Lesson 1.7 – Sigma Notation

Sigma: Σ : greek letter for "S" → stands for "sum"

upper bound

$$\sum_{k=3}^{8}(k+1) = (3+1)+(4+1)+(5+1)+(6+1)+(7+1)+(8+1)$$
$$= 4+5+6+7+8+9 = 39$$

index → $k=3$

lower bound

- Means "Sum from the term when K=3 to when K=8"
- Number of terms in the series: $8-3+1=6$

  Top # − Bottom # + 1
- If there is no top number, then this is considered an infinite series

Example #1: Determine the number of terms in the following series. Then determine the sum.

*not arithmetic or geometric

$$\sum_{j=2}^{4} 2j^2 = 2(2)^2 + 2(3)^2 + 2(4)^2$$
$$= 2(4) + 2(9) + 2(16)$$
$$= 8 + 18 + 32$$
$$= 58$$

# of terms:
$= 4-2+1$
$= 3$

Example #2: Determine the sum of the following

$$\sum_{g=1}^{8}(3g-1) = (3(1)-1) + (3(2)-1) + (3(3)-1) + (3(4)-1) + (3(5)-1) + \ldots + (3(8)-1)$$

$$= 2 + 5 + 8 + \ldots + 23$$

Arithmetic

$t_1 = 2$

$n = 8$

$t_n = 23$

$d = 3$

$$S_n = \frac{n(t_1 + t_n)}{2}$$

$$S_8 = \frac{8(2+23)}{2}$$

$$S_8 = 4(25)$$

$$\boxed{S_8 = 100}$$

Example #3: Determine the sum of the following sigma notation

$$\sum_{g=3}^{7} 2^g = 2^3 + 2^4 + 2^5 + \ldots + 2^7$$

$$= 8 + 16 + 32 + \ldots + 128$$

geometric

$r = 2$

$t_1 = 8$

$n = 7 - 3 + 1 = 5$

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

$$S_5 = \frac{8(1-2^5)}{1-2}$$

$$= \frac{8(1-32)}{-1}$$

$$= -8(-31)$$

$$= 248$$

Example #4: Write the standard sigma notation for the given series

-4+3+10+17+...+94

Arithmetic
$d=7$
$t_1=-4$
$t_n=94$
$n=?$

$t_n=t_1+(n-1)d$
$94=-4+(n-1)7$
$94=-4+7n-7$
$94=-11+7n$
$+11 \quad +11$
$\frac{105}{7}=\frac{7n}{7}$
$\boxed{n=15}$

$$\sum_{x=1}^{15}(-4+7(x-1))$$

Example #5: Write the standard sigma notation for the given series

-4+8-16+...-1024

Geometric
$r=-2$
$t_1=-4$
$t_n=-1024$
$n=?$

$t_n=t_1\cdot r^{n-1}$
$\frac{-1024}{-4}=\frac{(-4)(-2)^{n-1}}{-4}$
$256=(-2)^{n-1}$
$256=2^8=(-2)^{9-1}$ *guess & check*
$n=9$

$$\sum_{K=1}^{9}-4(-2)^{K-1}$$