1.3 Geometric Sequences

A geometric sequence is a sequence of terms where the ratio of consecutive terms is a constant. Therefore, each term is formed by multipiying the previous term by this constant, referred to as the common ratio.

Example: 3, 6, 12, 24, . . . Find the common ratio

$r = \frac{6}{3} = 2$ $r = \frac{12}{6} = 2$ $r = \frac{24}{12} = 2$

$\therefore \boxed{r=2}$

> NOTE: The common ratio can be found by dividing any term by the preceding term.

Consider the sequence 2, -6, 18, -54 , . . .	The general term of a geometric sequence is:
$r = -3$ $t_1 = 2$ $t_2 = 2(-3) = -6$ $t_3 = 2(-3)(-3)$ $t_4 = 2(-3)(-3)(-3)$ $t_n = 2(-3)^{n-1}$ $t_n = t_1 \cdot r^{n-1}$	$t_n = t_1 \cdot r^{n-1}$ r = common ratio t_1 = first term n = number of terms

Geometric sequences can be divergent or convergent.

- 2, 8, 32, 128, ... is divergent because the terms do not approach a constant value.
- $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{32}, \ldots$ is convergent because the terms approach a constant value (in this case zero).

Example #1: Given the geometric sequence: 5, 10, 20, 40, . . .

a) Find an expression for the general term.

$t_n = t_1 \cdot r^{n-1}$

$t_1 = 5$

$r = \frac{10}{5} = 2$

$t_n = 5(2)^{n-1}$

b) Find the 10th term.

$t_n = 5(2)^{n-1}$ *BEDMAS*

$t_{10} = 5(2)^{10-1}$

$= 5(2)^9$

$= 5(512)$

$t_{10} = 2560$

Example #2: In a geometric sequence, the third term is 54 and the sixth term is -1458. Determine the values of t_1 and r and list the first three terms of the sequence.

$t_3 = 54$

$t_6 = -1458$

__ __ 54 __ __ -1458

xr xr xr

$\frac{54r^3}{54} = \frac{-1458}{54}$

$\sqrt[3]{r^3} = \sqrt[3]{-27}$

$\boxed{r = -3}$

$t_n = t_1 \cdot r^{n-1}$

$54 = t_1(-3)^{3-1}$

$54 = t_1(-3)^2$

$\frac{54}{9} = \frac{9t_1}{9}$

$\boxed{t_1 = 6}$

∴ 1st three terms: 6, −18, 54

Example #3: Create a geometric sequence whose 6th term is 27.

$\frac{1}{9}$ $\frac{1}{3}$ 1 3 9 27

t_6

÷3 ÷3 ÷3 ÷3 ÷3

*work backwards

*note: there are infinitely many solutions.

∴ choose a ratio that is a factor of 27

i.e. $r = 3$

Example #4: In a finite geometric sequence, $t_1 = 7$ and $t_5 = 567$.

a. Determine t_2 and t_6

$t_1 = 7$

$t_5 = 567$

$r = ?$

$t_n = t_1 \cdot r^{n-1}$

$567 = 7 \cdot r^{5-1}$

$\frac{567}{7} = \frac{7r^4}{7}$

$81 = r^4$

$\pm\sqrt[4]{81} = \sqrt[4]{r^4}$

$r = \pm 3$

$t_1 = 7$

$t_2 = \pm 21$

$t_3 = 63$

$t_4 = \pm 189$

$t_5 = 567$

$t_6 = \pm 1701$

b. The last term is 45 927. How many terms are in the sequence?

$t_n = 45927$

$t_1 = 7$

$r = \pm 3$

$t_n = t_1 \cdot r^{n-1}$

$\frac{45927}{7} = \frac{7 \cdot (3)^{n-1}}{7}$

$6561 = 3^{n-1}$

guess & check!

$3^7 = 2187$

$3^8 = 6561$

∴ $n - 1 = 8$

$n = 9$

Example #5: A car was purchased for \$15 874. The resale value of the same car three years later was \$13 610. Assuming the depreciation is a geometric sequence, what will be the value of the car 6 years after purchase

\$15 874	____	____	13 610	____	____	____
t_1	t_2 (1yr later)	t_3 (2yrs)	t_4 (3yrs)	t_5 (4yrs)	t_6 (5yrs)	t_7 (6yrs)

$t_1 = 15874$ $\qquad t_n = t_1 \cdot r^{n-1}$

$t_4 = 13610$ $\qquad 13610 = 15\,874 \cdot r^{4-1}$

$$\frac{13610}{15874} = \frac{15874 \cdot r^3}{15874}$$

$$\sqrt[3]{0.85738} = \sqrt[3]{r^3}$$

$$0.95 = r$$

6 years after purchase is t_7, $n=7$

$$t_n = t_1 \cdot r^{n-1}$$

$$t_7 = 15874 \cdot (0.95)^{7-1}$$

$$t_7 = 15874\,(0.95)^6$$

$$\boxed{t_7 = \$11\,668.85}$$