2.5 Solving Radical Equations

When solving a radical equations follow the following steps:

1. Identify and state any restrictions.
2. Isolate the radical on one side of the equation.
3. Square each side.
4. Solve for the variable in the remaining equation.
5. Perform a Check / Identify extraneous roots (roots that do not actually satisfy the original equation) and reject them.

Example #1: Solve each equation. Verify your solutions

a) $\frac{3}{4} = \frac{4\sqrt{x}}{4}$

Restrictions: $\boxed{x \geq 0}$

$\left(\frac{3}{4}\right)^2 = (\sqrt{x})^2$

$\boxed{\frac{9}{16} = x}$

Check:

$\frac{3}{4} = \frac{\cancel{4}\sqrt{\frac{9}{16}}}{\cancel{4}}$

$\frac{3}{4} = \frac{3}{4}$ ✓

b) $2\sqrt{x+1} - 7 = 13$

$+7 \quad +7$

Restrictions: $x + 1 \geq 0$

$-1 \quad -1$

$\boxed{x \geq -1}$

$\frac{\cancel{2}\sqrt{x+1}}{\cancel{2}} = \frac{20}{2}$

$(\sqrt{x+1})^2 = (10)^2$

$x + 1 = 100$

$-1 \quad -1$

$\boxed{x = 99}$

Check: $2\sqrt{99+1} - 7 = 13$

$2\sqrt{100} - 7 = 13$

$2(10) - 7 = 13$

$20 - 7 = 13$

$13 = 13$ ✓

c) $4\sqrt{x}+3=5\sqrt{x}+1$ Restrictions: $x \geq 0$

$-4\sqrt{x} \quad -4\sqrt{x}$

$3=\sqrt{x}+1$

$-1 \quad -1$

$(2)^2=(\sqrt{x})^2$

$\boxed{4=x}$

Check:

$4\sqrt{4}+3=5\sqrt{4}+1$

$4(2)+3=5(2)+1$

$8+3=10+1$

$11=11$ ✓

d) $(\sqrt{x-1})^2=(\sqrt{2x+3})^2$ Restrictions: $x-1 \geq 0 \rightarrow x \geq 1$

$2x+3 \geq 0 \rightarrow x \geq \frac{-3}{2} \rightarrow -1.5$

$x \geq 1$ → more restrictive so it trumps the other one!

$x-1=2x+3$

$-x \quad -x$

$-1=x+3$

$-3 \quad -3$

$\boxed{-4=x}$

NOTE: this is not greater than one, so it is rejected

Check:

$\sqrt{-4-1}=\sqrt{2(-4)+3}$

$\sqrt{-5}=\sqrt{-8+3}$

→ can't take the sq. root of a negative!

*we call the "rejected" answers <u>extraneous roots</u>