3.2 Solving Quadratic Equations by Factoring

Quadratic Equation

A quadratic equation is any equation that can be written in the form
$\boldsymbol{ax^2 + bx + c = 0}$, where a, b, c are constants and $a \neq 0$

When an equation contains quadratic (x^2) and linear (x) terms it cannot be solved by isolating the variable. The strategy we must use depends on the following Zero Product Property:

- If the product of two numbers is zero, then either number or both numbers equals zero.
- That is, if $ab=0$, then $a=0$, or $b=0$, or both.

Example #1: Solve each equation, then verify the solution.

a) $x^2 + x - 56 = 0$

-56: 8, -7

$(x+8)(x-7)=0$

$x+8=0$ or $x-7=0$

$-8 \quad -8$ $\qquad$ $+7 \quad +7$

$\boxed{x=-8}$ $\qquad$ $\boxed{x=7}$

check:

$x^2+x-56=0$

$(-8)^2+(-8)-56=0$

$64-8-56=0$

$0=0$ ✓

$x^2+x-56=0$

$7^2+7-56=0$

$49+7-56=0$

$0=0$ ✓

b) $(3x + 1)(x - 6) = 22$

$3x^2 - 18x + x - 6 = 22$

* Always set equation equal to zero*

$3x^2 - 17x - 6 = 22$
$\quad\quad\quad -22 \quad -22$

$3x^2 - 17x - 28 = 0$

$3(-28) = -84$

$4 \quad -21$

$\underbrace{3x^2 + 4x}\underbrace{-21x - 28} = 0$

* $x(3x + 4) - 7(3x + 4) = 0$

$(x - 7)(3x + 4) = 0$

$x - 7 = 0$ or $3x + 4 = 0$

$\boxed{x = 7}$

$\quad -4 \quad -4$

$\frac{3x}{3} = \frac{-4}{3}$

$\boxed{x = \frac{-4}{3}}$

Check:

$(3(7) + 1)(7 - 6) = 22$

$(22)(1) = 22$

$22 = 22$ ✓

$\left[3\left(\frac{-4}{3}\right) + 1\right]\left[\left(\frac{-4}{3}\right) - 6\right] = 22$

$(-4 + 1)\left(\frac{-4}{3} - 6\right) = 22$

$(-3)\left(\frac{-22}{3}\right) = 22$

$22 = 22$ ✓

c) $3x^2 + 75 = -30x$

$3x^2 + 30x + 75 = 0$

* Always set equation to zero*

$3(x^2 + 10x + 25) = 0$

$3(x + 5)(x + 5) = 0$

$x + 5 = 0$ or $x + 5 = 0$

$\boxed{x = -5}$ $\boxed{x = -5}$

Check:

$3(-5)^2 + 75 = -30(-5)$

$3(25) + 75 = 150$

$150 = 150$ ✓

d) $5x^2 = -20x$

$5x^2 + 20x = 0$

$5x(x + 4) = 0$

$5x = 0$ or $x + 4 = 0$

$\boxed{x = 0}$ $\boxed{x = -4}$

Check:

$5(0) = -20(0)$

$0 = 0$ ✓

$5(-4)^2 = -20(-4)$

$5(16) = 80$

$80 = 80$ ✓

isolate radical first!

e) $\sqrt{6-x}+4=x$
$-4 \quad -4$

$(\sqrt{6-x})^2=(x-4)^2$

$6-x=(x-4)(x-4)$

$6-x=x^2-4x-4x+16$
$-6+x \quad +x \quad -6$

$0=x^2-7x+10$

$0=(x-5)(x-2)$

10
-2 -5

$x-5=0$ $\quad$ $x-2=0$

$\boxed{x=5}$ $\quad$ $\boxed{x=2}$

Recall: state restrictions

$6-x\geq 0$
$-6 \quad -6$

$-x \geq -6$

$\frac{-x}{-1} \quad \frac{-6}{-1}$

$x\leq 6$

* when (×) or (÷) by a negative, reverse the inequality

Check: ① Do they check restrictions? ✓
② Check in original equation

$\sqrt{6-5}+4=5$
$\sqrt{1}+4=5$
$5=5$ ✓

$\sqrt{6-2}+4=2$
$\sqrt{4}+4=2$
$2+4=2$
$6\neq 2$

∴ reject → $x=2$ is not a solution!

All solutions of equations should be verified by substitution into the original equation. Sometimes a solution of a quadratic equation produces an extraneous root, which means the number is a root to the equation but is not a solution to the problem.