3.3 Using Square Roots to Solve Quadratic Equations

In a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, when $b=0$ it becomes the equation $ax^2 + c = 0$. If this equation has a solution, it can be solved by using square roots.

Example #1: Solve each equation. Verify the solution.

a) $3x^2 - 7 = 8$

$+7 \quad +7$

$\frac{3x^2}{3} = \frac{15}{3}$

$\sqrt{x^2} = \sqrt{5}$

$\boxed{x = \pm\sqrt{5}}$

Check:

$3x^2 - 7 = 8$

$3(\sqrt{5})^2 - 7 = 8$

$3(5) - 7 = 8$

$15 - 7 = 8$

$8 = 8$ ✓

$3x^2 - 7 = 8$

$3(-\sqrt{5})^2 - 7 = 8$

$3(5) - 7 = 8$

$15 - 7 = 8$

$8 = 8$ ✓

b) $\sqrt{(x+3)^2} = \sqrt{20}$

$x + 3 = \pm\sqrt{20}$

$-3 \quad -3$

$\boxed{x = \pm\sqrt{20} - 3}$

Check:

$(x+3)^2 = 20$

$((\cancel{-3} + \sqrt{20}) + \cancel{3})^2 = 20$

$(\sqrt{20})^2 = 20$

$20 = 20$ ✓

$(x+3)^2 = 20$

$[(\cancel{-3} - \sqrt{20} + \cancel{3}]^2 = 20$

$(-\sqrt{20})^2 = 20$

$20 = 20$ ✓

Not all quadratic equations have roots that are real roots.

$3x^2 + 18 = 0$

$\frac{3x^2}{3} = \frac{-18}{3}$

$\sqrt{x^2} = \sqrt{-6}$ not a real # !

$x = \pm\sqrt{-6}$

∴ we say this has no real roots since $x^2 \geq 0$

Some quadratic equations, for example $2x^2 - 20x + 6 = 0$, cannot be solved by factoring. We use the strategy of completing the square to try to solve the equation.

Steps for Completing the Square: $2x^2 - 20x + 6 = 0$

1. Remove the coefficient of x^2 term by multiplying or dividing

$$\frac{2x^2 - 20x + 6}{2} = \frac{0}{2}$$

$$x^2 - 10x + 3 = 0$$

2. Divide the x term by 2, square it. Now add and subtract the result. (Note that this is the equivalent of adding zero.)

$(\frac{1}{2})(-10) = \frac{-10}{2} = -5$

$\hookrightarrow (-5)^2 = 25$

$$x^2 - 10x + 3 = 0$$

$$(\underbrace{x^2 - 10x + 25}, -25) + 3 = 0$$

$$(x^2 - 10x + 25) - 22 = 0$$

3. Factor the perfect square.

$$(x-5)(x-5) - 22 = 0$$

$$(x-5)^2 - 22 = 0$$

4. Isolate the perfect square and take the square root of both sides to solve.

$$(x-5)^2 - 22 = 0$$

$$+22 \quad +22$$

$$\sqrt{(x-5)^2} = \sqrt{22}$$

$$x - 5 = \pm\sqrt{22}$$

$$+5 \quad +5$$

$$\boxed{x = 5 \pm \sqrt{22}}$$

Example 2: Solve each equation by completing the square.

a) $x^2 + 4x - 3 = 0$

$\frac{1}{2}(4) = 2 \to 2^2 = 4$

$x^2 + 4x + 4 - 4 - 3 = 0$

$(x^2 + 4x + 4) - 7 = 0$

$(x+2)^2 - 7 = 0$

$+7 \quad +7$

$\sqrt{(x+2)^2} = \sqrt{7}$

$x + 2 = \pm\sqrt{7}$

$-2 \quad -2$

$\rightarrow \boxed{x = -2 \pm \sqrt{7}}$

b) $\frac{-5x^2 - 10x + 2}{-5} = \frac{0}{-5}$ * always make sure the leading coefficient is positive 1

$\frac{1}{2}(2) = 1 \to 1^2 = 1$

$x^2 + 2x - \frac{2}{5} = 0$

$x^2 + 2x + 1 - 1 - \frac{2}{5} = 0$

$(x^2 + 2x + 1) \frac{-5}{5} - \frac{2}{5} = 0$

$(x+1)^2 - \frac{7}{5} = 0$

$(x+1)^2 = \frac{7}{5}$

$x + 1 = \pm\sqrt{\frac{7}{5}}$

$-1 \quad -1$

$\boxed{x = -1 \pm \sqrt{\frac{7}{5}}}$

c) $2 \times \left(\frac{1}{2}x^2 + 3x - \frac{9}{2}\right) = 0 \times 2$

$\frac{2(x^2 + 6x - 9)}{2} = \frac{0}{2}$

$\frac{1}{2}(6) = 3 \to 3^2 = 9$

$(x^2 + 6x + 9) - 9 - 9 = 0$

$(x^2 + 6x + 9) - 18 = 0$

$(x+3)^2 - 18 = 0$

$\sqrt{(x+3)^2} = \sqrt{18}$

$x + 3 = \pm\sqrt{18}$

$\boxed{x = -3 \pm \sqrt{18}}$