

Pre-Calculus II 7.1

1. Note: A rational : $\frac{\text{Polynomial}}{\text{Polynomial}}$

a.) Rational b.) Rational c.) Rational

d.) Not Rational \rightarrow " \sqrt{x} " not a polynomial

e.) Rational f.) Not Rational \rightarrow " 2^x " not a polynomial

2. a.) $6w = 0$
 $w = 0$

b.) $x^2 + 3$

$x^2 = -3$ Not Possible

\therefore there are no non-permissible values. $x \in \mathbb{R}$.

c.) $b - 1 = 0$
 $b = 1$

d.) $m^2 - 1 = 0$

$(m+1)(m-1) = 0$

$m = -1$ $m = 1$

e.) $x^3 - 8 = 0$

$x^3 = 8$

$x = \sqrt[3]{8}$

$x = 2$

f.) $2x(x+4)$

$2x = 0$ $x + 4 = 0$

$x = 0$ $x = -4$

g.) $5 = 0$ \Leftarrow Not possible

\therefore there are no NPVs

$x \in \mathbb{R}$.

h.) $(x+2)(x-1) = 0$

$x + 2 = 0$ $x - 1 = 0$

$x = -2$ $x = 1$

i.) $(x+1)(x+2) = 0$

$x + 1 = 0$ $x + 2 = 0$

$x = -1$ $x = -2$

j.) $3x^2 + 6x$

$3x(x+2)$

$3x = 0$ $x + 2 = 0$

$x = 0$ $x = -2$

PC II 7.1 con't. 2.

2. k.) $x^2 + 7x + 12 = 0$
 $(x+3)(x+4) = 0$
 $x+3=0 \quad x+4=0$
 $x=-3 \quad x=-4$

l.) $3x^2 + 8x + 5 = 0$ $mn=3(5)$
 $3x^2 + 3x + 5x + 5 = 0$ $=15$
 $3x(x+1) + 5(x+1) = 0$ $3 \hat{3}$
 $(3x+5)(x+1) = 0$
 $3x+5=0 \quad x+1=0$
 $x = -\frac{5}{3} \quad x = -1$

m.) $x^2 + 16 = 0$
 $x^2 = -16$ Not Possible
 \therefore there are no NPV's
 $x \in \mathbb{R}$

n.) $x^2 - 25 = 0$
 $x^2 = 25$
 $x = \pm\sqrt{25}$
 $x = \pm 5$

o.) $x^3 + 1 = 0$
 $x^3 = -1$
 $x = \sqrt[3]{-1}$
 $x = -1$

3. a.) $\frac{3m^4}{6m}$
 $= \frac{m^3}{2}$

b.) $\frac{-12a^3b^5}{4a^2b^7}$
 $= -\frac{3a}{b^2}$

c.) $\frac{15x^2y}{-18x^3y^2z}$
 $= \frac{5}{-6xyz}$

d.) $\frac{4x+8}{2x+4}$
 $= \frac{4(x+2)}{2(x+2)}$
 $= 2$

e.) $\frac{2x-10}{3x-15}$
 $= \frac{2(x-5)}{3(x-5)}$
 $= \frac{2}{3}$

f.) $\frac{3a+12}{6a+24}$
 $= \frac{3(a+4)}{6(a+4)}$
 $= \frac{1}{2}$

PC11 7.1 con't...3

$$4. a.) \frac{x-5}{10-2x} = \frac{x-5}{-x-5} = \frac{2(5-x)}{-2(x-5)} = \frac{(x-5)}{-2(x-5)} = -\frac{1}{2}$$

$$b.) \frac{9-3x}{x-3} = \frac{3(3-x)}{x-3} = \frac{-3(x-3)}{x-3} = -3$$

$$c.) \frac{2x^2-10x}{4x-20} = \frac{2x(x-5)}{4(x-5)} = \frac{x}{2}$$

$$d.) \frac{3x^2-6x}{14-7x} = \frac{3x(x-2)}{7(2-x)} = \frac{3x(x-2)}{-7(x-2)} = -\frac{3x}{7}$$

$$e.) \frac{10xy-15x^2y}{6x^2-4x} = \frac{5xy(2-3x)}{2x(3x-2)} = \frac{-5xy(3x-2)}{2x(3x-2)} = -\frac{5y}{2}$$

$$f.) \frac{60a^2b^2-24ab}{16ab-40a^2b^2} = \frac{12ab(5ab-2)}{8ab(2-5ab)} = \frac{-12ab(2-5ab)}{8ab(2-5ab)} = -\frac{3}{2}$$

$$5. a.) \frac{x-3}{x^2+3x-18} = \frac{(x-3)}{(x+6)(x-3)} = \frac{1}{x+6}$$

$$b.) \frac{m^2-7m+10}{m-2} = \frac{(m-5)(m-2)}{(m-2)} = m-5$$

$$c.) \frac{x+4}{x^2-16} = \frac{(x+4)}{(x+4)(x-4)} = \frac{1}{x-4}$$

$$d.) \frac{a^2+5a-14}{a^2-6a+8} = \frac{(a+7)(a-2)}{(a-4)(a-2)} = \frac{a+7}{a-4}$$

$$e.) \frac{4a^2-16b^2}{4a^2-8ab} = \frac{4(a^2-4b^2)}{4a(a-2b)} = \frac{4(a+2b)(a-2b)}{4a(a-2b)} = \frac{a+2b}{a}$$

$$f.) \frac{x^2+6x+8}{x^2+7x+12} = \frac{(x+4)(x+2)}{(x+3)(x+4)} = \frac{x+2}{x+3}$$

PC II 7.1 con't... 4

$$\begin{aligned}
 6. a) \quad & \frac{m^2 - 9mn + 20n^2}{3m^2 - 15mn} \\
 & = \frac{(m-4n)(m-5n)}{3m(m-5n)} \\
 & = \frac{m-4n}{3m}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad & \frac{x^2 + 9xy + 18y^2}{2x^2 + 12xy} \\
 & = \frac{(x+6y)(x+3y)}{2x(x+6y)} \\
 & = \frac{x+3y}{2x}
 \end{aligned}$$

$$\begin{aligned}
 c) \quad & \frac{8t^2 - 32}{2t^2 + 12t + 16} \\
 & = \frac{8(t^2 - 4)}{2(t^2 + 6t + 8)} \\
 & = \frac{8(t+2)(t-2)}{2(t+4)(t+2)} \\
 & = \frac{4(t-2)}{t+4}
 \end{aligned}$$

$$\begin{aligned}
 d) \quad & \frac{3m^2 - 15m}{3m^2 - 16m + 5} \quad \begin{array}{l} mn=3(5) \\ =15 \\ \times \\ -15 -1 \end{array} \\
 & = \frac{3m(m-5)}{3m^2 - 15m - 1m + 5} \\
 & = \frac{3m(m-5)}{3m(m-5) - 1(m-5)} \\
 & = \frac{3m(m-5)}{(3m-1)(m-5)} \\
 & = \frac{3m}{3m-1}
 \end{aligned}$$

$$\begin{aligned}
 e) \quad & \frac{2x^2 + 3xy + y^2}{3x^2 + 2xy - y^2} \quad \begin{array}{l} mn=2 \\ 1 \quad 2 \\ mn=-3 \\ 3 \quad -1 \end{array} \\
 & = \frac{2x^2 + 1xy + 2xy + y^2}{3x^2 + 3xy - 1xy - y^2} \\
 & = \frac{x(2x+y) + y(2x+y)}{3x(x+y) - y(x+y)} \\
 & = \frac{(2x+y)(x+y)}{(3x-y)(x+y)} \\
 & = \frac{2x+y}{3x-y}
 \end{aligned}$$

$$\begin{aligned}
 f) \quad & \frac{x^2 - 5x - 6}{36 - x^2} \\
 & = \frac{(x-6)(x+1)}{(6-x)(6+x)} \\
 & = \frac{(x-6)(x+1)}{-(x-6)(6+x)} \\
 & = \frac{x+1}{-(6+x)} \\
 & = -\frac{(x+1)}{(x+6)} = \frac{-x-1}{x+6} = \frac{x+1}{-x-6}
 \end{aligned}$$

PC 11 7.1 con't... 5

7. a) $\frac{25-x^2}{2x^2-9x-5}$ $mn = -10$
 $-10 = -10 + 1$

$$= \frac{-(x^2-25)}{2x^2-10x+1x-5}$$

$$= \frac{-(x+5)(x-5)}{2x(x-5)+1(x-5)}$$

$$= \frac{-(x+5)(x-5)}{(2x+1)(x-5)}$$

$$= -\frac{(x+5)}{(2x+1)} = -\frac{x+5}{2x+1}$$

b) $\frac{16+x^2}{x^4+5x^2+4}$

$$= \frac{16+x^2}{(x^2+4)(x^2+1)}$$

Note: Nothing factors further as each binomial is a SUM of squares not a difference.
 ∴ It already is simplified

c) $\frac{x^4-5x^2+4}{x^2+x-2}$

$$= \frac{(x^2-1)(x^2-4)}{(x-1)(x+2)}$$

$$= \frac{(x+1)(x-1)(x+2)(x-2)}{(x-1)(x+2)}$$

$$= (x+1)(x-2)$$

d) $\frac{x^2-4xy+4y^2}{x^4-16y^4}$

$$= \frac{(x-2y)(x+2y)}{(x^2+4y^2)(x^2-4y^2)}$$

$$= \frac{(x-2y)(x-2y)}{(x^2+4y^2)(x+2y)(x-2y)}$$

$$= \frac{x-2y}{(x^2+4y^2)(x+2y)}$$

e) $\frac{x^4-y^4}{(x^2+y^2)(x^2-5xy+4y^2)}$

$$= \frac{(x^2+y^2)(x^2-y^2)}{(x^2+y^2)(x^2-y^2)}$$

$$= \frac{(x^2+y^2)(x-y)(x+4y)}{(x^2+y^2)(x+y)(x-y)}$$

$$= \frac{(x^2+y^2)(x-y)(x-4y)}{(x^2+y^2)(x-y)(x-4y)}$$

$$= \frac{x+y}{x-4y}$$

f) $\frac{16x^4-y^4}{(4x^2+y^2)(2x^2+3xy-2y^2)}$ $mn = -2$
 $-2 = -1 + 1$

$$= \frac{(4x^2+y^2)(4x^2-y^2)}{(4x^2+y^2)(2x^2+3xy-2y^2)}$$

$$= \frac{(4x^2+y^2)(2x+y)(2x-y)}{(4x^2+y^2)(2x(x+2y)-1y(x+2y))}$$

$$= \frac{(4x^2+y^2)(2x+y)(2x-y)}{(4x^2+y^2)(2x+y)(2x-y)}$$

$$= \frac{2x+y}{x+2y}$$

PC11 7.1 com 4... 6

8. a) $x \neq 3, x \neq -2$
 $x = 3 \quad x = -2$
 $x - 3 = 0 \quad x + 2 = 0$
 $(x - 3)(x + 2) = 0$
 $x^2 - x - 6 = 0$

Rational expression:

$$\frac{1}{x^2 - x - 6}$$

note: Numerator can be any polynomial.

b) $x \neq -\frac{1}{2}, x \neq \frac{2}{3}$
 $x = -\frac{1}{2} \quad x = \frac{2}{3}$
 $2x = -1 \quad 3x = 2$
 $2x + 1 = 0 \quad 3x - 2 = 0$
 $(2x + 1)(3x - 2)$

$$6x^2 - 4x + 3x - 2 = 0$$

$$6x^2 - x - 2 = 0$$

Rational expression:

$$\frac{1}{6x^2 - x - 2}$$

c) $x \in \mathbb{R} \rightarrow$ Any denominator that can never equal zero. ie. $\frac{1}{x^2 + 1}, \frac{1}{x^2 + 2}, \frac{1}{x^4 + 1}$

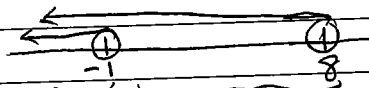
9. $A(p, 3) \quad B(2p + 1, p - 5)$
 $x_1 \quad y_1 \quad x_2 \quad y_2$

a) $m = \frac{y_2 - y_1}{x_2 - x_1}$
 $= \frac{p - 5 - 3}{2p + 1 - p}$
 $m = \frac{p - 8}{p + 1}$

b) For the slope to be negative,
 $p - 8 < 0$ or $p + 1 < 0$ but
 not both < 0 .

$$p - 8 < 0 \quad p + 1 < 0$$

$$p < 8 \quad p < -1$$



both are
 negative
 if $p < -1$

only numerator is
 negative when
 $-1 < p < 8$

c) NPV: $p = -1$
 When $p = -1$, the
 slope is $\frac{-8}{0}$ which
 is undefined.
 $\therefore H$ is a vertical
 line.

\therefore Slope is negative when
 $-1 < p < 8$

PC.11 7.1 con't... 7

10. a) $\frac{(x+2)^2 - (x+2) - 20}{x^2 - 9} \longrightarrow \text{let } m = x+2$

$$= \frac{(x-3)(x+6)}{(x+3)(x-3)}$$
$$= \frac{x+6}{x+3}$$

$$m^2 - m - 20$$
$$(m-5)(m+4)$$
$$(x+2-5)(x+2+4)$$
$$(x-3)(x+6)$$

b) $\frac{4(x^2-9)^2 - (x-3)^2}{x^2 - 6x + 9}$

$$= \frac{[2(x^2-9)]^2 - [x-3]^2}{(x-3)(x-3)}$$

$$= \frac{(2(x^2-9) + (x-3))(2(x^2-9) - (x-3))}{(x-3)(x-3)}$$

$$= \frac{(2x^2 - 18 + x - 3)(2x^2 - 18 - x + 3)}{(x-3)(x-3)}$$

$$= \frac{(2x^2 + x - 21)(2x^2 - x - 15)}{(x-3)(x-3)}$$

$$= \frac{(2x^2 + 7x - 6x - 21)(2x^2 - 6x + 5x - 15)}{(x-3)(x-3)}$$

$$= \frac{(x(2x+7) - 3(2x+7))(2x(x-3) + 5(x-3))}{(x-3)(x-3)}$$

$$= \frac{(2x+7)(x-3)(2x+5)(x-3)}{(x-3)(x-3)}$$

$$= (2x+7)(2x+5)$$

OR// Another way to simplify numerator:

$$4(x^2-9)^2 - (x-3)^2$$

$$= 4(x^2-9)(x^2-9) - (x-3)^2$$

$$= 4(x+3)(x-3)(x+3)(x-3) - (x-3)^2$$

$$= (x-3)^2(4(x+3)(x+3) - 1)$$

$$= (x-3)^2(4(x^2+6x+9) - 1)$$

$$= (x-3)^2(4x^2+24x+36-1)$$

$$= (x-3)^2(4x^2+24x+35)$$

$$= (x-3)^2(4x^2+14x+10x+35)$$

$$= (x-3)^2(2x(2x+7) + 5(2x+5))$$

$$= (x-3)^2(2x+5)(2x+7)$$

Then: $\frac{(x-3)^2(2x+5)(2x+7)}{(x-3)^2}$

$$= (2x+5)(2x+7)$$

PC 11 7.1 cont... 8

$$10. c) \frac{(x^2-x)^2 - 8(x^2-x) + 12}{(x^2-4)^2 - (x-2)^2} \longrightarrow \text{let } m = x^2 - x$$

$$\frac{m^2 - 8m + 12}{(m-6)(m-2)}$$

$$= \frac{(x^2-x-6)(x^2-x-2)}{(x^2-x-6)(x^2-x-2)}$$

$$\frac{[(x^2-4) + (x-2)][(x^2-4) - (x-2)]}{(x^2-4 + x - 2)(x^2-4 - x + 2)}$$

$$= \frac{(x-3)(x+2)(x-2)(x+1)}{(x^2-4+x-2)(x^2-4-x+2)}$$

$$= \frac{(x-3)(x+2)(x-2)(x+1)}{(x^2+x-6)(x^2-x-2)}$$

$$= \frac{(x-3)(x+2)(x-2)(x+1)}{(x+3)(x-2)(x-2)(x+1)}$$

$$= \frac{(x-3)(x+2)}{(x+3)(x-2)}$$

or// Another way to simplify denominator

$$(x^2-4)^2 - (x-2)^2$$

$$= ((x+2)(x-2))^2 - (x-2)^2$$

$$= (x-2)^2 ((x+2)^2 - 1)$$

$$= (x-2)^2 (x^2 + 4x + 4 - 1)$$

$$= (x-2)(x-2)(x^2 + 4x + 3)$$

$$= (x-2)(x-2)(x+3)(x+1)$$

$$d) \frac{(x^2+4x+4)^2 - 10(x^2+4x+4) + 9}{(2x+1)^2 - (x+2)^2}$$

Numerator: let $m = x^2 + 4x + 4$

$$m^2 - 10m + 9$$

$$= (m-9)(m-1)$$

$$= (x^2+4x+4-9)(x^2+4x+4-1)$$

$$= (x^2+4x-5)(x^2+4x+3)$$

$$= (x+5)(x-1)(x+3)(x+1)$$

Denominator:

$$(2x+1)^2 - (x+2)^2$$

$$= ((2x+1) + (x+2))((2x+1) - (x+2))$$

$$= (2x+1+x+2)(2x+1-x-2)$$

$$= (3x+3)(x-1)$$

$$= 3(x+1)(x-1)$$

Put it together:

$$\frac{(x+5)(x-1)(x+3)(x+1)}{3(x+1)(x-1)}$$

$$= \frac{(x+5)(x+3)}{3}$$

Pre-Calculus II 7.2

$$1. a) \frac{5}{8} \cdot \frac{2a}{3}$$

$$= \frac{10a}{24}$$

$$= \frac{5a}{12}$$

$$b) \frac{m^2}{4} \cdot \frac{2}{m}$$

$$= \frac{2m^2}{4m}$$

$$= \frac{m}{2}$$

$$c) \frac{9x^2}{4y^3} \cdot \frac{8y}{-3x}$$

$$= \frac{72x^2y}{-12xy^3}$$

$$= -\frac{6x}{y^2}$$

$$d) \frac{-4 \cdot 9b^2}{1 \cdot 6a}$$

$$= -\frac{36b^2}{6a}$$

$$= -\frac{6b^2}{a}$$

$$e) \frac{3m^2x}{6mn} \cdot \frac{4mn}{5m^2n^2}$$

$$= \frac{12}{30n} \left(= \frac{12m^3n^2}{30m^3n^3} \right)$$

$$= \frac{2}{5n}$$

$$f) \frac{-5x^2y}{(2xy)^3} \cdot \frac{-12x^2y^2}{-6xy}$$

$$= \frac{-5}{8x^3y^3} \cdot 2x^2y^2$$

$$= \frac{-10}{8xy} = \frac{-5}{4xy}$$

$$2. a) \frac{x^2}{14} \div \frac{x}{7}$$

$$= \frac{x^2}{14} \cdot \frac{7}{x}$$

$$= \frac{x}{2}$$

$$b) \frac{-6xy}{15} \div \frac{2x^2}{5}$$

$$= \frac{-2xy}{5} \cdot \frac{5}{2x^2}$$

$$= \frac{-y}{x}$$

$$c) \frac{2m}{9n} \div \frac{-4m}{3n^2}$$

$$= \frac{2m}{9n} \cdot \frac{3n^2}{-4m}$$

$$= \frac{6mn^2}{-36mn} = \frac{-1n}{6}$$

$$d) \frac{5a^2}{12b} \div 25a$$

$$= \frac{5a^2}{12b} \cdot \frac{1}{25a}$$

$$= \frac{a^2}{60ab}$$

$$= \frac{a}{60b}$$

$$e) \frac{4c^2d}{8cd} \div \frac{3c^2d^3}{6cd^3}$$

$$= \frac{4c^2d}{8cd} \cdot \frac{6cd^3}{3c^2d^3}$$

$$= \frac{1}{2} \cdot \frac{2}{1}$$

$$= 1$$

$$f) \frac{12mn^2}{9m} \div \frac{(3mn)^2}{6mn^2}$$

$$= \frac{4n}{3} \cdot \frac{6mn^2}{9m^2n^2}$$

$$= \frac{24mn^3}{27m^2n^2}$$

$$= \frac{8n}{9m}$$

PC II 7.2 cont'd. 2.

$$3. a.) \frac{6x}{(x+1)} \cdot \frac{2(x+1)}{3x^2}$$

$$= \frac{12x}{3x^2}$$

$$= \frac{4}{x}$$

NPV'S $x+1 \neq 0$ $x \neq 0$
 $x \neq -1$

$$b.) \frac{2(4a+3)^2}{4a} \cdot \frac{12a^3}{3(4a+3)}$$

$$= \frac{24a^3(4a+3)}{12a}$$

$$= 2a^2(4a+3)$$

NPV'S $4a \neq 0$ $4a+3 \neq 0$
 $a \neq 0$ $4a \neq -3$
 $a \neq -\frac{3}{4}$

$$c.) \frac{3(5-c)}{4c} \cdot \frac{4c+1}{6(c-5)}$$

$$= \frac{-3(c-5)}{4c} \cdot \frac{4c+1}{6(c-5)}$$

$$= \frac{-3(4c+1)}{24c}$$

$$= \frac{-1(4c+1)}{8c} = \frac{-4c-1}{8c}$$

NPV'S: $4c \neq 0$ $c-5 \neq 0$
 $c \neq 0$ $c \neq 5$

$$d.) \frac{3(x-2)}{4(x+5)} \div \frac{6(x-2)}{x+5}$$

$$= \frac{3(x-2)}{4(x+5)} \cdot \frac{(x+5)}{6(x-2)}$$

$$= \frac{3}{24}$$

$$= \frac{1}{8}$$

NPV'S $x+5 \neq 0$ $x-2 \neq 0$
 $x \neq -5$ $x \neq 2$

$$e.) \frac{(a-3)^2}{14(a-1)} \div \frac{2(a-3)}{7(1-a)}$$

$$= \frac{(a-3)^2}{14(a-1)} \cdot \frac{-7(1-a)}{2(a-3)}$$

$$= \frac{-7(a-3)}{28}$$

$$= \frac{-(a-3)}{4} = \frac{-a+3}{4}$$

NPV'S $a-1 \neq 0$ $a-3 \neq 0$
 $a \neq 1$ $a \neq 3$

$$f.) \frac{50(x-6)}{24(y-2)} \div \frac{-25(x-6)}{12(2-y)}$$

$$= \frac{50(x-6)}{24(y-2)} \cdot \frac{-12(2-y)}{-25(x-6)}$$

$$= \frac{250(-12)}{224(-25)}$$

$$= \frac{2}{2} = 1$$

NPV'S $y-2 \neq 0$ $x-6 \neq 0$
 $y \neq 2$ $x \neq 6$

PC II 7.2. Con't... 3.

$$\begin{aligned}
 4. a) \quad & \frac{3a^3}{a^2-9} \cdot \frac{2a-6}{2a^2} \\
 & = \frac{3a^3}{(a+3)(a-3)} \cdot \frac{2(a-3)}{2a^2} \\
 & = \frac{3a}{a+3}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad & \frac{x^2-121}{x^2-4} \cdot \frac{x+2}{x-11} \\
 & = \frac{(x+11)(x-11)}{(x+2)(x-2)} \cdot \frac{(x+2)}{(x-11)} \\
 & = \frac{x+11}{x-2}
 \end{aligned}$$

$$\begin{aligned}
 c) \quad & \frac{15m}{2m+6} \div \frac{10m}{3m+9} \\
 & = \frac{15m}{2m+6} \cdot \frac{3m+9}{10m} \\
 & = \frac{15m}{2(m+3)} \cdot \frac{3(m+3)}{10m} \\
 & = \frac{45}{20} \\
 & = \frac{9}{4}
 \end{aligned}$$

$$\begin{aligned}
 d) \quad & \frac{5x-10}{6x+6} \div \frac{2x-4}{x+1} \\
 & = \frac{5x-10}{6x+6} \cdot \frac{x+1}{2x-4} \\
 & = \frac{5(x-2)}{6(x+1)} \cdot \frac{x+1}{2(x-2)} \\
 & = \frac{5}{12}
 \end{aligned}$$

$$\begin{aligned}
 e) \quad & \frac{4a^2-10}{a-3b} \div \frac{6a^2-15}{2a^2-18b^2} \\
 & = \frac{4a^2-10}{a-3b} \cdot \frac{2a^2-18b^2}{6a^2-15} \\
 & = \frac{4a^2-10}{a-3b} \cdot \frac{2a^2-18b^2}{3(2a^2-5)} \\
 & = \frac{2(2a^2-5)}{a-3b} \cdot \frac{2(a^2-9b^2)}{3(2a^2-5)} \\
 & = \frac{4(a+3b)(a-3b)}{3(a-3b)} \\
 & = \frac{4(a+3b)}{3} \\
 & = \frac{4a+12b}{3}
 \end{aligned}$$

$$\begin{aligned}
 f) \quad & \frac{y+2}{ay-by} \div \frac{y^2+2y}{ay^2-by^2} \\
 & = \frac{y+2}{ay-by} \cdot \frac{ay^2-by^2}{y^2+2y} \\
 & = \frac{y+2}{ay-by} \cdot \frac{y^2(a-b)}{y^2(y+2)} \\
 & = \frac{y+2}{y(a-b)} \cdot \frac{y^2(a-b)}{y(y+2)} \\
 & = \frac{y^2}{y^2} \\
 & = 1
 \end{aligned}$$

PC11 7.2 cont. 4

$$5. a) \frac{a^2-3a-10}{25-a^2} \div \frac{a+2}{a+5}$$

$$= \frac{(a-5)(a+2)}{(5-a)(5+a)} \cdot \frac{(a+5)}{(a+2)}$$

$$= \frac{(a-5)(a+2)}{-\cancel{(a-5)}(5+a)} \cdot \frac{(a+5)}{\cancel{(a+2)}}$$

$$= -1$$

$$b) \frac{x^2+x-2}{x^2-x} \cdot \frac{x^2+x}{x^2-1}$$

$$= \frac{(x+2)(x-1)}{x(x-1)} \cdot \frac{x(x+1)}{(x+1)(x-1)}$$

$$= \frac{x+2}{x-1}$$

$$c) \frac{x^2-2x-15}{x^2-9} \cdot \frac{3-x}{x-5}$$

$$= \frac{(x-5)(x+3)}{(x+3)(x-3)} \cdot \frac{-(x-3)}{(x-5)}$$

$$= -1$$

$$d) \frac{a^2+11ab+30b^2}{a^2-25b^2} \cdot \frac{3a^2-15ab}{6a^2+36ab}$$

$$= \frac{(a+5b)(a+6b)}{(a+5b)(a-5b)} \cdot \frac{3a(a-5b)}{6a(a+6b)}$$

$$= \frac{1}{2}$$

$$e) \frac{x^2+5xy+6y^2}{x^2+4xy-5y^2} \cdot \frac{x^2+3xy-10y^2}{x^2+xy-6y^2}$$

$$= \frac{(x+3y)(x+2y)}{(x+5y)(x-y)} \cdot \frac{(x+5y)(x-2y)}{(x+3y)(x-2y)}$$

$$= \frac{x+2y}{x-y}$$

$$f) \frac{m^2-9mn+14n^2}{m^2+7mn+12n^2} \div \frac{3m^2-21mn}{4m^3+16m^2n}$$

$$= \frac{(m-7n)(m-2n)}{(m+4n)(m+3n)} \cdot \frac{4m^2(m+4n)}{3m(m-7n)}$$

$$= \frac{4m(m-2n)}{3(m+3n)}$$

$$6. a) \frac{x+2y}{x-3y} \cdot \frac{x^2-9y^2}{x^2-4y^2} \div \frac{x+3y}{x-2y}$$

$$= \frac{(x+2y)}{(x-3y)} \cdot \frac{(x+3y)(x-3y)}{(x-2y)(x+2y)} \cdot \frac{(x-2y)}{(x+3y)}$$

$$= 1$$

$$b) \frac{(3a+7b)^2}{2a-5b} \cdot \frac{4a^2-25b^2}{9a^2-49b^2} \div \frac{2a+5b}{3a-7b}$$

$$= \frac{(3a+7b)(3a+7b)}{(2a-5b)} \cdot \frac{(2a+5b)(2a-5b)}{(3a+7b)(3a-7b)} \cdot \frac{(3a-7b)}{(2a+5b)}$$

$$= 3a+7b$$

PC11 7.2 cont... 5.

$$\begin{aligned}
 \text{b. c.) } & \frac{3x+6}{5-x} \div \frac{x^2-4}{x^2-8x+15} \cdot \frac{x^2-x-2}{x^2+x-12} \\
 & = \frac{3x+6}{5-x} \cdot \frac{x^2-8x+15}{x^2-4} \cdot \frac{x^2-x-2}{x^2+x-12} \\
 & = \frac{3(x+2)}{-(x-5)} \cdot \frac{(x-3)(x-5)}{(x+2)(x-2)} \cdot \frac{(x-2)(x+1)}{(x+4)(x-3)} \\
 & = \frac{-3(x+1)}{x+4}
 \end{aligned}$$

$$\begin{aligned}
 \text{d.) } & \frac{3x^2+3x-6}{x^2y-7xy} \div \frac{x^2-6x}{x^2} \cdot \frac{x^2y-13xy+42y}{6x^2+12x} \\
 & = \frac{3(x^2+x-2)}{xy(x-7)} \cdot \frac{x^2}{x^2-6x} \cdot \frac{y(x^2-13x+42)}{6x(x+2)} \\
 & = \frac{3(x+2)(x-1)}{xy(x-7)} \cdot \frac{x^2}{x(x-6)} \cdot \frac{y(x-6)(x-7)}{6x(x+2)} \\
 & = \frac{3x^2(x-1)}{6x^3} \\
 & = \frac{x-1}{2x}
 \end{aligned}$$

$$\begin{aligned}
 \text{e.) } & \frac{2m^2-7m-15}{2m^2-10m} \div \frac{4m^2-9}{6} \cdot \frac{3-2m}{1} \\
 & = \frac{2m^2-10m+3m-15}{2m(m-5)} \cdot \frac{6}{4m^2-9} \cdot \frac{3-2m}{1} \\
 & = \frac{2m(m-5)+3(m-5)}{2m(m-5)} \cdot \frac{6}{(2m+3)(2m-3)} \cdot \frac{-1(2m-3)}{1} \\
 & = \frac{(2m+3)(m-5)}{2m(m-5)} \cdot \frac{6}{(2m+3)(2m-3)} \cdot \frac{-1(2m-3)}{1} \\
 & = \frac{-6}{2m} \\
 & = \frac{-3}{m}
 \end{aligned}$$

PC11 7.2 cont... 6

6. f.) $\frac{8x^2-2x-3}{x^2-1} \div \frac{2x^2-3x-2}{2x-2} \div \frac{3-4x}{x+1}$

$mn = -4$
 $-4 \div 1$

$mn = -24$
 $-6 \div 4$

$$= \frac{8x^2-6x+4x-3}{(x+1)(x-1)} \cdot \frac{2x-2}{2x^2-4x+1x-2} \div \frac{3-4x}{x+1}$$

$$= \frac{2x(4x-3)+1(4x-3)}{(x+1)(x-1)} \cdot \frac{2(x-1)}{2x(x-2)+1(x-2)} \cdot \frac{x+1}{3-4x}$$

$$= \frac{(2x+1)(4x-3)}{(x+1)(x-1)} \cdot \frac{2(x-1)}{(2x+1)(x-2)} \cdot \frac{(x+1)}{-1(4x-3)}$$

$$= \frac{-2}{x-2}$$

7. a.) $\frac{\frac{1}{x}+4}{\frac{1}{x}-4}$ lcd = x

$$= \frac{(\frac{1}{x}+4) \cdot x}{(\frac{1}{x}-4) \cdot x}$$

$$= \frac{1+4x}{1-4x}$$

b.) $\frac{x-\frac{1}{4}}{x+\frac{1}{4}}$ lcd = 4

$$= \frac{(x-\frac{1}{4}) \cdot 4}{(x+\frac{1}{4}) \cdot 4}$$

$$= \frac{4x-1}{4x+1}$$

c.) $\frac{\frac{8}{x}+2}{\frac{4}{x^2}-1}$ lcd = x^2

$$= \frac{x^2 \cdot \frac{8}{x} + 2x^2}{x^2 \cdot \frac{4}{x^2} - 1x^2}$$

$$= \frac{8x + 2x^2}{4 - x^2}$$

d.) $\frac{5+\frac{2}{5x}}{3-\frac{3}{2x}}$ lcd = 10x

$$= \frac{5(10x) + \frac{2}{5x}(10x)}{3(10x) + \frac{3}{2x}(10x)}$$

$$= \frac{50x + 4}{30x + 15}$$

PC II 7.2 cont. 7

$$\begin{aligned}
 8. a) \quad & \frac{x^2 - xy - 12y^2}{x^2 - 2xy - 3y^2} \cdot \frac{x^2 + 5xy + 4y^2}{x^2 - 16y^2} \rightarrow \frac{(a+b) + 3(a-b)}{(a+b) - 3(a-b)} \\
 & = \frac{(x-4y)(x+3y)}{(x-3y)(x+y)} \cdot \frac{(x+4y)(x+y)}{(x+4y)(x-4y)} \\
 & = \frac{x+3y}{x-3y} \quad x=a+b \quad y=a-b \\
 & = \frac{a+b+3a-3b}{a+b-3a+3b} \\
 & = \frac{4a-2b}{-2a+4b} \\
 & = \frac{2(2a-b)}{2(-a+2b)} \\
 & = \frac{2a-b}{2b-a}
 \end{aligned}$$

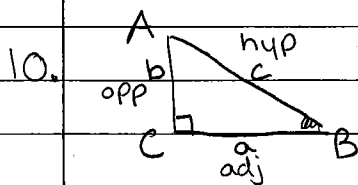
$$\begin{aligned}
 b) \quad & \left(\frac{3x-21y}{6x+12y} \right)^2 \div \frac{x^2-49y^2}{2x^2+8xy+8y^2} \\
 & = \left(\frac{3(x-7y)}{6(x+2y)} \right)^2 \cdot \frac{2(x^2+4xy+4y^2)}{x^2-49y^2} \\
 & = \frac{3^2(x-7y)(x-7y)}{6^2(x+2y)(x+2y)} \cdot \frac{2(x+2y)(x+2y)}{(x+7y)(x-7y)} \\
 & = \frac{9(x-7y)(2)}{36(x+7y)} \\
 & = \frac{18(x-7y)}{36(x+7y)} \\
 & = \frac{x-7y}{2(x+7y)} \quad x=a+b \quad y=a-b \\
 & \rightarrow \frac{(a+b) - 7(a-b)}{2((a+b) + 7(a-b))} \\
 & = \frac{a+b-7a+7b}{2(a+b+7a-7b)} \\
 & = \frac{-6a+8b}{2(8a-6b)} \\
 & = \frac{2(-3a+4b)}{2(8a-6b)} \\
 & = \frac{4b-3a}{8a-6b}
 \end{aligned}$$

PC II 7.2 cont... 8

9. $M(p-1, 2p+3)$ $N(2p-5, p+1)$
 x_1 y_1 x_2 y_2

a.) Slope = $\frac{y_2 - y_1}{x_2 - x_1}$
 $= \frac{(p+1) - (2p+3)}{(2p-5) - (p-1)}$
 $= \frac{p+1-2p-3}{2p-5-p+1}$
 $m = \frac{-p-2}{p-4}$

b.) $m = \frac{-p-2}{p-4} \perp m = -\left(\frac{p-4}{-p-2}\right)$
 $= \frac{-(p-4)}{-p-2} = \frac{-p+4}{-p-2} = \frac{4-p}{-p-2}$
or $= \frac{p-4}{p+2}$



a.) $\tan B = \frac{\text{opp}}{\text{adj}} = \frac{b}{a}$

c.) $\tan B = \frac{\sin B}{\cos B}$

b.) $\frac{\sin B}{\cos B} = \frac{\frac{b}{c}}{\frac{a}{c}}$
 $= \frac{b}{c} \div \frac{a}{c}$
 $= \frac{b}{c} \cdot \frac{c}{a}$
 $= \frac{b}{a}$

Pre-Calculus II 7.3/7.4

1. a.) $\frac{3}{2a} - 4$ lcd = $2a$

$$= \frac{3}{2a} - \frac{4(2a)}{2a}$$

$$= \frac{3 - 8a}{2a}$$

NPV:
 $2a \neq 0$
 $a \neq 0$

b.) $\frac{7}{y+1} - 2$ lcd = $y+1$

$$= \frac{7}{y+1} - \frac{2(y+1)}{y+1}$$

$$= \frac{7 - 2y - 2}{y+1}$$

$$= \frac{5 - 2y}{y+1}$$

NPV:
 $y+1 \neq 0$
 $y \neq -1$

c.) $\frac{x-2}{x+4}$ lcd = $x+4$

$$= \frac{x(x+4)}{x+4} - \frac{2}{x+4}$$

$$= \frac{x^2 + 4x - 2}{x+4}$$

NPV:
 $x+4 \neq 0$
 $x \neq -4$

d.) $\frac{4}{x-1} - (x-2)$ lcd = $x-1$

$$= \frac{4}{x-1} - \frac{(x-2)(x-1)}{x-1}$$

$$= \frac{4 - (x^2 - 3x + 2)}{x-1}$$

$$= \frac{4 - x^2 + 3x - 2}{x-1}$$

$$= \frac{-x^2 + 3x + 2}{x-1}$$

NPV:
 $x-1 \neq 0$
 $x \neq 1$

e.) $\frac{x-5}{x-3} + \frac{2}{x-3}$ lcd = $x-3$

$$= \frac{x(x-3)}{x-3} - \frac{5(x-3)}{x-3} + \frac{2}{x-3}$$

$$= \frac{x^2 - 3x - 5x + 15 + 2}{x-3}$$

$$= \frac{x^2 - 8x + 17}{x-3}$$

NPV:
 $x-3 \neq 0$
 $x \neq 3$

f.) $\frac{2}{x-4} - \frac{x-8}{x-4}$ lcd = $x-4$

$$= \frac{2}{x-4} - \frac{x(x-4)}{x-4} - \frac{8(x-4)}{x-4}$$

$$= \frac{2 - x^2 + 4x - 8x + 32}{x-4}$$

$$= \frac{-x^2 - 4x + 34}{x-4}$$

NPV:
 $x-4 \neq 0$
 $x \neq 4$

PC 11 7.3/7.4 con't...2.

2. a) $\frac{x-3}{x-2} + \frac{1}{x-3}$ lcd = $(x-2)(x-3)$

$$= \frac{(x-3)(x-3)}{(x-2)(x-3)} + \frac{1(x-2)}{(x-3)(x-2)}$$

$$= \frac{x^2 - 6x + 9 + x - 2}{(x-2)(x-3)}$$

$$= \frac{x^2 - 5x + 7}{(x-2)(x-3)}$$

b.) $\frac{x+6}{x-3} + \frac{x-4}{x-5}$ lcd = $(x-3)(x-5)$

$$= \frac{(x+6)(x-5)}{(x-3)(x-5)} + \frac{(x-4)(x-3)}{(x-5)(x-3)}$$

$$= \frac{x^2 - 5x + 6x - 3 + x^2 - 3x - 4x + 12}{(x-3)(x-5)}$$

$$= \frac{2x^2 + 6x + 9}{(x-3)(x-5)}$$

c.) $\frac{2}{x} - \frac{3x}{x-2}$ lcd = $x(x-2)$

$$= \frac{2(x-2)}{x(x-2)} - \frac{3x(x)}{(x-2)(x)}$$

$$= \frac{2x - 4 - 3x^2}{x(x-2)}$$

$$= \frac{-3x^2 + 2x - 4}{x(x-2)}$$

d.) $\frac{3y}{2(y+9)} + \frac{5y}{3(y+9)}$ lcd = $6(y+9)$

$$= \frac{3y}{2(y+9)} \cdot \frac{3}{3} + \frac{5y}{3(y+9)} \cdot \frac{2}{2}$$

$$= \frac{9y + 10y}{6(y+9)}$$

$$= \frac{19y}{6(y+9)}$$

e.) $\frac{5}{3(a-7)} - \frac{2}{3(a+1)}$ { lcd = $3(a-7)(a+1)$

$$= \frac{5(a+1)}{3(a-7)(a+1)} - \frac{2(a-7)}{3(a+1)(a-7)}$$

$$= \frac{5a+5-2a+14}{3(a-7)(a+1)}$$

$$= \frac{3a+19}{3(a-7)(a+1)}$$

f.) $\frac{3x}{x-2} - \frac{4x}{x-3}$ lcd = $(x-2)(x-3)$

$$= \frac{3x(x-3)}{(x-2)(x-3)} - \frac{4x(x-2)}{(x-3)(x-2)}$$

$$= \frac{3x^2 - 9x - 4x^2 + 8x}{(x-2)(x-3)}$$

$$= \frac{-x^2 - 1x}{(x-2)(x-3)}$$

PC11 7.3/7.4 con't...3

$$\begin{aligned}
 3. a) \quad & \frac{5x}{10x-15} - \frac{4x}{16x-24} \\
 & = \frac{5x}{5(2x-3)} - \frac{4x}{8(2x-3)} \\
 & = \frac{x}{2x-3} - \frac{x}{2(2x-3)} \\
 & = \frac{2x}{2(2x-3)} - \frac{x}{2(2x-3)} \\
 & = \frac{2x-x}{2(2x-3)} \\
 & = \frac{x}{2(2x-3)}
 \end{aligned}$$

$$\begin{aligned}
 \text{lcd} &= 2(2x-3) \\
 \text{NPI: } & 2x-3 \neq 0 \\
 & 2x \neq 3 \\
 & x \neq \frac{3}{2}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad & \frac{2x+5}{3x-12} - \frac{2x}{4-x} \\
 & = \frac{2x+5}{3(x-4)} - \frac{2x}{-(x-4)} \\
 & = \frac{2x+5}{3(x-4)} + \frac{2x}{(x-4)} \cdot \frac{3}{3} \\
 & = \frac{2x+5+6x}{3(x-4)} \\
 & = \frac{8x+5}{3(x-4)}
 \end{aligned}$$

$$\begin{aligned}
 \text{lcd} &= 3(x-4) \\
 \text{NPI: } & x-4 \neq 0 \\
 & x \neq 4
 \end{aligned}$$

$$\begin{aligned}
 c) \quad & \frac{x-7}{x^2-2x-15} - \frac{3x}{x-5} \\
 & = \frac{x-7}{(x-5)(x+3)} - \frac{3x(x+3)}{(x-5)(x+3)} \\
 & = \frac{x-7-3x^2-9x}{(x-5)(x+3)} \\
 & = \frac{-3x^2-8x-7}{(x-5)(x+3)}
 \end{aligned}$$

$$\begin{aligned}
 \text{lcd} &= (x-5)(x+3) \\
 \text{NPI: } & x-5 \neq 0 \quad x+3 \neq 0 \\
 & x \neq 5 \quad x \neq -3
 \end{aligned}$$

PC II 7.3/7.4 cont. 4

3. d.) $\frac{4x+1}{x+3} + \frac{x-6}{x^2-9}$

$$= \frac{4x+1}{x+3} + \frac{x-6}{(x+3)(x-3)}$$

$$\text{lcd} = (x+3)(x-3)$$

$$= \frac{(4x+1)(x-3) + (x-6)}{(x+3)(x-3)}$$

$$\text{NPV: } \begin{array}{ll} x+3 \neq 0 & x-3 \neq 0 \\ x \neq -3 & x \neq 3 \end{array}$$

$$= \frac{4x^2 - 12x + x - 3 + x - 6}{(x+3)(x-3)}$$

$$= \frac{4x^2 - 10x - 9}{(x+3)(x-3)}$$

e.) $\frac{3x}{x-1} - \frac{2x}{x^2+x-2}$

$$= \frac{3x}{x-1} - \frac{2x}{(x+2)(x-1)}$$

$$\text{lcd} = (x+2)(x-1)$$

$$= \frac{3x(x+2) - 2x}{(x-1)(x+2)}$$

$$\text{NPV: } \begin{array}{ll} x+2 \neq 0 & x-1 \neq 0 \\ x \neq -2 & x \neq 1 \end{array}$$

$$= \frac{3x^2 + 6x - 2x}{(x-1)(x+2)}$$

$$= \frac{3x^2 + 4x}{(x-1)(x+2)}$$

f.) $\frac{8x-3}{x^2-7x+12} - \frac{2x+1}{x-4}$

$$= \frac{8x-3}{(x-4)(x-3)} - \frac{(2x+1)(x-3)}{(x-4)(x-3)}$$

$$\text{lcd} = (x-4)(x-3)$$

$$= \frac{8x-3 - (2x^2 - 6x + x - 3)}{(x-4)(x-3)}$$

$$\text{NPV: } \begin{array}{ll} x-4 \neq 0 & x-3 \neq 0 \\ x \neq 4 & x \neq 3 \end{array}$$

$$= \frac{8x-3 - 2x^2 + 6x - x + 3}{(x-4)(x-3)}$$

$$= \frac{-2x^2 + 13x}{(x-4)(x-3)}$$

PC 11 7.3/7.4 cont...5

$$\begin{aligned}
 \text{4. a.) } & \frac{x+3}{x^2+11x+24} - \frac{2x+10}{x^2+11x+30} \\
 & = \frac{(x+3)}{(x+8)(x+3)} - \frac{2(x+5)}{(x+5)(x+6)} \\
 & = \frac{1}{x+8} - \frac{2}{x+6} \\
 & = \frac{1(x+6) - 2(x+8)}{(x+8)(x+6)} \\
 & = \frac{x+6 - 2x-16}{(x+8)(x+6)} \\
 & = \frac{-x-10}{(x+8)(x+6)}
 \end{aligned}$$

$$\begin{aligned}
 \text{b.) } & \frac{m-4}{m^2-8m+16} + \frac{3m+21}{m^2+12m+35} \\
 & = \frac{m-4}{(m-4)(m-4)} + \frac{3(m+7)}{(m+7)(m+5)} \\
 & = \frac{1}{m-4} + \frac{3}{m+5} \\
 & = \frac{1(m+5) + 3(m-4)}{(m-4)(m+5)} \\
 & = \frac{m+5 + 3m-12}{(m-4)(m+5)} \\
 & = \frac{4m-7}{(m-4)(m+5)}
 \end{aligned}$$

$$\begin{aligned}
 \text{c.) } & \frac{3x+9}{x^2+5x+6} - \frac{2x-2}{x^2+x-2} \\
 & = \frac{3(x+3)}{(x+3)(x+2)} - \frac{2(x-1)}{(x+2)(x-1)} \\
 & = \frac{3}{x+2} - \frac{2}{x+2} \\
 & = \frac{1}{x+2}
 \end{aligned}$$

$$\begin{aligned}
 \text{d.) } & \frac{5m+25}{2m^2+13m+15} - \frac{10m-20}{m^2-4} \\
 & = \frac{5(m+5)}{(2m+3)(m+5)} - \frac{10(m-2)}{(m+2)(m-2)} \\
 & = \frac{5}{2m+3} - \frac{10}{m+2} \\
 & = \frac{5(m+2) - 10(2m+3)}{(2m+3)(m+2)} \\
 & = \frac{5m+10-20m-30}{(2m+3)(m+2)} \\
 & = \frac{-15m-20}{(2m+3)(m+2)}
 \end{aligned}$$

$2m^2+13m+15$
 $= 2m^2+10m+3m+15$
 $= 2m(m+5)+3(m+5)$
 $= (2m+3)(m+5)$

$$\begin{aligned}
 \text{e.) } & \frac{4x^2-20x}{x^2+2x-35} + \frac{3x-6}{3x^2-10x+8} \\
 & = \frac{4x(x-5)}{(x+7)(x-5)} + \frac{3(x-2)}{(3x-4)(x-2)} \\
 & = \frac{4x}{x+7} + \frac{3}{3x-4} \\
 & = \frac{4x(3x-4) + 3(x+7)}{(x+7)(3x-4)} \\
 & = \frac{12x^2-16x+3x+21}{(x+7)(3x-4)}
 \end{aligned}$$

$$\begin{aligned}
 & = \frac{-15m-20}{(2m+3)(m+2)} \\
 & = \frac{12x^2-13x+21}{(x+7)(3x-4)}
 \end{aligned}$$

$3x^2-10x+8$
 $3x^2-6x-4x+8$
 $3x(x-2)-4(x-2)$
 $(3x-4)(x-2)$

PC 11 7.3/7.4 con't...6

$$\begin{aligned}
 4. f.) \quad & \frac{2x}{3x^2-11x+6} - \frac{3x-12}{3x^2-14x+8} \\
 & = \frac{2x}{(3x-2)(x-3)} - \frac{3(x-4)}{(3x-2)(x-4)} \\
 & = \frac{2x}{(3x-2)(x-3)} - \frac{3(x-3)}{(3x-2)(x-3)} \\
 & = \frac{2x-3x+9}{(3x-2)(x-3)} \\
 & = \frac{-x+9}{(3x-2)(x-3)}
 \end{aligned}$$

$$\begin{aligned}
 & 3x^2-11x+6 \\
 & = 3x^2-9x-2x+6 \\
 & = 3x(x-3)-2(x-3) \\
 & = (3x-2)(x-3)
 \end{aligned}$$

$$\begin{aligned}
 & 3x^2-14x+8 \\
 & = 3x^2-12x-2x+8 \\
 & = 3x(x-4)-2(x-4) \\
 & = (3x-2)(x-4)
 \end{aligned}$$

$$\begin{aligned}
 5. a.) \quad & \frac{3x^2+6xy}{3x} - \frac{4y^2-2xy}{2y} \\
 & = \frac{3x(x+2y)}{3x} - \frac{2y(2y-x)}{2y} \\
 & = x+2y-2y+x \\
 & = 2x
 \end{aligned}$$

$$\begin{aligned}
 b.) \quad & \frac{x^2-5xy+6y^2}{x-3y} - \frac{x^2-xy-12y^2}{x-4y} \\
 & = \frac{(x-3y)(x-2y)}{(x-3y)} - \frac{(x-4y)(x+3y)}{(x-4y)} \\
 & = x-2y-x-3y \\
 & = -5y
 \end{aligned}$$

$$\begin{aligned}
 c.) \quad & \frac{a-b}{a^2+2ab-3b^2} + \frac{a+b}{a^2-2ab-3b^2} \\
 & = \frac{(a-b)}{(a+3b)(a-b)} + \frac{(a+b)}{(a-3b)(a+b)} \\
 & = \frac{1}{a+3b} + \frac{1}{a-3b} \\
 & = \frac{a-3b}{(a+3b)(a-3b)} + \frac{a+3b}{(a-3b)(a+3b)} \\
 & = \frac{2a}{(a+3b)(a-3b)}
 \end{aligned}$$

$$\begin{aligned}
 d.) \quad & \frac{b}{3-a} - \frac{b}{a+3} + \frac{1}{a^2-9} \\
 & = \frac{b}{-(a-3)} - \frac{b}{(a+3)} + \frac{1}{(a+3)(a-3)} \\
 & = \frac{-b(a+3) - b(a-3) + 1}{(a-3)(a+3)} \\
 & = \frac{-ab-3b-ab+3b+1}{(a-3)(a+3)} \\
 & = \frac{-2ab+1}{(a-3)(a+3)}
 \end{aligned}$$

PC 11 7.3/7.4 continue

$$6. \quad A \left(\frac{p-1}{2}, \frac{p}{3} \right) \quad B \left(\frac{p}{3}, \frac{2p-3}{4} \right)$$

$x_1 \quad y_1 \qquad \qquad x_2 \quad y_2$

$$\begin{aligned} \text{a.) Slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{\frac{2p-3}{4} - \frac{p}{3}}{\frac{p}{3} - \frac{p-1}{2}} \\ &= \frac{\frac{3(2p-3)}{12} - \frac{4}{12} \left(\frac{p}{3} \right)}{\frac{4}{12} \left(\frac{p}{3} \right) - \frac{6}{12} \left(\frac{p-1}{2} \right)} \\ &= \frac{\frac{6p-9}{12} - \frac{4p}{12}}{\frac{4p}{12} - \frac{6p+6}{12}} \end{aligned} \quad \rightarrow \quad \begin{aligned} &= \frac{6p-9-4p}{12} \div \frac{4p-6p+6}{12} \\ &= \frac{2p-9}{12} \cdot \frac{12}{-2p+6} \\ &= \frac{2p-9}{-2p+6} \\ &= \frac{2p-9}{-2(p-3)} \quad \text{or} \quad \frac{-2p+9}{2(p-3)} \quad p \neq 3 \end{aligned}$$

b.) When $p=3$, the denominator = 0, therefore the slope is undefined so the line is vertical.

c.) If $p < 3$ then $-2p+9 > 0$
and $2(p-3) < 0$
∴ the slope is negative.

PC11 7.3/7.4 cont... 8

$$7. \left(\frac{p}{p-x} + \frac{q}{q-x} + \frac{r}{r-x} \right) - \left(\frac{x}{p-x} + \frac{x}{q-x} + \frac{x}{r-x} \right)$$

$$= \frac{p(q-x)(r-x) + q(p-x)(r-x) + r(p-x)(q-x)}{(p-x)(q-x)(r-x)} - \frac{(x(q-x)(r-x) + x(p-x)(r-x) + x(p-x)(q-x))}{(p-x)(q-x)(r-x)}$$

$$= \frac{p(qr - qx - rx + x^2) + q(pr - px - rx + x^2) + r(pq - px - qx + x^2)}{(p-x)(q-x)(r-x)} - \frac{(x(qr - qx - rx + x^2) + x(pr - px - rx + x^2) + x(pq - px - qx + x^2))}{(p-x)(q-x)(r-x)}$$

$$= \frac{pqr - pqx - prx + px^2 + pqr - pqx - qrx + qx^2 + pqr - prx - qrx + rx^2}{(p-x)(q-x)(r-x)} - \frac{(qrx - qx^2 - rx^2 + x^3 + prx - px^2 - rx^2 + x^3 + pqx - px^2 - qx^2 + x^3)}{(p-x)(q-x)(r-x)}$$

$$= \frac{3pqr - 2pqx - 2prx + px^2 - 2qrx + qx^2 + rx^2}{(p-x)(q-x)(r-x)} - \frac{(qrx - 2qx^2 - 2rx^2 + 3x^3 + prx - 2px^2 + pqx)}{(p-x)(q-x)(r-x)}$$

$$= \frac{3pqr - 2pqx - 2prx + px^2 - 2qrx + qx^2 + rx^2 - pqx - prx + 2px^2 - qrx + 2qx^2 + 2rx^2 - 3x^3}{(p-x)(q-x)(r-x)}$$

$$= \frac{3pqr - 3pqx - 3prx + 3px^2 - 3qrx + 3qx^2 + 3rx^2 - 3x^3}{pqr - pqx - prx + px^2 - qrx + qx^2 + rx^2 - x^3}$$

$$= \frac{3(pqr - pqx - prx + px^2 - qrx + qx^2 + rx^2 - x^3)}{pqr - pqx - prx + px^2 - qrx + qx^2 + rx^2 - x^3}$$

$$= 3$$

Pre-Calculus II 7.5

1. a) $\frac{2}{a} = 4$ lcd = a

$\frac{2}{a} = \frac{4a}{a}$ NPVs:
a ≠ 0

$\frac{2}{4} = \frac{4a}{4}$

$a = \frac{1}{2}$

b.) $-15 = \frac{-3m}{2}$

$2 \cdot -15 = \frac{-3m}{2} \cdot 2$

$\frac{-30}{-3} = \frac{-3m}{-3}$

$10 = m$

*The equation is defined for all real values of x.
x ∈ ℝ

c.) $\frac{90}{x} = \frac{2x}{5}$ lcd = 5x

$\frac{5(90)}{5(x)} = \frac{2x(x)}{5(x)}$ NPVs
5x ≠ 0

$\frac{450}{5x} = \frac{2x^2}{5x}$ x ≠ 0

$\frac{450}{2} = \frac{2x^2}{2}$

$225 = x^2$

$x = \pm \sqrt{225}$

$x = \pm 15$

d.) $\frac{2}{x} - \frac{7}{2x} = 6$ lcd = 2x

$\frac{2(2)}{2(x)} - \frac{7}{2x} = 6(\frac{2x}{2x})$ NPVs
2x ≠ 0

$\frac{4-7}{2x} = \frac{12x}{2x}$ x ≠ 0

$\frac{-3}{12} = \frac{12x}{12}$

$x = -\frac{1}{4}$

e.) $\frac{x}{4} - \frac{7}{4} = \frac{2}{x}$ lcd = 4x

$\frac{x(x)}{x(4)} - \frac{x(7)}{x(4)} = \frac{4(2)}{4(x)}$

$\frac{x^2-7x}{4x} = \frac{8}{4x}$ NPVs:
4x ≠ 0

$x^2-7x=8$

$x^2-7x-8=0$

$(x-8)(x+1)=0$

$x-8=0$ $x+1=0$

$x=8$ $x=-1$

f.) $\frac{2x}{7} - \frac{5}{7} = \frac{1}{x}$ lcd = 7x

$\frac{x(2x)}{x(7)} - \frac{x(5)}{x(7)} = \frac{7(1)}{7(x)}$

$\frac{2x^2-5x}{7x} = \frac{7}{7x}$ NPVs:
7x ≠ 0

$2x^2-5x=7$

$2x^2-5x-7=0$ mn = -14
-7+7

$2x^2-7x+2x-7=0$

$x(2x-7)+1(2x-7)=0$

$(2x-7)(x+1)=0$

$x = \frac{7}{2}$ $x = -1$

PC II 7.5 con't...2

2. a) $\frac{3x-2}{2} + 4 = \frac{13}{x} - \frac{1-6x}{4}$ lcd = 4x

$$\frac{2x(3x-2)}{2x(2)} + \frac{4x(4)}{4x} = \frac{13\left(\frac{4}{4}\right)}{x(4)} - \frac{x(1-6x)}{x(4)}$$

$$6x^2 - 4x + 16x = 52 - x + 6x^2$$

$$\begin{array}{r} 6x^2 - 4x + 16x \\ -6x^2 \end{array} = \begin{array}{r} 52 - x + 6x^2 \\ -6x^2 \end{array}$$

$$\begin{array}{r} 12x = 52 - x \\ +x \qquad +x \\ \hline 13x = 52 \\ \frac{13x}{13} \quad \frac{52}{13} \\ \hline x = 4 \end{array}$$

b) $\frac{2x-3}{3x-4} = \frac{2x+7}{3x+4}$

$$(2x-3)(3x+4) = (2x+7)(3x-4)$$

$$\begin{array}{r} 6x^2 + 8x - 9x - 12 \\ -6x^2 \end{array} = \begin{array}{r} 6x^2 - 8x + 21x - 28 \\ -6x^2 \end{array}$$

$$\begin{array}{r} -1x - 12 = 13x - 28 \\ -13x + 12 \quad -13x + 12 \\ \hline -14x = -16 \\ \frac{-14x}{-14} \quad \frac{-16}{-14} \\ \hline x = \frac{8}{7} \end{array}$$

c) $\frac{3-x}{x-2} = 1 - \frac{2x-5}{x+2}$

lcd = (x-2)(x+2)

$$\frac{(3-x)(x+2)}{(x-2)(x+2)} = \frac{1(x-2)(x+2)}{(x-2)(x+2)} - \frac{(2x-5)(x-2)}{(x+2)(x-2)}$$

$$3x+6-x^2-2x = x^2+2x-2x-4 - (2x^2-4x-5x+10)$$

$$\begin{array}{r} x+6-x^2 \\ -x-6+x^2 \end{array} = \begin{array}{r} x^2-4-2x^2+4x+5x-10 \\ -x \qquad -6 \end{array}$$

$$0 = 8x - 20 \rightarrow x = \frac{20}{8} = \frac{5}{2}$$

$$20 = 8x$$

d) $\frac{2(x-1)}{x-3} = \frac{x-4}{x-5} + 1$ lcd = (x-3)(x-5)

$$\frac{2(x-1)(x-5)}{(x-3)(x-5)} = \frac{(x-4)(x-3)}{(x-5)(x-3)} + \frac{1(x-3)(x-5)}{(x-3)(x-5)}$$

$$2(x^2-5x-x+5) = x^2-3x-4x+12 + x^2-5x-3x+15$$

$$\begin{array}{r} 2x^2 - 10x - 2x + 10 \\ -2x^2 + 15x - 27 \end{array} = \begin{array}{r} 2x^2 - 15x + 27 \\ -2x^2 + 15x - 27 \end{array}$$

$$3x - 17 = 0 \rightarrow x = \frac{17}{3}$$

PCII 7.5 cont...3

$$2. e.) 1 - \frac{x-5}{5x-1} = \frac{4(x-3)}{5x-2} \quad \text{lcd} = (5x-1)(5x-2)$$

$$\frac{1(5x-1)(5x-2) - (x-3)(5x-2)}{(5x-1)(5x-2)} = \frac{4(x-3)(5x-1)}{(5x-1)(5x-2)}$$

$$25x^2 - 10x - 5x + 2 - (5x^2 - 2x - 25x + 10) = 4(5x^2 - x - 15x + 3)$$

$$25x^2 - 15x + 2 - 5x^2 + 27x - 10 = 20x^2 - 4x - 60x + 12$$

$$\cancel{20x^2} + 12x - 8 = \cancel{20x^2} - 64x + 12$$

+64x +8 +64x +8

$$76x = 20 \quad \rightarrow \quad x = \frac{20}{76} = \frac{5}{19}$$

$$f.) \frac{x}{x-3} + \frac{2}{x+3} = 0 \quad \text{lcd} = (x-3)(x+3)$$

$$\frac{x(x+3)}{(x-3)(x+3)} + \frac{2(x-3)}{(x+3)(x-3)} = 0 \frac{(x+3)(x-3)}{(x+3)(x-3)}$$

$$x^2 + 3x + 2x - 6 = 0$$

$$x^2 + 5x - 6 = 0$$

$$(x+6)(x-1) = 0 \quad \rightarrow \quad x = -6, x = 1$$

$$g.) \frac{x^2}{x^2-4} = \frac{2x}{x+2} \quad \text{lcd} = (x+2)(x-2)$$

$$\frac{x^2}{(x+2)(x-2)} = \frac{2x(x-2)}{(x+2)(x-2)}$$

$$x^2 = 2x^2 - 4x$$

-x^2 -x^2

$$0 = x^2 - 4x$$

$$0 = x(x-4)$$

$$x = 0 \quad x - 4 = 0$$

$$x = 4$$

PC II 7.5 cont'd. 4

$$2. h.) \frac{3x^2}{x^2-1} = \frac{x}{x+1} + \frac{x}{1-x}$$

$$\frac{3x^2}{(x+1)(x-1)} = \frac{x}{(x+1)} - \frac{x}{(x-1)} \quad \text{lcd} = (x+1)(x-1)$$

$$\frac{3x^2}{(x+1)(x-1)} = \frac{x(x-1)}{(x+1)(x-1)} - \frac{x(x+1)}{(x-1)(x+1)}$$

$$3x^2 = x^2 - x - x^2 - x$$

$$3x^2 = -2x$$

$$3x^2 + 2x = 0$$

$$x(3x+2) = 0$$

$$x=0 \quad 3x+2=0$$

$$x = -\frac{2}{3}$$

$$i.) \frac{9x^2}{x^2-25} = \frac{4x}{x-5} + \frac{x}{x+5}$$

$$\text{lcd} = (x+5)(x-5)$$

$$\frac{9x^2}{(x+5)(x-5)} = \frac{4x(x+5)}{(x-5)(x+5)} + \frac{x(x-5)}{(x+5)(x-5)}$$

$$9x^2 = 4x^2 + 20x + x^2 - 5x$$

$$9x^2 = 5x^2 + 15x$$

$$-5x^2 \quad -5x^2$$

$$4x^2 - 15x = 0$$

$$x(4x-15) = 0$$

$$x=0 \quad 4x-15=0$$

$$x = \frac{15}{4}$$

$$3. a.) \frac{5}{x+1} + \frac{4}{3} = \frac{x+1}{x-1}$$

$$\text{lcd} = 3(x+1)(x-1)$$

$$\frac{5(3(x-1))}{(x+1)(3)(x-1)} + \frac{4(x+1)(x-1)}{3(x+1)(x-1)} = \frac{(x+1)(3)(x+1)}{(x-1)(3)(x+1)}$$

$$15(x-1) + 4(x^2 - x + x - 1) = (3x+3)(x+1)$$

$$15x - 15 + 4x^2 - 4 = 3x^2 + 3x + 3x + 3$$

$$4x^2 + 15x - 19 = 3x^2 + 6x + 3$$

$$-3x^2 \quad -6x - 3 \quad -3x^2 - 6x - 3$$

$$x^2 + 9x - 22 = 0$$

$$(x+11)(x-2) = 0$$

$$x = -11 \quad x = 2$$

NPV'S

$$x+1 \neq 0 \rightarrow x \neq -1$$

$$x-1 \neq 0 \rightarrow x \neq 1$$

PC11 7.5 cont...5

3. b.) $\frac{2m+3}{m+3} + \frac{1}{2} = \frac{m+1}{m-1}$

lcd = $2(m+3)(m-1)$

NPV's

$$\frac{(2m+3)(2)(m-1)}{(m+3)(2)(m-1)} + \frac{1(m+3)(m-1)}{2(m+3)(m-1)} = \frac{(m+1)(2)(m+3)}{(m-1)(2)(m+3)}$$

$$(4m+6)(m-1) + (m^2-m+3m-3) = (2m+2)(m+3)$$

$$4m^2-4m+6m-6 + m^2+2m-3 = 2m^2+6m+2m+6$$

$$5m^2+4m-9 = 2m^2+8m+6$$

$$5m^2+4m-9-2m^2-8m-6=0$$

$$3m^2-4m-15=0$$

$$3m^2-9m+5m-15=0$$

$$3m(m-3)+5(m-3)=0$$

$$\rightarrow (m-3)(3m+5)=0$$

$$m-3=0 \quad 3m+5=0$$

$$m=3 \quad m=-\frac{5}{3}$$

$m+3 \neq 0$
 $\hookrightarrow m \neq -3$
 $m-1 \neq 0$
 $\hookrightarrow m \neq 1$

c.) $\frac{2x-3}{x-1} - \frac{x-1}{x+2} = \frac{2x-5}{x+2} + \frac{2-x}{1-x}$

* Recall $1-x = -(x-1)$

$$\frac{(2x-3)(x+2)}{(x-1)(x+2)} - \frac{(x-1)(x-1)}{(x+2)(x-1)} = \frac{(2x-5)(x-1)}{(x+2)(x-1)} - \frac{(2-x)(x+2)}{(x-1)(x+2)}$$

$$2x^2+4x-3x-6 - (x^2-x-x+1) = 2x^2-2x-5x+5 - (2x+4-x^2-2x)$$

$$2x^2+x-6-x^2+2x-1 = 2x^2-7x+5-2x-4+x^2+2x$$

$$x^2+3x-7 = 3x^2-7x+1$$

$$0 = 3x^2-7x+1-x^2-3x+7 \rightarrow 2(x-4)(x-1)=0$$

$$0 = 2x^2-10x+8$$

$$0 = 2(x^2-5x+4)$$

$$x-4=0 \quad x-1=0$$

$$x=4 \quad x=1$$

* Note: NPV's $x-1 \neq 0$ $x+2 \neq 0$

$x \neq 1$ $x \neq -2$

\therefore Solution: $x=4$.

PC11 7.5 cont...6

$$4 \ a) \frac{a^2+2a}{a^2+3a+2} = \frac{a-4}{3a-12} + \frac{a^2-6a+5}{a^2-2a-15}$$

$$\frac{a(a+2)}{(a+2)(a+1)} = \frac{a-4}{3(a-4)} + \frac{(a-5)(a-1)}{(a-5)(a+3)}$$

Note: NPVs

$$a+2 \neq 0 \rightarrow a \neq -2$$

$$a+1 \neq 0 \rightarrow a \neq -1$$

$$a-4 \neq 0 \rightarrow a \neq 4$$

$$a-5 \neq 0 \rightarrow a \neq 5$$

$$a+3 \neq 0 \rightarrow a \neq -3$$

$$\frac{a}{a+1} = \frac{1}{3} + \frac{a-1}{a+3}$$

$$\frac{3(a+3)a}{3(a+3)(a+1)} = \frac{1(a+1)(a+3)}{3(a+1)(a+3)} + \frac{3(a+1)(a-1)}{3(a+1)(a+3)}$$

$$\frac{3(a+3)a}{3(a+3)(a+1)} = \frac{1(a+1)(a+3)}{3(a+1)(a+3)} + \frac{3(a+1)(a-1)}{3(a+1)(a+3)}$$

$$(3a+9)(a) = a^2+3a+a+3 + 3(a^2-a+a-1)$$

$$3a^2+9a = a^2+4a+3+3a^2-3$$

$$0 = a^2 - 5a$$

$$0 = a(a-5)$$

$$\nearrow a=0, a=5 \text{ (this is an NPV)}$$

$$\therefore a=0.$$

$$b.) \frac{8x+24}{x^2+x-6} - \frac{20x^2-180}{x^4-13x^2-36} = 1$$

Note: NPVs

$$x+3 \neq 0 \rightarrow x \neq -3$$

$$x-2 \neq 0 \rightarrow x \neq 2$$

$$x^2-4 \neq 0 \rightarrow x \neq \pm 2$$

$$x^2-9 \neq 0 \rightarrow x \neq \pm 3$$

$$\frac{8(x+3)}{(x+3)(x-2)} - \frac{20(x^2-9)}{(x^2-4)(x^2-9)} = 1$$

$$\frac{8}{(x-2)} - \frac{20}{(x+2)(x-2)} = 1$$

$$\frac{8(x+2)}{(x-2)(x+2)} - \frac{20}{(x+2)(x-2)} = \frac{1(x+2)(x-2)}{(x+2)(x-2)}$$

$$8x+16-20 = x^2-4$$

$$0 = x^2-4-8x-16+20$$

$$0 = x^2-8x$$

$$0 = x(x-8)$$

$$x=0 \cdot x-8=0$$

$$x=8$$

PC11 7.5 con't., 7

$$4) c) \frac{x^2-x}{x-1} + \frac{6x-18}{x^2+2x-15} = \frac{6x^2+2x-12}{3x^2+x-6}$$

$$\frac{x(x-1)}{(x-1)} + \frac{6(x-3)}{(x+5)(x-3)} = \frac{2(3x^2+x-6)}{(3x^2+x-6)}$$

$$x + \frac{6}{x+5} = 2$$

lcd = (x+5)

$$\frac{x(x+5)}{x+5} + \frac{6}{x+5} = \frac{2(x+5)}{x+5}$$

$$x^2+5x+6 = 2x+10$$

$$x^2+5x+6-2x-10=0$$

$$x^2+3x-4=0$$

$$\rightarrow (x+4)(x-1)=0$$

$$x = -4 \quad x = 1$$

reject. (this is an NPV)

$$\therefore x = -4$$

Note: NPV'S

$$x-1 \neq 0 \rightarrow x \neq 1$$

$$x+5 \neq 0 \rightarrow x \neq -5$$

$$x-3 \neq 0 \rightarrow x \neq 3$$

$$3x^2+x-6 \neq 0$$

using the quadratic formula

$$x \neq -1.6, 1.3$$

$$d) \frac{2x^2+3x-2}{2x^2+5x+2} + \frac{x^2-1}{x^2+2x-3} = \frac{3x^2-4x+1}{2x^2-x-1} + \frac{x^2-x-6}{6x^2-6x-36}$$

$$\frac{2x^2+4x-x-2}{2x^2+4x+x+2} + \frac{(x+1)(x-1)}{(x+3)(x-1)} = \frac{3x^2-3x-x+1}{2x^2-2x+x-1} + \frac{(x-3)(x+2)}{6(x^2-x-6)}$$

$$\frac{2x(x+2)-1(x+2)}{2x(x+2)+1(x+2)} + \frac{(x+1)(x-1)}{(x+3)(x-1)} = \frac{3x(x-1)-1(x-1)}{2x(x-1)+1(x-1)} + \frac{(x-3)(x+2)}{6(x-3)(x+2)}$$

$$\frac{(2x-1)(x+2)}{(x+2)(2x+1)} + \frac{(x+1)(x-1)}{(x+3)(x-1)} = \frac{(3x-1)(x-1)}{(2x+1)(x-1)} + \frac{(x-3)(x+2)}{6(x-3)(x+2)}$$

$$\frac{2x-1}{2x+1} + \frac{x+1}{x+3} = \frac{3x-1}{2x+1} + \frac{1}{6}$$

lcd = 6(x+3)(2x+1)

NPV'S
x+2 ≠ 0 → x ≠ -2
2x+1 ≠ 0 → x ≠ -1/2
x+3 ≠ 0 → x ≠ -3

$$\frac{(2x-1)(6)(x+3) + (x+1)(6)(2x+1)}{6(x+3)(2x+1)} = \frac{(3x-1)(6)(x+3) + (x+3)(2x+1)}{6(x+3)(2x+1)}$$

$$(12x-6)(x+3) + (6x+6)(2x+1) = (18x-6)(x+3) + 2x^2+x+6x+3$$

$$12x^2+36x-6x-18+12x^2+6x+12x+6 = 18x^2+54x-6x-18+2x^2+7x+3$$

$$24x^2+48x-12 = 20x^2+55x-15$$

$$24x^2+48x-12-20x^2-55x+15=0$$

$$4x^2-7x+3=0$$

$$4x^2-4x-3x+3=0$$

$$4x(x-1)-3(x-1)=0$$

$$(4x-3)(x-1)=0$$

$$\rightarrow 4x-3=0 \quad x-1=0$$

$$4x=3 \quad x=1 \leftarrow \text{reject, NPV}$$

$$x = \frac{3}{4}$$

$$x \neq 1$$

$$\therefore x = \frac{3}{4}$$

PC 11 7.5 cont... 8

5. $\frac{1}{a} - \frac{1}{b} = \frac{4}{5}$ $b = \frac{1}{a}$

$$\frac{\frac{1}{a} - \frac{1}{b}}{\frac{1}{a} + \frac{1}{b}} = \frac{4}{5}$$

$$\frac{\left(\frac{1}{a} - \frac{1}{\frac{1}{a}}\right)}{\left(\frac{1}{a} + \frac{1}{\frac{1}{a}}\right)} = \frac{4}{5}$$

$$\frac{\left(\frac{1}{a} - 1 \div \frac{1}{a}\right)}{\left(\frac{1}{a} + 1 \div \frac{1}{a}\right)} = \frac{4}{5}$$

$$\frac{\left(\frac{1}{a} - 1 \cdot \frac{a}{1}\right)}{\left(\frac{1}{a} + 1 \cdot \frac{a}{1}\right)} = \frac{4}{5}$$

$$\frac{\left(\frac{1}{a} - a\right)}{\left(\frac{1}{a} + a\right)} = \frac{4}{5}$$

$$\frac{\left(\frac{1-a^2}{a}\right)}{\left(\frac{1+a^2}{a}\right)} = \frac{4}{5}$$

$$\left(\frac{1-a^2}{a}\right) \div \left(\frac{1+a^2}{a}\right) = \frac{4}{5}$$

$$\left(\frac{1-a^2}{a}\right) \cdot \left(\frac{a}{1+a^2}\right) = \frac{4}{5}$$

$$\frac{1-a^2}{1+a^2} = \frac{4}{5}$$

$$\frac{1-a^2}{1+a^2} = \frac{4}{5}$$

$$5(1-a^2) = 4(1+a^2)$$

$$5 - 5a^2 = 4 + 4a^2$$

$$-5 - 4a^2 \quad -5 \quad -4a^2$$

$$\frac{-9a^2}{-9} = \frac{-1}{-9}$$

$$a^2 = \frac{1}{9}$$

$$a = \pm \sqrt{\frac{1}{9}}$$

$$a = \pm \frac{1}{3}$$

PC11 7.5 cont...9

$$b. a.) \frac{1}{x} - \frac{1}{y} = a$$

$$\frac{1}{x} = a + \frac{1}{y}$$

$$\rightarrow x \neq 0, y \neq 0$$

$$\frac{1}{x} = \frac{ay + 1}{y}$$

$$x = \frac{y}{ay + 1}$$

$$b.) d = v_0 t + \frac{1}{2} g t^2$$

$$d - \frac{1}{2} g t^2 = v_0 t$$

$$\frac{2d - g t^2}{2} = v_0 t$$

$$\frac{1}{t} \left(\frac{2d - g t^2}{2} \right) = v_0 t \left(\frac{1}{t} \right) \rightarrow t \neq 0$$

$$\frac{2d - g t^2}{2t} = v_0$$

$$c.) I = \frac{E}{R + \frac{r}{n}}$$

$$\rightarrow R + \frac{r}{n} \neq 0 \rightarrow R \neq -\frac{r}{n}$$

$$I = \frac{E}{\frac{Rn + r}{n}}$$

$$\rightarrow n \neq 0$$

$$I = E \div \frac{Rn + r}{n}$$

$$I = E \cdot \frac{n}{(Rn + r)}$$

$$I(Rn + r) = En$$

$$IRn + Ir = En$$

$$Ir = En - IRn$$

$$Ir = n(E - IR)$$

$$\frac{Ir}{E - IR} = n$$

$$E - IR$$

$$\rightarrow E - IR \neq 0 \rightarrow E \neq IR$$

Pre-Calculus 11 7.6

1. Let x = Average speed of train

| | Distance | Avg Speed | Time |
|----------|----------|-----------|-------------------|
| Train | 1200 | x | $\frac{1200}{x}$ |
| Airplane | 1200 | $8x$ | $\frac{1200}{8x}$ |

Time the train takes = 14h + time airplane takes

$$\frac{1200}{x} = 14 + \frac{1200}{8x} \quad \text{lcd} = 8x$$

$$\frac{8(1200)}{8(x)} = \frac{8x(14)}{8x} + \frac{1200}{8x}$$

$$\frac{9600}{8x} = \frac{112x + 1200}{8x}$$

$$9600 = 112x + 1200$$

$$-1200 \quad -1200$$

$$\begin{aligned} 8400 &= 112x \\ 112 & \quad 112 \\ 75 &= x \quad (\text{train}) \end{aligned}$$

$$(\text{airplane}) 8x = 8(75) = 600$$

\therefore Speed of the train is 75 km/h and the average speed of the airplane is 600 km/h

2. Let x = one number

then $12-x$ is the other number

$$\frac{1}{x} + \frac{1}{12-x} = \frac{3}{8} \quad \text{lcd} = 8x(12-x)$$

$$\frac{8(12-x)(1)}{8(12-x)(x)} + \frac{8x(1)}{8x(12-x)} = \frac{3x(12-x)}{8x(12-x)}$$

$$96 - 8x + 8x = 36x - 3x^2$$

$$-36x + 3x^2 \quad -36x + 3x^2$$

$$3x^2 - 36x + 96 = 0$$

$$3(x^2 - 12x + 32) = 0$$

$$(x-8)(x-4) = 0$$

$$x=8 \quad x=4$$

$$\begin{aligned} x &= 8 & x &= 4 \\ 12-8 &= 4 & 12-4 &= 8 \end{aligned}$$

\therefore The two numbers are 4 and 8

PC11 7.6 con't... 2

3. Let x hours represent the time it takes Elaine to paint the room herself

| | | |
|--------------------------------------|-------------------------|--------------------------|
| In One Hour: Amount Painted together | Amount Painted by Jerry | Amount Painted by Elaine |
| $\frac{1}{3}$ | $\frac{1}{5}$ | $\frac{1}{x}$ |

$$\frac{1}{3} = \frac{1}{5} + \frac{1}{x} \quad \text{lcd} = 15x \quad \rightarrow \quad 5x = 3x + 15$$

$$\frac{5x(1)}{5x(3)} = \frac{3x(1)}{3x(5)} + \frac{15(1)}{15(x)}$$

$$\frac{5x}{15x} = \frac{3x + 15}{15x}$$

$$\frac{2x}{2} = \frac{15}{2} \quad \rightarrow \quad x = 7.5$$

\therefore Elaine would take 7.5 hours.

4. Let x represent the rate of ascent of the elevator

| | | | |
|--------------|----------|-----------|-----------------------|
| | Distance | Speed | Time |
| Travels up | 160m | x | $\frac{160}{x}$ |
| Travels down | 160m | $x + 0.7$ | $\frac{160}{x + 0.7}$ |

Elevator goes up. Stops for 36 seconds. Travels down.

Total time = 2.5 min = 150 seconds

a) $\frac{160}{x} + 36 + \frac{160}{x + 0.7} = 150 \quad \text{lcd} = x(x + 0.7)$

b) $\frac{160(x + 0.7)}{x(x + 0.7)} + \frac{36x(x + 0.7)}{x(x + 0.7)} + \frac{160x}{(x + 0.7)(x)} = \frac{150x(x + 0.7)}{x(x + 0.7)}$

$$160x + 112 + 36x^2 + 25.2x + 160x = 150x^2 + 105x$$

$$345.2x + 112 + 36x^2 = 150x^2 + 105x$$

$$-345.2x - 112 - 36x^2 - 36x^2 - 345.2x - 112$$

$$0 = 114x^2 - 240.2x - 112$$

con't... \rightarrow

PC 11 7.6 cont... 3

4. cont...

Note: to get rid of decimals, multiply by 10.

$$0 = \frac{1140x^2 - 2402x - 1120}{2}$$

$$0 = 570x^2 - 1201x - 560 \quad \leftarrow \text{this is simplified a, b, c integers}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1201) \pm \sqrt{(-1201)^2 - 4(570)(-560)}}{2(570)}$$

$$x = \frac{1201 \pm 1649}{1140}$$

$$x = \frac{1201 - 1649}{1140}$$

$$= -0.39 \quad (\text{Answer can't be negative})$$

$$x = \frac{1201 + 1649}{1140}$$

$$x = 2.5$$

\therefore The rate of ascent is 2.5 m/s.

$$c) \frac{2.5 \text{ m}}{\text{Sec}} \cdot \frac{1 \text{ km}}{1000 \text{ m}} \times \frac{60 \text{ Sec}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ hr}} = \frac{9 \text{ km}}{\text{h}}$$

5. Let x represent the speed of the plane in calm air.

| | | | |
|---------------|--------------|-----------|------------------------|
| | { Distance } | { Speed } | { Time = $\frac{d}{s}$ |
| With Headwind | 1200 | $x - 50$ | $\frac{1200}{x - 50}$ |
| Calm Air | 1200 | x | $\frac{1200}{x}$ |

Time with headwind $-\frac{1}{2}$ hour = Time in Calm Air.

$$\frac{1200}{x-50} - \frac{1}{2} = \frac{1200}{x} \quad \text{lcd} = 2x(x-50) \quad x = \frac{-(-50) \pm \sqrt{(-50)^2 - 4(1)(120000)}}{2(1)}$$

$$\frac{2x(1200)}{2x(x-50)} - \frac{1(x(x-50))}{2(x(x-50))} = \frac{1200(2(x-50))}{x(2(x-50))}$$

$$= \frac{50 \pm 694.6}{2}$$

$$\begin{array}{r} 2400x - x^2 + 50x = 2400x - 120000 \\ -2400x \quad \quad \quad -2400x \end{array}$$

$$= 372.3 \quad \text{or} \quad = -322.3$$

\leftarrow can't be negative

$$0 = x^2 - 50x - 120000$$

\therefore The plane's speed is 372 km/h in Calm Air.

PC 11 7.6 con 4... 4

6. $\frac{x+6}{(x+1)-2} = \frac{9}{2}$ $2(x+6) = 9(x-1)$ If $x=3$
 $\frac{x+6}{x-1} = \frac{9}{2}$ $2x+12 = 9x-9$ then $x+1=4$
 "cross multiply" $-2x+9 \quad -2x+9$
 $\frac{21}{7} = \frac{7x}{7}$ \therefore The two
 $3 = x$ consecutive numbers
 are 3 and 4

7. Let x be one number
 then $x+1$ is the next consecutive number

$$\frac{1}{x} + \frac{1}{x+1} = \frac{11}{30} \quad \text{kd} = 30x(x+1)$$

$$0 = 11x^2 - 55x + 6x - 30$$

$$0 = 11x(x-5) + 6(x-5)$$

$$0 = (11x+6)(x-5)$$

$$11x+6=0 \quad x-5=0$$

$$x = -\frac{6}{11} \quad x=5$$

not an integer $x+1=6$

$$\frac{1}{x} \left(\frac{30(x+1)}{30(x+1)} \right) + \frac{1}{(x+1)} \left(\frac{30x}{30x} \right) = \frac{11}{30} \left(\frac{x(x+1)}{x(x+1)} \right)$$

$$\frac{30x+30+30x}{30x(x+1)} = \frac{11x^2+11x}{30x(x+1)}$$

$$60x+30 = 11x^2+11x$$

$$-60x \quad -30 \quad -60x \quad -30$$

$$0 = 11x^2 - 49x - 30$$

\therefore The two consecutive integers are 5 & 6.

8. Let x represent the number of students who went on the trip

| | Cost | # of students | Cost per student |
|------------------|-------|---------------|-------------------|
| Originally Going | \$540 | $x+6$ | $\frac{540}{x+6}$ |
| Actually Going | \$540 | x | $\frac{540}{x}$ |

Original Cost Per Student + \$3 per student = New Cost Per Student

$$\frac{540}{x+6} + 3 = \frac{540}{x}$$

$$\frac{540}{(x+6)} \left(\frac{x}{x} \right) + \frac{3(x(x+6))}{x(x+6)} = \frac{540(x+6)}{x(x+6)}$$

$$540x + 3x^2 + 18x = 540x + 3240$$

$$3x^2 + 18x - 3240 = 0$$

$$3(x^2 + 6x - 1080) = 0$$

$$3(x+36)(x-30) = 0$$

$$x = -36 \quad x = 30$$

cant be negative.

\therefore There were 30 students who went.

PC 11 7.6 cont., 5.

9. Let x represent the amount of time to fill pool with hose B

| In One Hour: | Together | Hose A | Hose B |
|-----------------------|---------------|---------------|---------------|
| Amount of pool filled | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{x}$ |

$$\frac{1}{3} + \frac{1}{x} = \frac{1}{2} \quad \text{lcd} = 6x$$

$$\frac{2x}{2x} \left(\frac{1}{3} \right) + \frac{6}{6} \left(\frac{1}{x} \right) = \frac{3x}{3x} \left(\frac{1}{2} \right)$$

$$\frac{2x+6}{6x} = \frac{3x}{6x}$$

$$2x+6 = 3x$$

$$\begin{array}{r} 2x+6 = 3x \\ -2x \quad -2x \\ \hline 6 = x \end{array}$$

\therefore It would take 6 hours to fill pool with hose B.