1.4 Geometric Series

A geometric series is a Sum of terms that form a geometric sequence.

- A geometric sequence is: 3, 9, 27, 81, ...
- The related geometric series is: 3 + 9 + 27 + 81 + ...

S_n - the sum of the first "n" terms of a series

The Sum of *n* Terms of an Geometric Series

For the geometric series $t_1 + t_1r + t_1r^2 + ... + t_1r^{n-1}$, the sum of *n* terms, S_n, is:

$$S_n = \frac{t_1(1-r^n)}{1-r}, \quad r \neq 1$$

where t_1 = the first term
r = the common ratio
n = the number of terms
S_n = the sum of the first n terms

Example #1: Determine the sum of the first 12 terms of this geometric series:

$3 + 12 + 48 + 192 + ...$

$t_1 = 3$
$r = 4$
$n = 12$

$$S_{12} = \frac{3(1-4^{12})}{1-4}$$

$$S_{12} = \frac{3(1-4^{12})}{-3}$$

$$S_{12} = 16\,777\,215$$

$$S_{12} = -(1-4^{12})$$
$$= -(1-16\,777\,216)$$
$$= -(-16\,777\,215)$$
$$= 16\,777\,215$$

Example #2: The sum of the first 14 terms of a geometric series is 16 383. The common ratio is -2. Determine the 1st term.

$S_{14} = 16383$
$t_1 = ?$
$r = -2$
$n = 14$

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

$$16383 = \frac{t_1(1-(-2)^{14})}{1-(-2)}$$

$$3 \times 16383 = \frac{t_1(-16383)}{3} \quad \times 3$$

$$\frac{49149}{-16383} = \frac{t_1(-16383)}{-16383}$$

$$t_1 = -3$$

Example #3: Calculate the sum of this geometric series: $-3 - 15 - 75 - \ldots - 46875$

$t_1 = -3$ $S_n = \frac{t_1(1-r^n)}{1-r}$

$r = 5$

$n = ?$ "Find n": $t_n = -46875$

$n = 7$ $t_n = t_1 \cdot r^{n-1}$

$\frac{-46875}{-3} = \frac{(-3)\cdot 5^{n-1}}{-3}$

$15625 = 5^{n-1}$

→ guess and check!

$15625 \neq 5^8 = 390625$

$15625 = 5^6 = 5^{7-1}$

$\therefore n = 7$

$S_7 = \frac{(-3)(1-5^7)}{1-5}$

$= \boxed{-58\,593}$

Example #4: The NCAA basketball tournament (March Madness) starts with 64 teams. The winners of each game continue to play until a final match determines the champion. What is the total number of games that will be played during the tournament?

There are two teams that play every game.

$\therefore$ Round one has $\frac{64}{2} = 32$ games

Round two has $\frac{32}{2} = 16$ games

$\therefore$ the final round has one match:

$t_1 = 32$ Find "n": $S_n = \frac{t_1(1-r^n)}{1-r}$

$r = \frac{1}{2}$ $t_n = t_1 \cdot r^{n-1}$ $= \frac{32(1-(\frac{1}{2})^6)}{1-\frac{1}{2}}$

$n = ?$ $1 = 32 \cdot (\frac{1}{2})^{n-1}$

$t_n = 1$ $\frac{1}{32} = \frac{1}{2}^{n-1}$ $= \frac{32(1-\frac{1}{64})}{\frac{1}{2}}$

guess + check:

$\frac{1}{32} = (\frac{1}{2})^5 = (\frac{1}{2})^{6-1}$ $= \frac{32(\frac{63}{64})}{\frac{1}{2}}$

$\therefore n = 6$ $= \boxed{63}$

$\therefore$ there are 63 games played