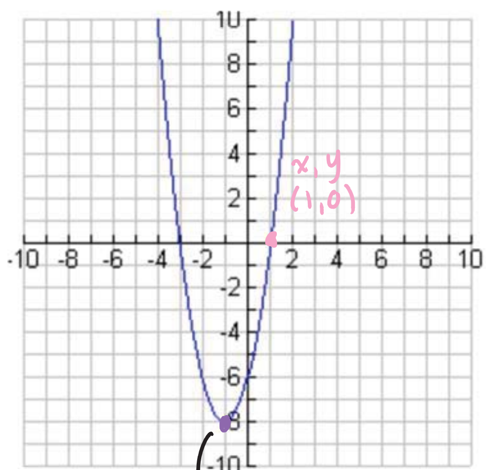
↳ multiply values by 2

③ move 4 right, 3 down



steps: 1, 3, 5
x 2 : 2, 6, 10

Example #5: The graph of a quadratic function is shown. What is the equation of the function?



vertex (-1, -8)

p = -1, q = -8

$y = a(x-p)^2 + q$

$y = a(x+1)^2 - 8$

TO FIND "a":

① sub in vertex (p, q)

② sub in (x, y) point

$0 = a(1+1)^2 - 8$

$0 = a(2)^2 - 8$

+8 +8

$8 = 4a$

$\frac{8}{4} = \frac{4a}{4}$

$a = 2$

$y = 2(x+1)^2 - 8$

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