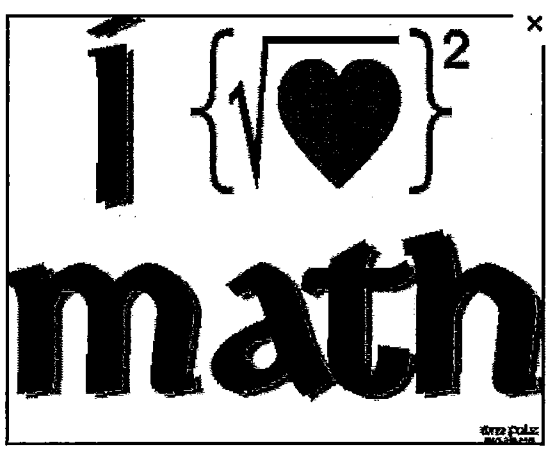
KEY

Chapter 2: Absolute Value and Radicals

Date	Topic
	2.2 Simplifying Radical Expressions Objective: • Simplify radical expressions with numerical or variable radicands
	2.3 Adding and Subtracting Radical Expressions Objective: • Simplify sums and differences of radical expressions
	2.4 Multiplying and Dividing Radical Expressions Objective: • Simplify produces and quotients of radical expressions
	2.5 Solving Radical Expressions Objectives: • Extend strategies for solving linear equations • Simplify radicals to solve equations involving radicals
	Review
	Test



2.2 Simplifying Radical Expressions

Recall: A radical is an expression involving the root sign:

index

$a\sqrt[n]{x}$ → whole thing is a radical ☺

radicand

coefficient

Multiplication Property	$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$	→ natural #s Where $n \in \mathbb{N}$ and $a, b, \sqrt[n]{a}, \sqrt[n]{b} \in \mathbb{R}$
Division Property	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$	

Review:

1. Express each mixed radical as an entire radical:

a) $4\sqrt{3}$

$=\sqrt{4^2}\sqrt{3}$

$=\sqrt{16}\sqrt{3}$

$=\sqrt{16\times 3}$ $=\sqrt{48}$

b) $x^2\sqrt{x}$

$=\sqrt{(x^2)^2} \cdot \sqrt{x}$

$=\sqrt{x^4 \cdot x}$

$=\sqrt{x^5}$

c) $2\sqrt[3]{3}$

$=\sqrt[3]{2^3} \cdot \sqrt[3]{3}$

$=\sqrt[3]{8 \cdot 3}$

$=\sqrt[3]{24}$

OR/ $\sqrt[3]{2 \cdot 2 \cdot 2 \cdot 3}$

$=\sqrt[3]{24}$

2. Convert each entire radical to a mixed radical in simplest form:

a) $\sqrt{75}$

$=\sqrt{25 \cdot 3}$

$=\sqrt{25} \cdot \sqrt{3}$

$=5\sqrt{3}$

b) $2\sqrt{48}$

$=2\sqrt{16 \cdot 3}$

$=2\sqrt{16} \cdot \sqrt{3}$

$=2(4) \cdot \sqrt{3}$

$=8\sqrt{3}$

c) $\sqrt[3]{x^7}$

$=\sqrt[3]{(x x x)(x x x)x}$

$=x \cdot x\sqrt[3]{x}$

$=x^2\sqrt[3]{x}$

$\sqrt[3]{x^3} = x$

Example #1: Arrange from greatest to least

a) $5\sqrt{2}$, $2\sqrt{5}$, $4\sqrt{3}$

Change to entire radicals

$\sqrt{25}\sqrt{2}$ $=\sqrt{50}$ ①

$\sqrt{4}\sqrt{5}$ $=\sqrt{20}$ ③

$\sqrt{16}\sqrt{3}$ $=\sqrt{48}$ ②

$5\sqrt{2}$, $4\sqrt{3}$, $2\sqrt{5}$

b) $9\sqrt[3]{2},\ 11\sqrt[3]{2},\ 8\sqrt[3]{2}$

$11\sqrt[3]{2},\ 9\sqrt[3]{2},\ 8\sqrt[3]{2}$

Since they are like radicals, compare coefficients

c) $3\sqrt[4]{8},\ 4\sqrt{2},\ 3\sqrt[3]{5}$

$\downarrow$ 5.0453 $\quad$ $\downarrow$ 5.6568 $\quad$ $\searrow$ 5.1299

$4\sqrt{2},\ 3\sqrt[3]{5},\ 3\sqrt[4]{8}$

Since the mixed radicals have different indices, use a calculator

Example #2:

a) Write $\sqrt[3]{\frac{-16}{135}}$ as a mixed radical.

$= \dfrac{\sqrt[3]{-16}}{\sqrt[3]{135}}$

$= \dfrac{\sqrt[3]{-8\cdot 2}}{\sqrt[3]{27\cdot 5}}$

$= \dfrac{-2\sqrt[3]{2}}{3\sqrt[3]{5}} = \dfrac{-2}{3}\sqrt[3]{\dfrac{2}{5}}$

b) Write $-3\sqrt[4]{\frac{2}{27}}$ as an entire radical

$= \sqrt[4]{(-3)^4}\sqrt[4]{2/27}$

$= \sqrt[4]{\frac{81}{1}}\ \sqrt[4]{\frac{2}{27}}$

$= \sqrt[4]{\frac{81 \times 2}{1 \times 27}}$

$= \sqrt[4]{\frac{162}{27}}$

$= \sqrt[4]{6}$

All square numbers are greater than or equal to 0, so the expression $\sqrt{x}$ is defined for $x \in \mathbb{R},\ x \geq 0$. The cube root of a positive number is positive, the cube root of a negative number is negative, and the cube root of zero is zero. Therefore, $\sqrt[3]{x}$ is defined for $x \in \mathbb{R}$.

↳ x is an element of the real #s

Example #3: For which values of the variable is each radical defined?

a) $\sqrt{27x^2}$

$x^2 \geq 0$

$x \geq 0$

$\therefore x \in \mathbb{R}$

b) $\sqrt[4]{-12x^3}$

*note: the 4th root is defined when the radicand is ≥ 0

$\uparrow$

since the coefficient is negative, x^3 must also be negative

$\therefore x^3 \leq 0$

$x \leq 0$

Example #4: For which values of the variable is each radical defined? Simplify the radical, if possible.

a) $\sqrt{45a^2}$ → since $\sqrt{a^2} = a$, the radical is defined for $a \in \mathbb{R}$

$= \sqrt{45}\sqrt{a^2}$

$= \sqrt{9 \cdot 5}\,(a)$

$= 3\sqrt{5}\,(a)$, $a \in \mathbb{R}$

$= 3a\sqrt{5}$

b) $\sqrt{-27b^9}$ → since $\sqrt{b^9}$ has an odd exponent, b could be + or −, but since we are taking a square root and we have a negative coefficient, the radical is defined when $b \leq 0$

$= \sqrt{-3(9)(b^8)b}$

$(bb)(bb)(bb)(bb)b$

$= 3b^4\sqrt{-3b}$, $b \leq 0$

c) $\sqrt[4]{7z}$ → since we have an even index, the radicand must be +

$\therefore z \geq 0$

↓

can't be simplified

d) $\sqrt[3]{24y^5}$ → since we have an odd index, we can have a + or − radicand $\therefore$ y can be anything!

↳ $y \in \mathbb{R}$

$= \sqrt[3]{8 \cdot 3 \cdot (yyy) \cdot yy}$

$= 2y\sqrt[3]{3y^2}$, $y \in \mathbb{R}$