1.2 Arithmetic Series

An arithmetic series is a ___SUM___ of terms that form an arithmetic sequence.

- An arithmetic sequence is: 2   5   8   11   ...
- The related arithmetic series is: 2 + 5 + 8 + 11   ...

S_n – ___the sum of the first "n" terms of a series___

For the arithmetic series above:

$S_1 = t_1$ $S_2 = t_1 + t_2$ $S_3 = t_1 + t_2 + t_3$ $S_4 = t_1 + t_2 + t_3 + t_4$

$S_1 = 2$ $S_2 = 2 + 5$ $S_3 = 2 + 5 + 8$ $S_4 = 2 + 5 + 8 + 11$

            $S_2 = 7$ $S_3 = 15$ $S_4 = 26$

These are examples of ___partial sums___. If there are few terms, S_n can be determined using mental math. Otherwise we will use the following formulas.

The Sum of n Terms of an Arithemetic Series

For an arithmetic series with 1st term, t_1, common difference, d, and n^{th} term, t_n, the sum of the first n terms, S_n, is:

$$S_n = \frac{n(t_1 + t_n)}{2} \qquad \text{or} \qquad S_n = \frac{n(2t_1 + d(n-1))}{2}$$

Notice that the second formula has the General Term of an Arithmetic Sequence formula substituted in for t_n in the first formula. The second formula is used when t_n is not known.

Example #1: Determine the (sum) of the first 40 terms of the arithmetic series:

     $1.5 + 3.5 + 5.5 + 7.5 + ...$

$t_1 = 1.5$

$d = 2$

$n = 40$

$S_n = ?$

*don't know t_{40}
so use second formula*

$S_n = \dfrac{n(2t_1 + d(n-1))}{2}$

$S_{40} = \dfrac{40((2)(1.5) + 2(40-1))}{2}$

$= 20(3 + 2(39))$

$= 20(3 + 78)$

$= 20(81)$

$= 1620$

Example #2: Determine the sum of the arithmetic series

$$15 + 18 + 21 + \ldots + 45$$

$t_1 = 15$

$t_n = 45$

$n = ?$

$S_n = \dfrac{n(t_1 + t_n)}{2}$

$S_n = \dfrac{11(15 + 45)}{2}$

$S_n = \dfrac{11(60)}{2}$

$\boxed{S_{11} = 330}$

need to find "n":

$d = 3$

$t_n = t_1 + (n-1)d$

$45 = 15 + (n-1)3$

$45 = 15 + 3n - 3$

$30 = 3n - 3$

$\dfrac{33}{3} = \dfrac{3n}{3}$

$\boxed{11 = n}$

Example #3: The sum of the first two terms of an arithmetic series is 13 and the sum of the first four terms is 46. Determine the first six terms and the sum of the first six terms