2.4 Multiplying and Dividing Radical Expressions

When multiplying radicals, you multiply coefficient by coefficient and radicand by radicand. You can only multiply radicals with the same index.

Example #1: Multiply the following. Answer in simplest form.

a) $2\sqrt{3} \times 4\sqrt{6}$

$= 2 \times 4\sqrt{3 \times 6}$

$= 8\sqrt{18}$

$= 8\sqrt{9 \cdot 2}$

$= 8(3)\sqrt{2}$

$= \boxed{24\sqrt{2}}$

b) $3\sqrt[3]{2x} \cdot 7\sqrt[3]{5x^2}$

$= 21\sqrt[3]{10x^3}$

$= \boxed{21x\sqrt[3]{10}}$

c) $(\sqrt{3} - 4\sqrt{5})(2 + \sqrt{5})$

$= 2\sqrt{3} + \sqrt{15} - 8\sqrt{5} - 4(5)$

$\sqrt{5} \cdot \sqrt{5} = \sqrt{25}$

$= 5$

$= \boxed{2\sqrt{3} + \sqrt{15} - 8\sqrt{5} - 20}$

FOIL

$(a+b)(c+d)$

Last
Inner

$(a+b)(c+d) = ac + ad + bc + bd$

F O I L

First
Outer

Note: When radicals have variables in the radicand, it is important to identify the values of the variable for which the expression is undefined. Any restriction on the variables should be determined from the original expression and not its simplified expression.

recall: difference of squares $(a+b)(a-b) = a^2 - b^2$

Example #2: Identify the values of the variable for which the expression is undefined, ~~then~~ expand and simplify.

$(3\sqrt{x} + \sqrt{y})(3\sqrt{x} - \sqrt{y}) - (\sqrt{x} + 5\sqrt{y})^2$

$x \geq 0$, $y \geq 0$ } restrictions

$= 9x - \cancel{3\sqrt{xy}} + \cancel{3\sqrt{xy}} - y - [(\sqrt{x} + 5\sqrt{y})(\sqrt{x} + 5\sqrt{y})]$

$= 9x - y - [x + 5\sqrt{xy} + 5\sqrt{xy} + 25y]$

$= 9x - y - x - 5\sqrt{xy} - 5\sqrt{xy} - 25y$

$= 8x - 26y - 10\sqrt{xy}$

Example #3: Divide the following. Answer in simplest form.

a) $\frac{8\sqrt{15}}{2\sqrt{3}}$

$= \frac{8}{2}\frac{\sqrt{15}}{\sqrt{3}}$

$= 4\sqrt{\frac{15}{3}}$

$= 4\sqrt{5}$

b) $\frac{2\sqrt{20}}{8\sqrt{5}}$

$= \frac{2}{8}\sqrt{\frac{20}{5}}$

$= \frac{1}{4}\sqrt{4}$

$= \frac{1}{4}\cdot(2)$

$= \frac{2}{4}$

$= \frac{1}{2}$

c) $\frac{\sqrt{24x^2}}{\sqrt{3x}}$

$= \sqrt{8x}$

$= \sqrt{4\cdot 2x}$

$= 2\sqrt{2x}$

A radical is in lowest terms when there is no radical in the denominator. To simplify an expression that has a radical in the denominator, you need to rationalize the denominator.

To Rationalize a Monomial Denominator:
1. Multiply top and bottom by the radical in the denominator
2. Simplify

Example #4: Simplify:

a) $\frac{(5\sqrt{7}+3)}{\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}}$

$= \frac{5(7)+3\sqrt{7}}{7}$

$= \frac{35+3\sqrt{7}}{7}$

b) $\frac{6\sqrt{2}-4\sqrt{3}}{\sqrt{18}}$

*always try to simplify denominator before rationalizing!

$= \frac{6\sqrt{2}-4\sqrt{3}}{\sqrt{9\cdot 2}}$

$= \frac{(6\sqrt{2}-4\sqrt{3})}{3\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{6(2)-4\sqrt{6}}{3(2)}$

$= \frac{\cancel{12}-\cancel{4}\sqrt{6}}{\cancel{6}}$

*if all 3 are divisible, then simplify!

$= \frac{6-2\sqrt{6}}{3}$

To Rationalize a Binomial Denominator

1. Determine a conjugate of the denominator.
2. Multiply top and bottom by the conjugate.
3. Simplify

The **conjugate** of $a+b$ is $\underline{a-b}$

Note that the product of conjugates is a difference of squares $(a+b)(a-b)=a^2-b^2$

Example #5: Simplify.

a) $\frac{(5\sqrt{3})}{(4-\sqrt{6})} \cdot \frac{(4+\sqrt{6})}{(4+\sqrt{6})}$

$= \frac{20\sqrt{3}+5\sqrt{18}}{16+\cancel{4\sqrt{6}}-\cancel{4\sqrt{6}}-6}$

$= \frac{20\sqrt{3}+5\sqrt{9\cdot 2}}{10}$

$= \frac{20\sqrt{3}+5(3)\sqrt{2}}{10}$

$= \frac{20\sqrt{3}+15\sqrt{2}}{10}$

$= \frac{4\sqrt{3}+3\sqrt{2}}{2}$

b) $\frac{(2\sqrt{3}-4\sqrt{6})}{(3\sqrt{2}+2\sqrt{3})} \cdot \frac{(3\sqrt{2}-2\sqrt{3})}{(3\sqrt{2}-2\sqrt{3})}$

$= \frac{6\sqrt{6}-4(3)-12\sqrt{12}+8\sqrt{18}}{9(2)+\cancel{6\sqrt{6}}-\cancel{6\sqrt{6}}-4(3)}$

$= \frac{6\sqrt{6}-12-12\sqrt{4\cdot 3}+8\sqrt{9\cdot 2}}{18-12}$

$= \frac{6\sqrt{6}-12-12(2)\sqrt{3}+8(3)\sqrt{2}}{6}$

$= \frac{6\sqrt{6}-12-24\sqrt{3}+24\sqrt{2}}{6}$ $\quad$ * every term is divisible by 6

$= \sqrt{6}-2-4\sqrt{3}+4\sqrt{2}$