## 3.5 Interpreting the Discriminant

Investigate: Using the quadratic formula, determine the solutions to the following:

| $2x^2+5x-3$ | $9x^2+6x+1$ | $3x^2+2x+4$ |
| --- | --- | --- |
| $a=2$ $b=5$ $c=-3$ <br> $x=\frac{-5\pm\sqrt{5^2-4(2)(-3)}}{2(2)}$ <br> $x=\frac{-5\pm\sqrt{49}}{4}$ <br> $x=\frac{-5\pm 7}{4}$ <br> $x=\frac{1}{2}$ $x=-3$ | $a=9$ $b=6$ $c=1$ <br> $x=\frac{-6\pm\sqrt{6^2-4(9)(1)}}{2(9)}$ <br> $x=\frac{-6\pm\sqrt{0}}{18}$ <br> $x=-\frac{1}{3}$ | $a=3$ $b=2$ $c=4$ <br> $x=\frac{-2\pm\sqrt{2^2-4(3)(4)}}{2(3)}$ <br> $x=\frac{-2\pm\sqrt{-44}}{6}$ can't sq. root a neg! <br> ∴ no real roots |

Depending of the quadratic equation there are 3 types of possible solutions. The expression $b^2-4ac$, is called the discriminant of the quadratic equation, because it discriminates among the types of possible solutions.

**Number of Roots of a Quadratic Equation**

The quadratic equations $ax^2+bx+c=0$ has:

- two real roots when $b^2-4ac>0$
- exactly one real root when $b^2-4ac=0$
- no real roots when $b^2-4ac<0$

**Example #1:** Without solving, determine whether the equation $9x^2-6x+1=0$ has one, two, or no real roots.

$a=9$ $b=-6$ $c=1$

$b^2-4ac$

$=(-6)^2-4(9)(1)$

$=36-36$

$=0$ ∴ It has one real root.

**Example #2:**

a. Determine the values of $k$ for which $2x^2 + 7x + k = 0$ has no real roots.

$a=2 \quad b=7 \quad c=k$

no real roots mean:

$b^2 - 4ac < 0$

$(7)^2 - 4(2)(k) < 0$

$49 - 8k < 0$

$+8k \quad +8k$

$\frac{49}{8} < \frac{8k}{8}$

$\boxed{\frac{49}{8} < k \text{ or } k > 6\tfrac{1}{8}}$

$\rightarrow \frac{-8k}{-8} < \frac{-49}{-8}$

$k > \frac{49}{8}$

∴ Recall: when you divide by a negative, you switch the sign!

b. Use one value of $k$ to write an equation that has no real roots.

no real roots when $k > 6\frac{1}{8}$

Let $k = 7$

$2x^2 + 7x + 7 = 0$

$a=2 \quad b=7 \quad c=7$

$x = \frac{-7 \pm \sqrt{(7)^2 - 4(2)(7)}}{2(2)}$

$x = \frac{-7 \pm \sqrt{49 - 56}}{4}$

$x = \frac{-7 \pm \sqrt{-7}}{4}$ → negative!

∴ no real roots