

3.2 Solving Quadratic Equations by Factoring**Quadratic Equation**

A quadratic equation is any equation that can be written in the form $ax^2 + bx + c = 0$, where a, b, c are constants and $a \neq 0$

When an equation contains quadratic (x^2) and linear (x) terms it cannot be solved by isolating the variable. The strategy we must use depends on the following Zero Product Property:

- If the product of two numbers is zero, then either number or both numbers equals zero.
- That is, if $ab = 0$, then $a = 0$, or $b = 0$, or both.

Example #1: Solve each equation, then verify the solution.

a) $x^2 + x - 56 = 0$

$(x+8)(x-7) = 0$

$$\begin{array}{c} -56 \\ / \quad \backslash \\ 8 \quad -7 \end{array}$$

$$\begin{array}{l} x+8=0 \quad \text{or} \quad x-7=0 \\ -8 \quad -8 \quad \quad \quad +7 \quad +7 \\ \boxed{x=-8} \quad \quad \quad \boxed{x=7} \end{array}$$

check:

$$\begin{array}{l} x^2 + x - 56 = 0 \\ (-8)^2 + (-8) - 56 = 0 \\ 64 - 8 - 56 = 0 \\ 0 = 0 \\ \checkmark \end{array}$$

$$\begin{array}{l} x^2 + x - 56 = 0 \\ 7^2 + 7 - 56 = 0 \\ 49 + 7 - 56 = 0 \\ 0 = 0 \\ \checkmark \end{array}$$

b) $(3x + 1)(x - 6) = 22$

$3x^2 - 18x + x - 6 = 22$

$3x^2 - 17x - 6 = 22$
 $-22 \quad -22$

$3x^2 - 17x - 28 = 0$

$3x^2 + 4x - 21x - 28 = 0$

$x(3x + 4) - 7(3x + 4) = 0$

$(x - 7)(3x + 4) = 0$

$x - 7 = 0$ or $3x + 4 = 0$

$x = 7$

$-4 \quad -4$
 $\frac{3x}{3} = \frac{-4}{3}$

$x = \frac{-4}{3}$

* Always set equation equal to zero*

$3(-28) = -84$

$4 \quad -21$

Check:

$(3(7) + 1)(7 - 6) = 22$

$(22)(1) = 22$

$22 = 22$

✓

$[3(\frac{-4}{3}) + 1][(\frac{-4}{3}) - 6] = 22$

$(-4 + 1)(\frac{-4}{3} - 6) = 22$

$(-3)(\frac{-22}{3}) = 22$

$22 = 22$

✓

c) $3x^2 + 75 = -30x$

$3x^2 + 30x + 75 = 0$

$3(x^2 + 10x + 25) = 0$

$3(x + 5)(x + 5) = 0$

$x + 5 = 0$ or $x + 5 = 0$

$x = -5$

$x = -5$

* Always set equation to zero*

Check:

$3(5)^2 + 75 = -30(-5)$

$3(25) + 75 = 150$

$150 = 150$

✓

d) $5x^2 = -20x$

$5x^2 + 20x = 0$

$5x(x + 4) = 0$

$5x = 0$ or $x + 4 = 0$

$x = 0$

$x = -4$

check:

$5(0) = -20(0)$

$0 = 0$

✓

$5(-4)^2 = -20(-4)$

$5(16) = 80$

$80 = 80$

✓

→ isolate radical first!

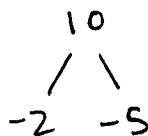
e) $\sqrt{6-x} + 4 = x$
 $\quad -4 \quad -4$

$(\sqrt{6-x})^2 = (x-4)^2$
 $6-x = (x-4)(x-4)$

$6-x = x^2 - 4x - 4x + 16$
 $-6 + x \quad \quad +x \quad \quad -6$

$0 = x^2 - 7x + 10$

$0 = (x-5)(x-2)$



$x-5=0$

$x=5$

$x-2=0$

$x=2$

Check: ① Do they check restrictions? ✓
 ② check in original equation

$\sqrt{6-5} + 4 = 5$
 $\sqrt{1} + 4 = 5$
 $5 = 5$
 ✓

$\sqrt{6-2} + 4 = 2$
 $\sqrt{4} + 4 = 2$
 $2 + 4 = 2$
 $6 \neq 2$

∴ reject $\ddot{\wedge}$ → $x=2$ is not a solution!

Recall: state restrictions

$6-x \geq 0$

$-6 \quad -6$

$-x \geq -6$

$\frac{-x}{-1} \geq \frac{-6}{-1}$

$x \leq 6$

* when (x) or (÷) by a negative, reverse the inequality

All solutions of equations should be verified by substitution into the original equation. Sometimes a solution of a quadratic equation produces an extraneous root, which means the number is a root to the equation but is not a solution to the problem.