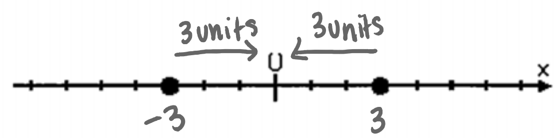
2.1 Absolute Value of a Real Number

Every real number can be represented as a point on a number line. The sign (+/-) of the number indicates its position relative to zero. The magnitude of the number indicates its distance from zero.

On the number line below, each of the numbers -3 and 3 is located 3 units from 0.

So, we say each number has an absolute value of 3.



In absolute value notion it is written as: $|-3| = 3$ $\quad$ $|3| = 3$

Below are some symbols that will be used this year and next for number sets you learned last year.

In addition, the symbol $\in$ (→element) shows membership in a set. So, the notion $x \in \mathbb{R}$ means that x is in the set of real numbers.

Using these conventions, the absolute value of a number a, $a \in \mathbb{R}$, is the distance from a to 0 on a number line.

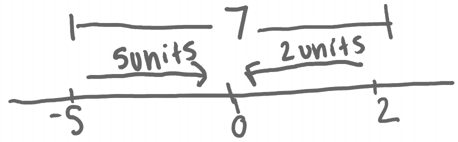
Number Set	Symbol
Natural Numbers	$\mathbb{N}$
Whole Numbers	$\mathbb{W}$
Integers	$\mathbb{Z}$
Rational Numbers	$\mathbb{Q}$
Irrational Numbers	$\overline{\mathbb{Q}}$
Real Numbers	$\mathbb{R}$

Example #1: Determine the absolute value of the following:

$|4.2| = 4.2$ $\qquad$ $|-6.1| = 6.1$

$\left|\frac{-3}{4}\right| = \frac{3}{4}$ $\qquad$ $|0| = 0$

Absolute value can be used to determine the distance between two points on a number line. To determine the distance between 2 and -5, count units.



7 units

Since distance is always positive, it does not matter which point you start with. Just remember always to subtract**. Then take the** absolute value **of the** difference**.**

<u>Example #2</u>: Use absolute value to determine the distance between each pair of numbers on a number line. Sketch a number line to illustrate the solution.

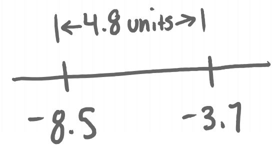
a. -3.7 and -8.5

$|-3.7-(-8.5)|$

$=|-3.7+8.5|$

$=|4.8|$

$=4.8$



b. $4\frac{2}{3}$ and $-5\frac{3}{5}$

$=\left|-5\frac{3}{5}-4\frac{2}{3}\right|$

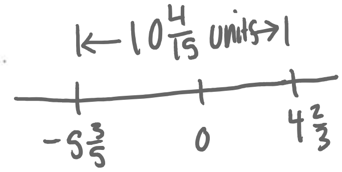
$=\left|-\frac{28}{5}-\frac{14}{3}\right|$

LCD: 15

$=\left|\frac{-84}{15}-\frac{70}{15}\right|$

$=\left|\frac{-154}{15}\right|$

$=\frac{154}{15} \longrightarrow 10\frac{4}{15}$



Consider the number 16: 16= 4^2 and 16= $(-4)^2$.

Therefore, 16 has two square roots: 4 and -4.

4 is called a principal square root of 16; it is written as $\sqrt{16}$ and represents the positive square root of 16.

$\boxed{x^2}$ $\sqrt{x^2}$

$\sqrt{x^2} = \pm x = |x| = x$

Example #3: Evaluate $\sqrt{(7-10)^2}$

$\sqrt{x^2} = |x|$

$= |7-10|$

$= |-3|$

$= 3$

Example #4: Evaluate: → always simplify absolute value signs first, then solve. Treat as "brackets" in BEDMAS

a. $|-26+13|-2|10-16|$

$= |-13| - 2|-6|$

$= 13 - 2(6)$

$= 13 - 12$

$= 1$

b. $|-2x^2-3x-4|$ when $x=2$

$= |-2(2)^2-3(2)-4|$

$= |-2(4)-6-4|$

$= |-8-10|$

$= |-18| \rightarrow = 18$