1.6 Infinite Geometric Series

An infinite geometric series has an infinite number of terms.

- There is no last term.
- A convergent series is a series with an infinite number of terms, in which the sequence ***approaches a fixed value***.
- A divergent series is a series with an infinite number of terms, in which the sequence ***does not approach a fixed value***.
- The value of r determines whether an infinite geometric series convergent or divergent.

$$\frac{t_1(1-0)}{1-0} = t_1$$

For a geometric series, $S_n = \dfrac{t_1(1-r^n)}{1-r}$, $r \neq 1$

When $-1 < r < 1$, r^n approaches 0 as n increases indefinitely.

The Sum of an Infinite Geometric Series

$$S_\infty = \frac{t_1}{1-r}$$

where t_1 = the first term
r = the common ratio *and* $-1 < r < 1$
S_∞ = the sum of the infinite series

Example #1: Determine whether each infinite geometric series converges or diverges. If it converges, determine its sum.

a) $1 - \frac{1}{3} + \frac{1}{9} - \ldots$ converges (approaching zero)

$$r = \frac{-\frac{1}{3}}{1} = -\frac{1}{3}$$

$$S_\infty = \frac{t_1}{1-r}$$

$$= \frac{1}{1-(-\frac{1}{3})}$$

$$= \frac{1}{1+\frac{1}{3}}$$

$$= \frac{1}{\frac{3}{3}+\frac{1}{3}}$$

$$= \frac{1}{4/3}$$

$$= \frac{1}{1} \div \frac{4}{3}$$

$$= \frac{1}{1} \times \frac{3}{4}$$

$$= \frac{3}{4}$$

b) $2 - 4 + 8 - \ldots$

$$r = \frac{-4}{2} = -2$$

∴ divergent
* no sum

Example #2: If the first tem in an infinite geometric series is 21 and the sum is 63, determine the common ratio.

$t_1 = 21$

$S_n = 63$

$r = ?$

$S = \frac{t_1}{1-r}$

$\times(1-r)\ 63 = \frac{21}{1-r} \times(1-r)$

$\frac{63(1-r)}{63} = \frac{21}{63}$

$1 - r = \frac{21}{63}$

$+r \quad +r$

$1 = \frac{21}{63} + r$

$63\times\ 1 - \frac{21}{63} = r$

$\frac{63}{63} - \frac{21}{63} = r \quad \div 21 = \frac{1}{3}$

$\frac{42}{63} = r$

$\frac{2}{3} = r$

Example #3: Determine a fraction that is equal to $0.1\overline{6}$

Recall expanded form:

$0.1\overline{6} = 0.1 + 0.06 + 0.006 + 0.0006 + 0.00006 + \ldots$

$= \frac{1}{10} + \underbrace{\frac{6}{100} + \frac{6}{1000} + \frac{6}{10000} + \ldots}$

these repeating terms form an infinite geometric series

$r = \frac{\frac{6}{1000}}{\frac{6}{100}} = \frac{6}{1000} \times \frac{100}{6} = \frac{6\cancel{00}}{60\cancel{00}} = \frac{6}{60} = \boxed{\frac{1}{10}}$

$S_\infty = \frac{t_1}{1-r}$

$= \frac{\frac{6}{100}}{1 - \frac{1}{10}}$

$= \frac{\frac{6}{100}}{\frac{10}{10} - \frac{1}{10}}$

$= \frac{\frac{6}{100}}{\frac{9}{10}}$

$= \frac{6}{10\cancel{0}} \times \frac{1\cancel{0}}{9} = \frac{6}{90} = \boxed{\frac{1}{15}}$

*add the sum fraction to the fraction that was not repeated:

$3\times\ \frac{1}{10} + \frac{1}{15} \times 2$

$\frac{3}{30} + \frac{2}{30} = \frac{5}{30} = \boxed{\frac{1}{6}}$