3.4 Developing and Applying the Quadratic Formula

When a quadratic equation is solved by completing the square, a formula is generated that can be used to solve any quadratic equation.

$$\frac{ax^2+bx+c=0}{a} \quad \leftarrow \text{dividing "a" from both sides}$$

$$x^2+\frac{b}{a}x+\frac{c}{a}=0 \quad \leftarrow \text{complete the square}$$

$\frac{1}{2}\left(\frac{b}{a}\right)=\frac{b}{2a}$

$\left(\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}$

$$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}-\frac{b^2}{4a^2}+\frac{c}{a}=0$$

$$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=\frac{b^2}{4a^2}-\frac{c}{a} \quad \cdot 4a \quad \cdot 4a$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{4ac}{4a^2} \quad \leftarrow \text{factor LS, simplify}$$

$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}} \quad \leftarrow \text{sq. root both sides}$$

$$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{\sqrt{4a^2}}$$

$$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

The Quadratic Formula

The solution of a quadratic equation, $ax^2+bx+c=0$, where a, b, and c are constant and $a\neq 0$, is given by the quadratic formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Example: Solve each equation.

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

a) $x^2+2x+7=0$

$a=1 \quad b=2 \quad c=7$

$$x = \frac{-(2) \pm \sqrt{(2)^2-4(1)(7)}}{2(1)}$$ ← sub in values for a, b, c

$$x = \frac{-2 \pm \sqrt{4-28}}{2}$$ ← simplify

$$x = \frac{-2 \pm \sqrt{-24}}{2}$$ ← can't sq. root a negative ∴ no real roots!

b) $x^2-2x-7=0$

$a=1 \quad b=-2 \quad c=-7$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2-4(1)(-7)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4+28}}{2}$$

$$x = \frac{2 \pm \sqrt{32}}{2}$$ → simplify radical!

$$x = \frac{2 \pm \sqrt{16 \cdot 2}}{2}$$

$$x = \frac{2 \pm 4\sqrt{2}}{2}$$ → All terms are divisible by 2! simplify → use the "÷" rule!

$$\boxed{x = 1 \pm 2\sqrt{2}}$$

c) $2x = 3(x-1)(x+1)$

$2x = 3(x^2 + x - x - 1)$

$2x = 3(x^2 - 1)$

$2x = 3x^2 - 3$

$-2x \quad -2x$

$0 = 3x^2 - 2x - 3$

$a=3 \quad b=-2 \quad c=-3$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-3)}}{2(3)}$$

$$x = \frac{2 \pm \sqrt{4 + 36}}{6}$$

$$x = \frac{2 \pm \sqrt{40}}{6}$$

$$x = \frac{2 \pm \sqrt{4 \cdot 10}}{6}$$

$$x = \frac{2 \pm 2\sqrt{10}}{6}$$

$$\boxed{x = \frac{1 \pm \sqrt{10}}{3}}$$

d) $\frac{2}{3}x^2 + 1 = \frac{5}{6}x$

$-\frac{5}{6}x \quad -\frac{5}{6}x$

$6 \times \left(\frac{2}{3}x^2 - \frac{5}{6}x + 1\right) = 0 \times 6$

$\frac{12}{3}x^2 - \frac{30}{6} + 6 = 0$

$4x^2 - 5x + 6 = 0$

$a=4 \quad b=-5 \quad c=6$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(6)}}{2(4)}$$

$$x = \frac{5 \pm \sqrt{25 - 96}}{8}$$ ← negative!

$\therefore$ no real roots