Pre-Calculus 11
4.1

1. a) yes b.) no c) yes d) yes e) yes f.) no

2. The "roots" of the equation are the "zeros" of the equation. The terms are interchangable. On the graph of the function, the roots or zeros are the x-intercepts.

3. a.) $f(x) = \frac{1}{2}x^2 + 4x - 2$
   i.) y-intercept $= -2$
   ii.) minimum
   (x^2 term positive)

b.) $y = 5x - 8 - 3x^2$
   i.) y-intercept $= -8$
   ii.) maximum
   (x^2 term negative)

c.) $y = 1.5x - 3x^2 + 5$
   i.) y-intercept $= 5$
   ii.) maximum
   (x^2 term negative)

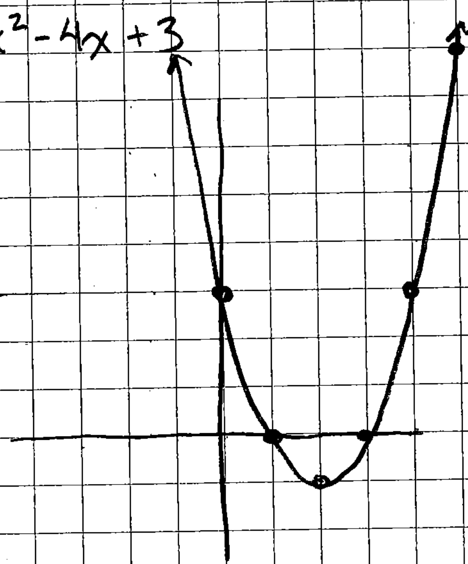
d.) $f(x) = 10 - 7x + x^2$
   i.) y-intercept $= 10$
   ii.) minimum
   (x^2 term positive)

4. a) i.) $(0,5)$ ii) $(-5,0)$ & $(-1,0)$
   iii.) $x = -3$ iv) $(-3,-4)$
   v) $\{x \mid x \in \mathbb{R}\}$ vi) $\{y \mid y \geq -4, y \in \mathbb{R}\}$

b.) i.) $(0,-8)$ ii) $(2,0)$ & $(4,0)$
   iii) $x = 3$ iv) $(3,1)$
   v) $\{x \mid x \in \mathbb{R}\}$ vi) $\{y \mid y \leq 1, y \in \mathbb{R}\}$

5. a.) $y = x^2 - 4x + 3$

x	y
-3	24
-2	15
-1	8
0	3
1	0
2	-1
3	0
4	3
5	8

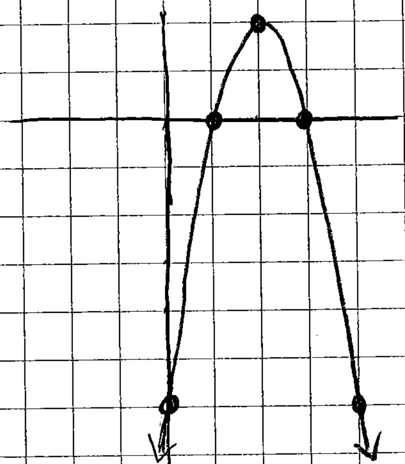


i) $(0,3)$
ii) $(1,0)$ & $(3,0)$
iii) $x = 2$
iv) $(2,-1)$
v) $\{x \mid x \in \mathbb{R}\}$
vi) $\{y \mid y \geq -1, y \in \mathbb{R}\}$

PC11 4.1 con't...2

5. b.) $y=-2x^2+8x-6$

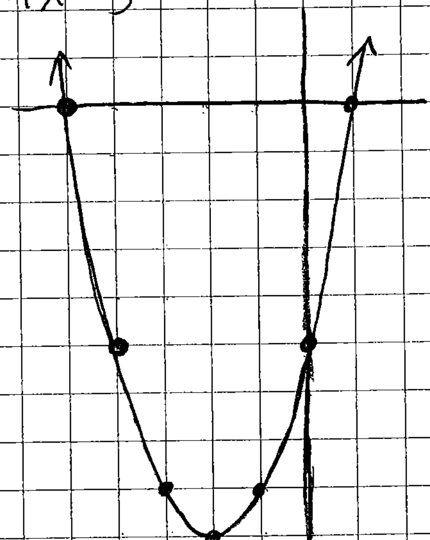
x	y
-1	-16
0	-6
1	0
2	2
3	0
4	-6
5	-16



i.) $(0,-6)$
ii) $(1,0)$ & $(3,0)$
iii) $x=2$
iv) $(2,2)$
v) $\{x \mid x \in \mathbb{R}\}$
vi) $\{y \mid y \leq 2, y \in \mathbb{R}\}$

c.) $y=x^2+4x-5$

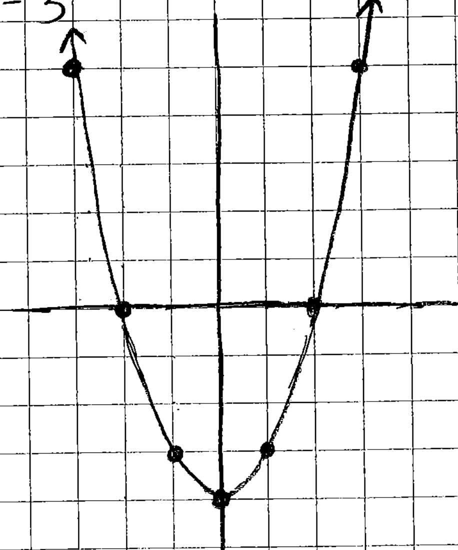
x	y
-6	7
-5	0
-4	-5
-3	-8
-2	-9
-1	-8
0	-5
1	0
2	7



i.) $(0,-5)$
ii.) $(-5,0)$ & $(1,0)$
iii) $x=-2$
iv) $(-2,-9)$
v) $\{x \mid x \in \mathbb{R}\}$
vi) $\{y \mid y \geq -9, y \in \mathbb{R}\}$

d.) $y=x^2-3$

x	y
-3	5
-2	0
-1	-3
0	-4
1	-3
2	0
3	5

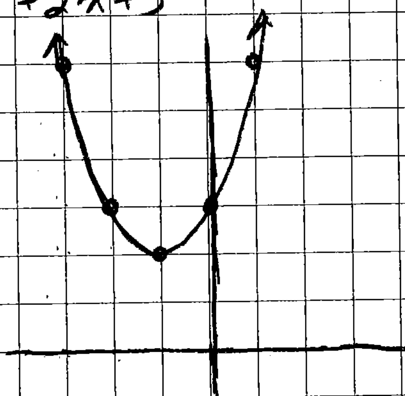


i.) $(0,-4)$
ii) $(-2,0)$ & $(2,0)$
iii) $x=0$
iv) $(0,-4)$
v) $\{x \mid x \in \mathbb{R}\}$
vi) $\{y \mid y \geq -4, y \in \mathbb{R}\}$

PC 11 4.1 con't... 3

5\. e.) $y = x^2 + 2x + 3$

x	y
-4	11
-3	6
-2	3
-1	2
0	3
1	6
2	11



i) $(0, 3)$

ii) there are no x-intercepts

iii) $x = -1$

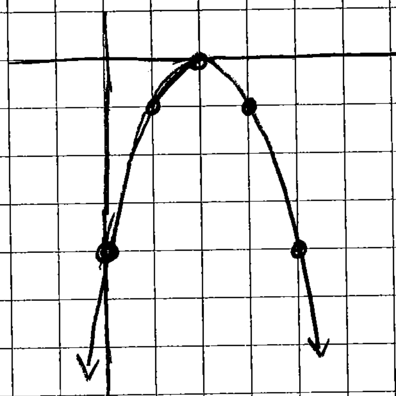
iv) $(-1, 2)$

v) $\{x \mid x \in \mathbb{R}\}$

vi) $\{y \mid y \geq 2, y \in \mathbb{R}\}$

f.) $y = -x^2 + 4x - 4$

x	y
-1	-9
0	-4
1	-1
2	0
3	-1
4	-4
5	-9



i) $(0, -4)$

ii) $(2, 0)$

iii) $x = 2$

iv) $(2, 0)$

v) $\{x \mid x \in \mathbb{R}\}$

vi) $\{y \mid y \leq 0, y \in \mathbb{R}\}$

6\. a.) $y = x^2 + 4x + 3$

i.) y-intercept $= 3$

ii) $x^2 + 4x + 3 = 0$

$(x+3)(x+1) = 0$

$x = -3 \quad x = -1$

$(-3, 0)$ & $(-1, 0)$

iii.) Axis $= \dfrac{-3 + -1}{2}$

$x = \dfrac{-4}{2}$

$x = -2$

iv) let $x = -2$

$y = x^2 + 4x + 3$

$y = (-2)^2 + 4(-2) + 3$

$y = 4 - 8 + 3$

$y = -1$

vertex $(-2, -1)$

v) $\{x \mid x \in \mathbb{R}\}$

vi.) x^2 is positive

so there is a minimum

$\{y \mid y \geq -1, y \in \mathbb{R}\}$

6. b) $y = x^2 + 2x - 3$

i.) y-intercept $= -3$

ii) $x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3 \quad x = 1$

$(-3, 0)$ + $(+1, 0)$

iii) Axis $= \frac{-3+1}{2}$

$x = \frac{-2}{2}$

$x = -1$

iv.) let $x = -1$

$y = x^2 + 2x + 3$

$y = (-1.0)^2 + 2(-1.0) + 3$

$y = 2$

vertex $(-1, 2)$

v.) $\{x \mid x \in \mathbb{R}\}$

vi.) x^2 is positive ↻

so there is a minimum

$\{y \mid y \geq 2, y \in \mathbb{R}\}$

c.) $y = 4x^2 - 8x - 5$

i.) y-intercept $= -5$

ii) $4x^2 - 8x - 5 = 0$

$4x^2 - 10x + 2x - 5 = 0$

$2x(2x-5) + 1(2x-5) = 0$

$(2x+1)(2x-5) = 0$

$x = \frac{-1}{2} \quad x = \frac{5}{2}$

$x = -0.5 \quad x = 2.5$

$(-0.5, 0)$ + $(2.5, 0)$

$mn = 4(-5)$
$= -20$
$-10 \; 2$

iii) Axis $= \frac{-0.5 + 2.5}{2}$

$x = \frac{2}{2}$

$x = 1$

iv.) let $x = 1$

$y = 4x^2 - 8x - 5$

$y = 4(1)^2 - 8(1) - 5$

$= 4 - 8 - 5$

$y = -9$

vertex $(1, -9)$

v.) $\{x \mid x \in \mathbb{R}\}$

vi.) x^2 is positive ↻

so there is a minimum

$\{y \mid y \geq -9, y \in \mathbb{R}\}$

6\. d.) $y = x^2 - 3x$

i.) y-intercept = 0

ii) $x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0 \quad x = 3$

$(0,0)$ & $(3,0)$

iii) Axis $= \frac{0+3}{2}$

$x = \frac{3}{2}$

$x = 1.5$

iv.) let $x = 1.5$

$y = x^2 - 3x$

$y = (1.5)^2 - 3(1.5)$

$y = -2.25$

$(0, -2.25)$

v.) $\{x \mid x \in \mathbb{R}\}$

vi.) x^2 is positive

so there is a minimum

$\{y \mid y \geq 1.5, y \in \mathbb{R}\}$

e.) $y = -x^2 + 4x - 4$

i.) y-intercept = -4

ii.) $-x^2 + 4x - 4 = 0$

$-1(x^2 - 4x + 4) = 0$

$-1(x-2)(x-2) = 0$

$x = 2 \quad x = 2$

$(2,0)$

iii) Axis:

$x = 2$

iv.) let $x = 2$

$y = -x^2 + 4x - 4$

$y = -(2)^2 + 4(2) - 4$

$= -4 + 8 - 4$

$= 0$

$(2,0)$

v.) $\{x \mid x \in \mathbb{R}\}$

vi) x^2 is negative

So there is a maximum

$\{y \mid y \leq 0, y \in \mathbb{R}\}$

PC 4.1 con't... 6

6. f.) $y = -x^2 + 4x - 3$

i.) y-intercept = -3

ii.) $-x^2 + 4x - 3 = 0$

$-1(x^2 - 4x + 3) = 0$

$-1(x-3)(x-1) = 0$

$x = 3 \quad x = 1$

$(3,0)$ + $(1,0)$

iii.) Axis $= \frac{3+1}{2}$

$x = \frac{4}{2}$

$x = 2$

iv.) let $x = 2$

$y = -x^2 + 4x - 3$

$y = -(2)^2 + 4(2) - 3$

$y = -4 + 8 - 3$

$y = 1$

$(2, 1)$

v.) $\{x \mid x \in \mathbb{R}\}$

vi.) x^2 is negative

So there is a maximum

$\{y \mid y \leq 1, y \in \mathbb{R}\}$

Pre-Calculus 11

4.4 Part II

1 a) $y=3(x-2)^2-5$

i.) opens up

ii.) Vertex $(2,-5)$

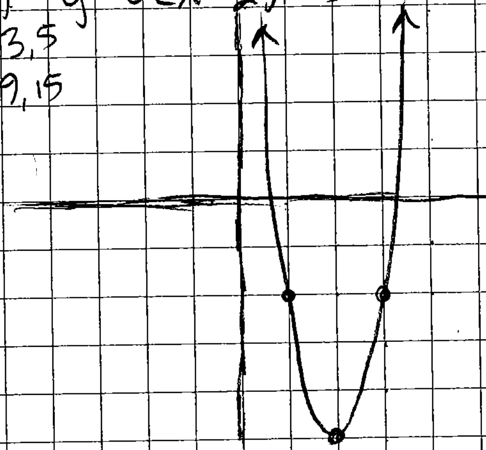
iii) Axis $x=2$

iv) d: $\{x \mid x \in \mathbb{R}\}$
r: $\{y \mid y \geq -5, y \in \mathbb{R}\}$

2a) $y=3(x-2)^2-5$

1, 3, 5

3, 9, 15



b.) $y=-4x^2+3$

i) opens down

ii.) Vertex $(0,3)$

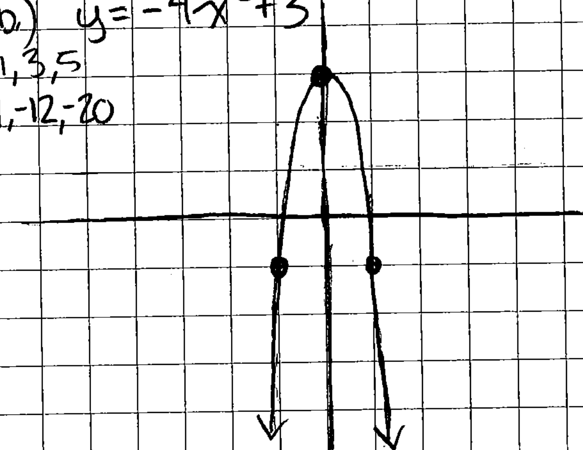
iii) Axis $x=0$

iv) d: $\{x \mid x \in \mathbb{R}\}$
r: $\{y \mid y \leq 3, y \in \mathbb{R}\}$

2, b.) $y=-4x^2+3$

1, 3, 5

-4, -12, -20



c.) $y=-\frac{1}{2}(x+3)^2$

i.) opens down

ii) Vertex $(-3,0)$

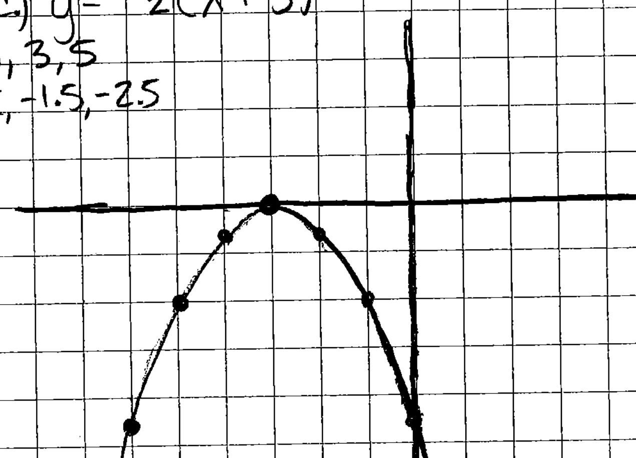
iii) Axis $x=-3$

iv) d: $\{x \mid x \in \mathbb{R}\}$
r: $\{y \mid y \leq 0, y \in \mathbb{R}\}$

2 c.) $y=-\frac{1}{2}(x+3)^2$

1, 3, 5

$-\frac{1}{2}$, -1.5, -2.5



PC 11 4.4 con't... 2

1. d.) $y = \frac{3}{4}(x-2)^2 + 1$

i.) ∪ opens up

ii) Vertex $(2, 1)$

iii) Axis $x = 2$

iv) d: $\{x \mid x \in \mathbb{R}\}$

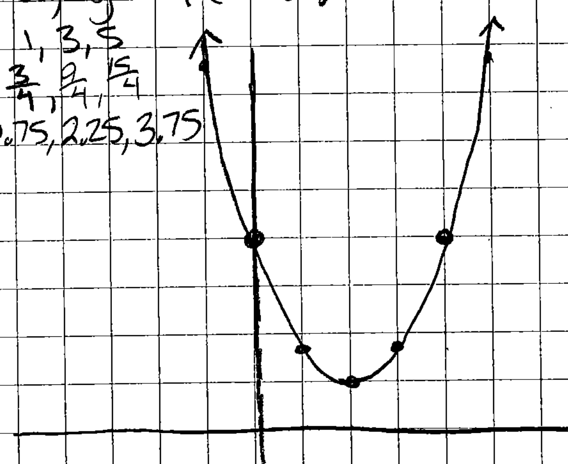
r: $\{y \mid y \geq 1, y \in \mathbb{R}\}$

2 d.) $y = \frac{3}{4}(x-2)^2 + 1$

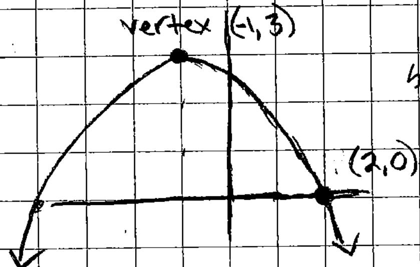
1, 3, 5

$\frac{3}{4}, \frac{9}{4}, \frac{15}{4}$

0.75, 2.25, 3.75



3.



$y = a(x-p)^2 + q$

subs vertex → $y = a(x-(-1))^2 + 3$

subs point → $0 = a(2+1)^2 + 3$

$0 = a(3)^2 + 3$

$0 = 9a + 3$

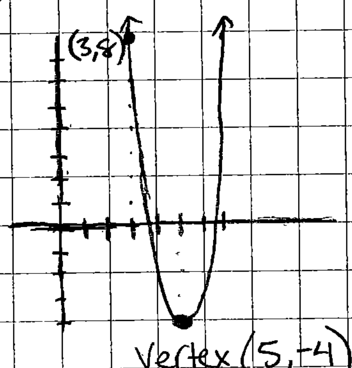
$-3 = 9a$

$-\frac{3}{9} = a$

$a = -\frac{1}{3}$

$y = -\frac{1}{3}(x+1)^2 + 3$

4.



$y = a(x-p)^2 + q$

subs vertex → $y = a(x-5)^2 - 4$

subs point → $8 = a(3-5)^2 - 4$

$8 = a(-2)^2 - 4$

$+4 \qquad +4$

$12 = a(4)$

$\frac{12}{4} = a$

$3 = a$

$y = 3(x-5)^2 - 4$

5. For each question, use vertex + choose one point.

a.) Vertex $(1, -4)$ Point $(0, -1)$

$y = a(x-p)^2 + q$

$y = a(x-1)^2 - 4$ ← subs vertex

$-1 = a(0-1)^2 - 4$ ← subs point

$-1 = a(1) - 4$ Solve for a

$+4 \qquad +4$

$3 = a$

$y = 3(x-1)^2 - 4$

PC 11 4.4 con't... 3

5 b.) Vertex $(2, -3)$ Point $(0, -2)$

$ (p, q) \qquad (x, y)$

$y = a(x-p)^2 + q$

$y = a(x-2)^2 - 3$ ← subs vertex

$-2 = a(0-2)^2 - 3$ ← subs point

$-2 = a(-2)^2 - 3$ solve for a

$+3 \qquad\qquad +3$

→ $1 = a(4)$

$\frac{1}{4} = a$

$y = \frac{1}{4}(x-2)^2 - 3$

c.) Vertex $(-3, -1)$ Point $(-1, -3)$

$y = a(x-p)^2 + q$

$y = a(x--3)^2 - 1$

$-3 = a(-1+3)^2 - 1$

$+1 \qquad\qquad +1$

$-2 = a(2)^2$

$-2 = a(4)$

$\frac{-2}{4} = a = -\frac{1}{2}$

$y = -\frac{1}{2}(x+3)^2 - 1$

d.) Vertex $(-3, -5)$ Point $(-1, -2)$

$y = a(x-p)^2 + q$

$y = a(x--3)^2 - 5$

$-2 = a(-1+3)^2 - 5$

$+5 \qquad\qquad +5$

$3 = a(2)^2$

$3 = a(4)$

$\frac{3}{4} = a$

$y = \frac{3}{4}(x+3)^2 - 5$

e.) Vertex $(-3, 4)$ Point $(0, 1)$

$y = a(x-p)^2 + q$

$y = a(x--3)^2 + 4$

$1 = a(0+3)^2 + 4$

$1 = a(9) + 4$

$-4 \qquad\qquad -4$

$-3 = a(9)$

$\frac{-3}{9} = a = -\frac{1}{3}$

$y = -\frac{1}{3}(x+3)^2 + 4$

f.) Vertex $(4, 6)$ Point $(0, 2)$

$y = a(x-p)^2 + q$

$y = a(x-4)^2 + 6$

$2 = a(0-4)^2 + 6$

$2 = a(-4)^2 + 6$

$-6 \qquad\qquad -6$

$-4 = a(16)$

$\frac{-4}{16} = a = -\frac{1}{4}$

$y = -\frac{1}{4}(x-4)^2 + 6$

PC11 4.4 con't... 4

5. g) Vertex $(-3,2)$ Point $(0,5)$

$y=a(x-p)^2+q$
$y=a(x--3)^2+2$
$5=a(0+3)^2+2$
$5=a(3)^2+2$
$-2 \qquad -2$
$3=a(9)$
$\frac{3}{9}=a=\frac{1}{3}$

$y=\frac{1}{3}(x+3)^2+2$

h.) Vertex $(-1,8)$ Point $(0,3)$

$y=a(x-p)^2+q$
$y=a(x--1)^2+8$
$3=a(0+1)^2+8$
$3=a(1)+8$
$-8 \qquad -8$
$-5=a$

$y=-5(x+1)^2+8$

i.) Vertex $(3,-4)$ Point $(0,5)$

$y=a(x-p)^2+q$
$y=a(x-3)^2-4$
$5=a(0-3)^2-4$
$5=a(-3)^2-4$
$+4 \qquad +4$
$9=a(9)$
$a=1$

$y=(x-3)^2-4$

j.) Vertex $(3,-2)$ Point $(1,-8)$

$y=a(x-p)^2+q$
$y=a(x-3)^2-2$
$-8=a(1-3)^2-2$
$-8=a(-2)^2-2$
$+2 \qquad +2$
$-6=a(4)$
$\frac{-6}{4}=a=-\frac{3}{2}$

$y=-\frac{3}{2}(x-3)^2-2$

6. Vertex $(0,0)$ Point $(\frac{3}{2},\frac{1}{3})$

$y=a(x-0)^2+0$
$y=ax^2$

$\frac{1}{3}=a(\frac{3}{2})^2$

$\frac{1}{3}=a\left(\frac{9}{4}\right)$

$\frac{4}{9}\cdot\frac{1}{3}=a\left(\cancel{\frac{9}{4}}\right)\cdot\cancel{\frac{4}{9}}$

$\frac{4}{27}=a$

$y=\frac{4}{27}x^2$

PC11 4.4 con't... 5

7. $y=(x+3)^2+q$ Point $(\underset{x}{1},\underset{y}{20})$

$20=(1+3)^2+q$

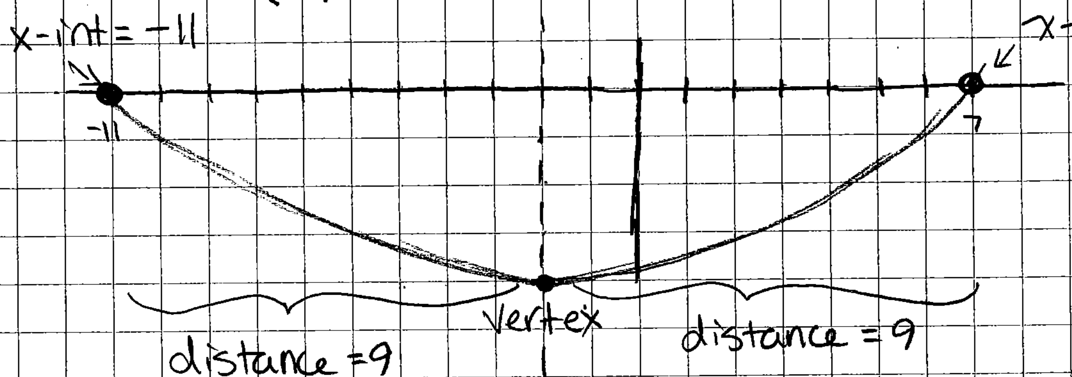
$20=(4)^2+q$

$20=16+q$

$-16 \quad -16$

$4=q$

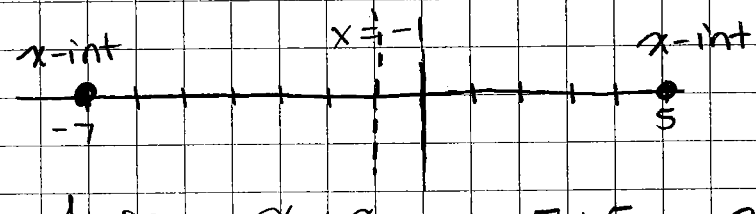
8. Vertex $(-2,-4)$ x-int $=7$



Axis of Symmetry $x=-2$

If one x-intercept is 9 units to the right of the axis of symmetry, the other x-intercept is 9 units to the left. $\therefore$ x-intercept $=-11$

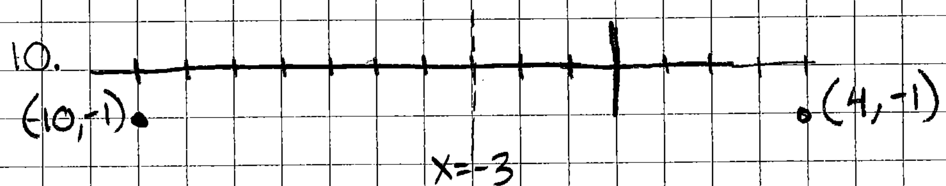
9.



The axis of symmetry will be right in the middle of the two x-intercepts.

$$\text{Axis} = \frac{x_1+x_2}{2} = \frac{-7+5}{2} = \frac{-2}{2} \Rightarrow x=-1$$

10.

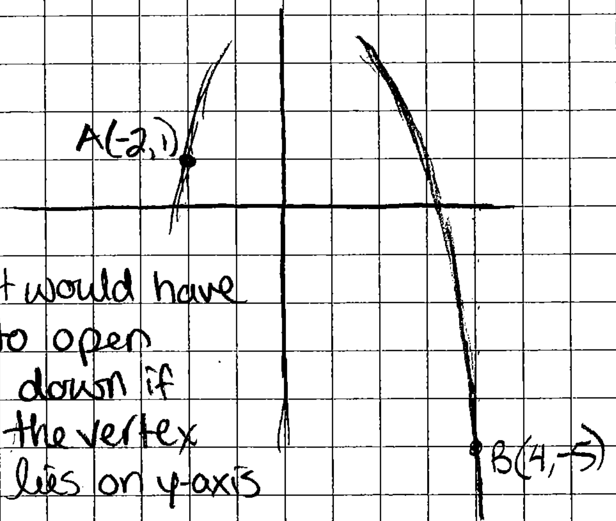


The axis of symmetry will be in the middle of the two x-values

$$\text{Axis} = \frac{x_1+x_2}{2} = \frac{-10+4}{2} = \frac{-6}{2} = -3 \Rightarrow x=-3$$

PC 11 4.4 con't...6

11. Axis of symmetry y-axis $\Rightarrow x=0$



It would have to open down if the vertex lies on y-axis

$y=a(x-p)^2+q$

$y=a(x-0)^2+q$

$y=ax^2+q$

A(-2,1)	B(4,-5)
$1=a(-2)^2+q$	$-5=a(4)^2+q$
$1=4a+q$	$-5=16a+q$

Solve the system of equations
(By elimination)

$$\begin{array}{r} 1=4a+q \\ -(-5=16a+q) \\ \hline 6=-12a \\ \frac{6}{-12}=a \\ -\frac{1}{2}=a \end{array}$$

Sub $a=-\frac{1}{2}$

$1=4a+q$

$1=4(-\frac{1}{2})+q$

$1=-2+q$

$+2 \quad +2$

$3=q$

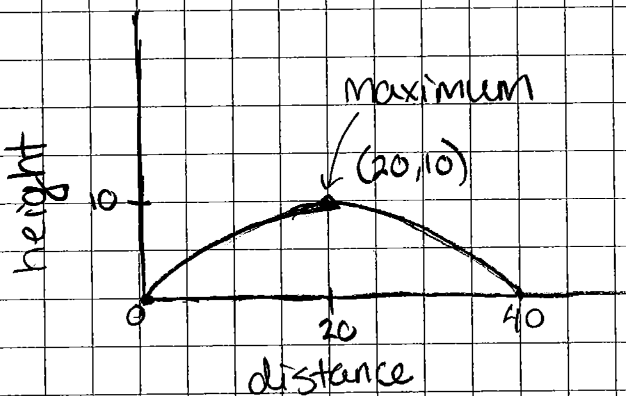
So $a=-\frac{1}{2}$, $q=3$

$y=ax^2+q$

$y=-\frac{1}{2}x^2+3$

PC 11 4.4 con't... 7

12. $h(d) = -0.025(d-20)^2 + 10$



a.) Maximum Height = 10 m

b.) Horizontal distance at maximum height = 20 m

c.) When the ball hits the ground, the height = 0m

$0 = -.025(d-20)^2 + 10$

$-10 \qquad\qquad -10$

$-10 = -.025(d-20)^2$

$\frac{-10}{-0.025} = (d-20)^2$

$400 = (d-20)^2$

$\pm\sqrt{400} = d-20$

$\pm 20 = d-20$

$d = 20 \pm 20$

$d = 20-20 \qquad d = 20+20$

$d = 0 \qquad d = 40\text{m}$

Starting point hits the ground.

Method 2: We know that the axis of symmetry is $x=20$, so if one x-intercept is 20 units left, the other x-intercept is 20 units right.

$\therefore$ distance = 40m

d.) $d = 10$

$h(10) = -0.025(10-20)^2 + 10$

height = 7.5 m

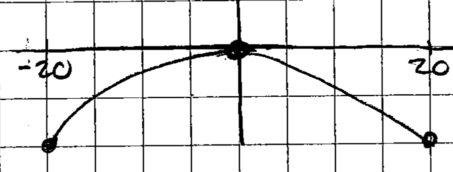
e.) $d = 34$

$h(34) = -0.025(34-20)^2 + 10$

height = 5.1 m

The opposing player would have to head the ball at a height of 5.1m so no, it is not possible.

f.)



$h = -0.025x^2$

Pre-Calculus 11 4.5

1. a) $x^2+10x+c$
$\hookrightarrow \frac{1}{2}(10)=5$
$\hookrightarrow 5^2=25$
$\therefore c=25$

b) $x^2-14x+c$
$\hookrightarrow \frac{1}{2}(-14)=-7$
$\hookrightarrow (-7)^2=49$
$\therefore c=49$

c.) $x^2+11x+c$
$\hookrightarrow \frac{1}{2}(11)=\frac{11}{2}$
$\hookrightarrow \left(\frac{11}{2}\right)^2=\frac{121}{4}$
$\therefore c=\frac{121}{4}$

d) $x^2+\frac{3}{4}x+c$
$\hookrightarrow \frac{1}{2}\left(\frac{3}{4}\right)=\frac{3}{8}$
$\hookrightarrow \left(\frac{3}{8}\right)^2=\frac{9}{64}$
$\therefore c=\frac{9}{64}$

e.) $x^2-1.2x+c$
$\hookrightarrow \frac{1}{2}(-1.2)=-0.6$
$\hookrightarrow (-0.6)^2=0.36$
$\therefore c=0.36$

f.) $x^2-10.3x+c$
$\hookrightarrow \frac{1}{2}(-10.3)=-5.15$
$\hookrightarrow (-5.15)^2=26.5225$
$\therefore c=26.5225$

2. a) $y=x^2+6x+8$ $\quad \frac{1}{2}(6)=3$
$=(x^2+6x)+8$ $\quad \hookrightarrow 3^2=9$
$=(x^2+6x+9-9)+8$
$=(x^2+6x+9)-9+8$
$y=(x+3)^2-1$

b) $y=x^2-10x+14$ $\quad \frac{1}{2}(-10)=-5$
$=(x^2-10x)+14$ $\quad \hookrightarrow (-5)^2=25$
$=(x^2-10x+25-25)+14$
$=(x^2-10x+25)-25+14$
$y=(x-5)^2-11$

c.) $y=x^2+2x-5$ $\quad \frac{1}{2}(2)=1$
$=(x^2+2x)-5$ $\quad \hookrightarrow 1^2=1$
$=(x^2+2x+1-1)-5$
$=(x^2+2x+1)-1-5$
$=(x+1)^2-6$

d.) $y=x^2-8x+1$ $\quad \frac{1}{2}(-8)=-4$
$=(x^2-8x)+1$ $\quad \hookrightarrow (-4)^2=16$
$=(x^2-8x+16-16)+1$
$=(x^2-8x+16)-16+1$
$=(x-4)^2-15$

e.) $y=x^2-4x-5$ $\quad \frac{1}{2}(-4)=-2$
$y=(x^2-4x)-5$ $\quad \hookrightarrow (-2)^2=4$
$=(x^2-4x+4-4)-5$
$=(x^2-4x+4)-4-5$
$y=(x-2)^2-9$

f.) $y=x^2+12x+30$ $\quad \frac{1}{2}(12)=6$
$=(x^2+12x)+30$ $\quad \hookrightarrow (6)^2=36$
$=(x^2+12x+36-36)+30$
$=(x^2+12x+36)-36+30$
$=(x+6)^2-6$

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3\. a) $y=2x^2+4x+7$ $\qquad \frac{1}{2}(2)=1$
$=2(x^2+2x)+7$ $\qquad \hookrightarrow (1)^2=1$
$=2(x^2+2x+1-1)+7$
$=2(x^2+2x+1)-2+7$
$y=2(x+1)^2+5$

b) $y=-2x^2+4x+5$ $\qquad \frac{1}{2}(-2)=-1$
$=-2(x^2-2x)+5$ $\qquad \hookrightarrow (-1)^2=1$
$=-2(x^2-2x+1-1)+5$
$=-2(x^2-2x+1)+2+5$
$y=-2(x-1)^2+7$

c) $y=3x^2-24x+40$ $\qquad \frac{1}{2}(-8)=-4$
$=3(x^2-8x)+40$ $\qquad \hookrightarrow (-4)^2=16$
$=3(x^2-8x+16-16)+40$
$=3(x^2-8x+16)-48+40$
$y=3(x-4)^2-8$

d) $y=-5x^2-20x-30$ $\qquad \frac{1}{2}(4)=2$
$=-5(x^2+4x)-30$ $\qquad \hookrightarrow (2)^2=4$
$=-5(x^2+4x+4-4)-30$
$=-5(x^2+4x-4)+20-30$
$y=-5(x+2)^2-10$

e) $y=-4x^2+24x-20$ $\qquad \frac{1}{2}(-6)=-3$
$=-4(x^2-6x)-20$ $\qquad \hookrightarrow (-3)^2=9$
$=-4(x^2-6x+9-9)-20$
$=-4(x^2-6x+9)+36-20$
$=-4(x-3)^2+16$

f) $y=-2x^2-16x-14$ $\qquad \frac{1}{2}(8)=4$
$=-2(x^2+8x)-14$ $\qquad \hookrightarrow (4)^2=16$
$=-2(x^2+8x+16-16)-14$
$=-2(x^2+8x+16)+32-14$
$=-2(x+4)^2+18$

4\. a) $y=4x^2+12x-5$ $\qquad \frac{1}{2}(3)=\frac{3}{2}$
$=4(x^2+3x)-5$ $\qquad \hookrightarrow (\frac{3}{2})=\frac{9}{4}$
$=4(x^2+3x+\frac{9}{4}-\frac{9}{4})-5$
$=4(x^2+3x+\frac{9}{4})-4(\frac{9}{4})-5$
$=4(x+\frac{3}{2})^2-9-5$
$y=4(x+\frac{3}{2})^2-14$

b) $y=-2x^2+14x-12$ $\qquad \frac{1}{2}(-7)=\frac{-7}{2}$
$=-2(x^2-7x)-12$ $\qquad \hookrightarrow (\frac{-7}{2})^2=\frac{49}{4}$
$=-2(x^2-7x+\frac{49}{4}-\frac{49}{4})-12$
$=-2(x^2-7x+\frac{49}{4})+2(\frac{49}{4})-12$
$=-2(x-\frac{7}{2})^2+\frac{49}{2}-\frac{24}{2}$
$=-2(x-\frac{7}{2})^2+\frac{25}{2}$

c) $y=3x^2+9x-2$ $\qquad \frac{1}{2}(3)=\frac{3}{2}$
$=3(x^2+3x)-2$ $\qquad \hookrightarrow (\frac{3}{2})^2=\frac{9}{4}$
$=3(x^2+3x+\frac{9}{4}-\frac{9}{4})-2$
$=3(x^2+3x+\frac{9}{4})-3(\frac{9}{4})-2$
$=3(x-\frac{3}{2})^2-\frac{27}{4}-\frac{8}{4}$
$=3(x-\frac{3}{2})^2-\frac{35}{4}$

d) $y=-2x^2+10x+3$ $\qquad \frac{1}{2}(-5)=\frac{-5}{2}$
$=-2(x^2-5x)+3$ $\qquad \hookrightarrow (-\frac{5}{2})^2=\frac{25}{4}$
$=-2(x^2-5x+\frac{25}{4}-\frac{25}{4})+3$
$=-2(x^2-5x+\frac{25}{4})+2(\frac{25}{4})+3$
$=-2(x-\frac{5}{2})^2+\frac{25}{2}+\frac{6}{2}$
$=-2(x-\frac{5}{2})^2+\frac{31}{2}$

PC11 4.5 con't...3

4 e.) $y=-5x^2-15x-5$ $\quad \frac{1}{2}(3)=\frac{3}{2}$ $\quad \hookrightarrow\left(\frac{3}{2}\right)^2=\frac{9}{4}$

$=-5(x^2+3x)-5$

$=-5\left(x^2+3x+\frac{9}{4}-\frac{9}{4}\right)-5$

$=-5\left(x^2+3x+\frac{9}{4}\right)+5\left(\frac{9}{4}\right)-5$

$=-5\left(x+\frac{3}{2}\right)^2+\frac{45}{4}-\frac{20}{4}$

$y=-5\left(x+\frac{3}{2}\right)^2+\frac{25}{4}$

f.) $y=6x^2+30x-10$ $\quad \frac{1}{2}(5)=\frac{5}{2}$ $\quad \hookrightarrow\left(\frac{5}{2}\right)^2=\frac{25}{4}$

$=6(x^2+5x)-10$

$=6\left(x^2+5x+\frac{25}{4}-\frac{25}{4}\right)-10$

$=6\left(x^2+5x+\frac{25}{4}\right)-\cancel{6}^{3}\left(\frac{25}{\cancel{4}_{2}}\right)-10$

$=6\left(x+\frac{5}{2}\right)^2-\frac{75}{2}-\frac{20}{2}$

$y=6\left(x+\frac{5}{2}\right)^2-\frac{95}{2}$

5. a.) $y=2x^2-5x+7$ $\quad \frac{1}{2}\left(-\frac{5}{2}\right)=-\frac{5}{4}$ $\quad \hookrightarrow\left(-\frac{5}{4}\right)^2=\frac{25}{16}$

$=2\left(x^2-\frac{5}{2}x\right)+7$

$=2\left(x^2-\frac{5}{2}x+\frac{25}{16}-\frac{25}{16}\right)+7$

$=2\left(x^2-\frac{5}{2}x+\frac{25}{16}\right)-2\left(\frac{25}{16}\right)+7$

$=2\left(x-\frac{5}{4}\right)^2-\frac{25}{8}+\frac{56}{8}$

$y=2\left(x-\frac{5}{4}\right)^2+\frac{31}{8}$

b.) $y=-3x^2+2x+2$ $\quad \frac{1}{2}\left(-\frac{2}{3}\right)=-\frac{2}{6}=-\frac{1}{3}$ $\quad \hookrightarrow\left(-\frac{1}{3}\right)^2=\frac{1}{9}$

$=-3\left(x^2-\frac{2}{3}x\right)+2$

$=-3\left(x^2-\frac{2}{3}x+\frac{1}{9}-\frac{1}{9}\right)+2$

$=-3\left(x^2-\frac{2}{3}x+\frac{1}{9}\right)+3\left(\frac{1}{9}\right)+2$

$=-3\left(x-\frac{1}{3}\right)^2+\frac{1}{3}+\frac{6}{3}$

$y=-3\left(x-\frac{1}{3}\right)^2+\frac{7}{3}$

c.) $y=2x^2-9x+18$ $\quad \frac{1}{2}\left(-\frac{9}{2}\right)=-\frac{9}{4}$ $\quad \hookrightarrow\left(-\frac{9}{4}\right)^2=\frac{81}{16}$

$=2\left(x^2-\frac{9}{2}x\right)+18$

$=2\left(x^2-\frac{9}{2}x+\frac{81}{16}-\frac{81}{16}\right)+18$

$=2\left(x^2-\frac{9}{2}x+\frac{81}{16}\right)-2\left(\frac{81}{16}\right)+18$

$=2\left(x-\frac{9}{4}\right)^2-\frac{81}{8}+\frac{144}{8}$

$y=2\left(x-\frac{9}{4}\right)^2+\frac{63}{8}$

d.) $y=3x^2-4x-6$ $\quad \frac{1}{2}\left(-\frac{4}{3}\right)=-\frac{2}{3}$ $\quad \hookrightarrow\left(-\frac{2}{3}\right)^2=\frac{4}{9}$

$=3\left(x^2-\frac{4}{3}x\right)-6$

$=3\left(x^2-\frac{4}{3}x+\frac{4}{9}-\frac{4}{9}\right)-6$

$=3\left(x^2-\frac{4}{3}x+\frac{4}{9}\right)-3\left(\frac{4}{9}\right)-6$

$=3\left(x-\frac{2}{3}\right)^2-\frac{4}{3}-\frac{18}{3}$

$y=3\left(x-\frac{2}{3}\right)^2-\frac{22}{3}$

e.) $y=-4x^2+10x-7$ $\quad \frac{1}{2}\left(-\frac{5}{2}\right)=-\frac{5}{4}$ $\quad \hookrightarrow\left(-\frac{5}{4}\right)^2=\frac{25}{16}$

$=-4\left(x^2-\frac{5}{2}x\right)-7$

$=-4\left(x^2-\frac{5}{2}x+\frac{25}{16}-\frac{25}{16}\right)-7$

$=-4\left(x^2-\frac{5}{2}x+\frac{25}{16}\right)+4\left(\frac{25}{16}\right)-7$

$=-4\left(x-\frac{5}{4}\right)^2+\frac{25}{4}-\frac{28}{4}$

$y=-4\left(x-\frac{5}{4}\right)^2-\frac{3}{4}$

f.) $y=-2x^2+5x$ $\quad \frac{1}{2}\left(-\frac{5}{2}\right)=-\frac{5}{4}$ $\quad \hookrightarrow\left(-\frac{5}{4}\right)^2=\frac{25}{16}$

$=-2\left(x^2-\frac{5}{2}x\right)$

$=-2\left(x^2-\frac{5}{2}x+\frac{25}{16}-\frac{25}{16}\right)$

$=-2\left(x^2-\frac{5}{2}x+\frac{25}{16}\right)+2\left(\frac{25}{16}\right)$

$y=-2\left(x-\frac{5}{4}\right)^2+\frac{25}{8}$

6. a.) $y=\frac{1}{2}x^2-2x+7$ $\quad \frac{1}{2}(-4)=-2$ $\quad \hookrightarrow(-2)^2=4$

$=\frac{1}{2}(x^2-4x)+7$

$=\frac{1}{2}(x^2-4x+4-4)+7$

$=\frac{1}{2}(x^2-4x+4)-\frac{1}{2}(4)+7$

$=\frac{1}{2}(x-2)^2-2+7$ $\rightarrow$ $y=\frac{1}{2}(x-2)^2+5$

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6. b.) $y=0.4x^2+2x+2.5$ $\quad \frac{1}{2}(5)=2.5$
$=0.4(x^2+5x)+2.5$ $\quad \hookrightarrow (2.5)^2=6.25$
$=0.4(x^2+5x+6.25-6.25)+2.5$
$=0.4(x^2+5x+6.25)-0.4(6.25)+2.5$
$=0.4(x+2.5)^2-2.5+2.5$
$y=0.4(x+2.5)^2$

c.) $y=\frac{3}{4}x^2-9x+7$ $\quad \frac{1}{2}(-12)=-6$
$=\frac{3}{4}(x^2-12x)+7$ $\quad \hookrightarrow (-6)^2=36$
$=\frac{3}{4}(x^2-12x+36-36)+7$
$=\frac{3}{4}(x^2-12x+36)-\frac{3}{4}(36)+7$
$=\frac{3}{4}(x-6)^2-27+7$
$y=\frac{3}{4}(x-6)^2-20$

7. a) $y=x^2-8x+10$ $\quad \frac{1}{2}(-8)=-4$
$y=(x^2-8x)+10$ $\quad (-4)^2=16$
$=(x^2-8x+16-16)+10$
$=(x^2-8x+16)-16+10$
$y=(x-4)^2-6$

b.) $y=2x^2-12x+9$ $\quad \frac{1}{2}(-6)=-3$
$=2(x^2-6x)+9$ $\quad \hookrightarrow (-3)^2=9$
$=2(x^2-6x+9-9)+9$
$=2(x^2-6x+9)-18+9$
$y=2(x-3)^2-9$

c.) $y=-3x^2-12x+16$ $\quad \frac{1}{2}(4)=2$
$=-3(x^2+4x)+16$ $\quad \hookrightarrow 2^2=4$
$=-3(x^2+4x+4-4)+16$
$=-3(x^2+4x+4)+12+16$
$y=-3(x+2)^2+28$

d.) $y=-1.5x^2-9x+7$ $\quad \frac{1}{2}(6)=3$
$=-1.5(x^2+6x)+7$ $\quad \hookrightarrow 3^2=9$
$=-1.5(x^2+6x+9-9)+7$
$=-1.5(x^2+6x+9)+13.5+7$
$y=-1.5(x+3)^2+20.5$

e.) $y=3x^2-4x+3$ $\quad \frac{1}{2}(-\frac{4}{3})=-\frac{2}{3}$
$=3(x^2-\frac{4}{3}x)+3$ $\quad \hookrightarrow (-\frac{2}{3})^2=\frac{4}{9}$
$=3(x^2-\frac{4}{3}x+\frac{4}{9}-\frac{4}{9})+3$
$=3(x^2-\frac{4}{3}x+\frac{4}{9})-3(\frac{4}{9})+3$
$=3(x-\frac{2}{3})^2-\frac{4}{3}+\frac{9}{3}$
$y=3(x-\frac{2}{3})^2-\frac{5}{3}$

f.) $y=\frac{1}{2}x^2-3x+5$ $\quad \frac{1}{2}(-6)=-3$
$=\frac{1}{2}(x^2-6x)+5$ $\quad \hookrightarrow (-3)^2=9$
$=\frac{1}{2}(x^2-6x+9-9)+5$
$=\frac{1}{2}(x^2-6x+9)-\frac{1}{2}(9)+5$
$=\frac{1}{2}(x-3)^2-\frac{9}{2}+\frac{10}{2}$
$y=\frac{1}{2}(x-3)^2+\frac{1}{2}$

PC11 4.5 con't...5

8. a) $y = ax^2 + bx + c$

$\frac{1}{2}\left(\frac{b}{a}\right) = \frac{b}{2a}$

$\hookrightarrow \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$

$= a\left(x^2 + \frac{b}{a}x\right) + c$

$= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c$

$= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - \frac{ab^2}{4a^2} + c$

$= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + \frac{c(4a)}{4a}$

$y = a\left(x + \frac{b}{2a}\right)^2 - \frac{(b^2 - 4ac)}{4a}$

$y = a\left(x - \left(\frac{-b}{2a}\right)\right)^2 + \left(\frac{-b^2 + 4ac}{4a}\right)$

$y = a(x - p)^2 + q$ $\qquad p = \frac{-b}{2a}$ $\qquad q = \frac{-b^2 + 4ac}{4a}$

b) Vertex $(p, q) \rightarrow \left(\frac{-b}{2a}, \frac{-b^2 + 4ac}{4a}\right)$

c) Axis of symmetry $\rightarrow$ $x = \frac{-b}{2a}$

d) y-intercept : let $x = 0$

$y = a\left(0 + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a}$

$= \frac{ab^2}{4a^2} - \frac{(b^2 - 4ac)}{4a}$

$= \frac{b^2 - (b^2 - 4ac)}{4a}$

$= \frac{b^2 - b^2 + 4ac}{4a}$

$= \frac{4ac}{4a} = c$ $\rightarrow (0, c)$

Method 2:

let $x = 0$

$y = ax^2 + bx + c$

$y = a(0)^2 + b(0) + c$

$y = c$

PC11 4.5 con't...6

9. $f(x) = ax^2 + bx + c$

From #8, $f(x) = a\left(x - \left(\frac{-b}{2a}\right)\right)^2 + \left(\frac{-b^2+4ac}{4a}\right)$

$f(x)$ has a minimum of 0, this means that the y-coordinate of the vertex is 0. $\Rightarrow q = 0$

So, $\frac{-b^2+4ac}{4a} = 0$

$-b^2 + 4ac = 0$

$-b^2 = -4ac$

$b^2 = 4ac$

$b = \pm\sqrt{4ac}$

$\boxed{b = \pm 2\sqrt{ac}}$

For there to be a minimum ∪, a must be positive
So, $\boxed{a > 0}$

Since $b = \pm 2\sqrt{ac}$ and $a > 0$, then c isn't negative because you can't take the square root of a negative number
So, $\boxed{c \geq 0}$

Conditions: $a > 0$, $c \geq 0$, $b = \pm 2\sqrt{ac}$

Pre-Calculus 11 4.6

1. a.) $y=(x-2)(x-4)$

$0=(x-2)(x-4)$

$x-2=0 \quad x-4=0$

$x=2 \quad x=4$

b.) $y=-(x-3)(x-5)$

$0=-(x-3)(x-5)$

$x-3=0 \quad x-5=0$

$x=3 \quad x=5$

c.) $y=3(x+2)(x-4)$

$0=3(x+2)(x-4)$

$x+2=0 \quad x-4=0$

$x=-2 \quad x=4$

2a.) $y=(x+5)(x-1)$

$0=(x+5)(x-1)$

$x+5=0 \quad x-1=0$

$x=-5 \quad x=1$

b.) $y=-2x(x-3)$

$0=-2x(x-3)$

$-2x=0 \quad x-3=0$

$x=0 \quad x=3$

c.) $y=\frac{1}{2}(x+4)(x-2)$

$0=\frac{1}{2}(x+4)(x-2)$

$x+4=0 \quad x-2=0$

$x=-4 \quad x=2$

3. a.) $y=(x+1)(x-3)$

i.) y-int: $x=0$

$y=(0+1)(0-3)$

$y=-3$

ii.) x-int: $y=0$

$0=(x+1)(x-3)$

$x=-1,\ x=3$

iii.) Axis

$x=\frac{x_1+x_2}{2}$

$x=\frac{-1+3}{2}$

$x=\frac{2}{2}$

$x=1$

iv.) $y=(x+1)(x-3)$

$y=(1+1)(1-3)$

$=(2)(-2)$

$y=-4$

$(1,-4)$ Vertex

b) $y=-2(x-2)(x-5)$

i.) y-int: $x=0$

$y=-2(0-2)(0-5)$

$=-2(-2)(-5)$

$y=-20$

ii.) x-int: $y=0$

$0=-2(x-2)(x-5)$

$x=2,\ x=5$

iii.) Axis

$x=\frac{x_1+x_2}{2}$

$x=\frac{2+5}{2}$

$x=\frac{7}{2}=3.5$

iv.) $y=-2(x-2)(x-5)$

$=-2(3.5-2)(3.5-5)$

$=-2(1.5)(-1.5)$

$y=4.5$

$(3.5, 4.5)$ Vertex

PC11 4.6 con't...2

3. c) $y=\frac{1}{2}(x+6)(x-2)$

i.) y-int: $x=0$

$y=\frac{1}{2}(0+6)(0-2)$

$=\frac{1}{2}(6)(-2)$

$y=-6$

ii.) x-int: $y=0$

$0=\frac{1}{2}(x+6)(x-2)$

$x=-6 \quad x=2$

iii.) Axis

$x=\frac{x_1+x_2}{2}$

$x=\frac{-6+2}{2}$

$x=\frac{-4}{2}$

$x=-2$

iv.) $y=\frac{1}{2}(x+6)(x-2)$

$y=\frac{1}{2}(-2+6)(-2-2)$

$=\frac{1}{2}(4)(-4)$

$y=-8$

$(-2,-8)$ Vertex

d.) $y=x^2+5x+6$

i.) y-int: $x=0$

$y=0^2+5(0)+6$

$y=6$

ii.) x-int: $y=0$

$0=x^2+5x+6$

$0=(x+3)(x+2)$

$x=-3 \quad x=-2$

iii) Axis

$x=\frac{x_1+x_2}{2}$

$x=\frac{-3+-2}{2}$

$x=\frac{-5}{2}$

$x=-2.5$

iv.) $y=x^2+5x+6$

$y=(-2.5)^2+5(-2.5)+6$

$y=-0.25$

$(-2.5,-0.25)$ Vertex

e.) $y=4x^2+8x+3$

i.) y-int: $x=0$

$y=4(0)^2+8(0)+3$

$y=3$

ii.) x-int: $y=0$

$0=4x^2+8x+3$

$0=4x^2+6x+2x+3$

$0=2x(2x+3)+1(2x+3)$

$0=(2x+1)(2x+3)$

$2x+1=0 \quad 2x+3=0$

$x=\frac{-1}{2} \quad x=\frac{-3}{2}$

$x=-0.5 \quad x=-1.5$

iii.) Axis

$x=\frac{x_1+x_2}{2}$

$x=\frac{-0.5+-1.5}{2}$

$x=-\frac{2}{2}$

$x=-1$

iv.) $y=4(-1)^2+8(-1)+3$

$=4-8+3$

$=-4+3$

$y=-1$

$(-1,-1)$ Vertex

PC11 4.6 con't... 3

3. f.) $y = -\frac{1}{4}x^2 - x - 1$

i.) y-int: $x=0$

$y = -\frac{1}{4}(0)^2 - 0 - 1$

$y = -1$

ii.) x-int: $y=0$

$0 = -\frac{1}{4}x^2 - x - 1$

$0 = -\frac{1}{4}(x^2+4x+4)$

$0 = -\frac{1}{4}(x+2)(x+2)$

$x=-2 \quad x=-2$

iii.) Axis

$x = \frac{x_1+x_2}{2}$

$x = \frac{-2+-2}{2}$

$x = \frac{-4}{2}$

$x = -2$

iv.) $y = -\frac{1}{4}x^2 - x - 1$

$y = -\frac{1}{4}(-2)^2 - (-2) - 1$

$y = -1 + 2 - 1$

$y = 0$

$(-2, 0)$ Vertex

4. Done on graph paper

5. x-intercepts 1 + 5 through point $(\underset{x}{7}, \underset{y}{-3})$

a) $y = a(x-x_1)(x-x_2)$

$y = a(x-1)(x-5)$ ← subs in x-intercepts

$-3 = a(7-1)(7-5)$ ← subs in point for x + y

$-3 = a(6)(2)$ ← solve for a

$\frac{-3}{12} = \frac{a(12)}{12}$

$-\frac{1}{4} = a$ → $y = -\frac{1}{4}(x-1)(x-5)$

b.) $y = -\frac{1}{4}(x-1)(x-5)$

$= -\frac{1}{4}(x^2 - 5x - 1x + 5)$

$= -\frac{1}{4}(x^2 - 6x + 5)$

$= -\frac{1}{4}x^2 + \frac{6}{4}x - \frac{5}{4}$

$y = -\frac{1}{4}x^2 + \frac{3}{2}x - \frac{5}{4}$

PC11 4.6 con't... 4

6. x-ints: -1 & 3 y-int: $6 \rightarrow (\underset{x}{0}, \underset{y}{6})$

a.) $y = a(x - x_1)(x - x_2)$

$y = a(x - -1)(x - 3)$

$6 = a(0+1)(0-3)$

$6 = a(1)(-3)$

$\frac{6}{-3} = \frac{a(-3)}{-3}$

$-2 = a$

$\therefore y = -2(x+1)(x-3)$

b.) $y = -2(x+1)(x-3)$

$= -2(x^2 - 3x + 1x - 3)$

$= -2(x^2 - 2x - 3)$

$y = -2x^2 + 4x + 6$

7. x-ints: 2 & 4 through point $(\underset{x}{5}, \underset{y}{9})$

a.) $y = a(x - x_1)(x - x_2)$

$y = a(x-2)(x-4)$

$9 = a(5-2)(5-4)$

$9 = a(3)(1)$

$\frac{9}{3} = \frac{a(3)}{3}$

$3 = a$

$\therefore y = 3(x-2)(x-4)$

b.) $y = 3(x-2)(x-4)$

$= 3(x^2 - 4x - 2x + 8)$

$= 3(x^2 - 6x + 8)$

$y = 3x^2 - 18x + 24$

8. a) x-intercepts -1 & 7

Note: Pick any nice point → point on graph $(\underset{x}{3}, \underset{y}{8})$

$y = a(x - x_1)(x - x_2)$

$y = a(x - -1)(x - 7)$

$8 = a(3+1)(3-7)$

$8 = a(4)(-4)$

$\frac{8}{-16} = \frac{a(-16)}{-16}$

$-\frac{1}{2} = a$

$\therefore y = -\frac{1}{2}(x+1)(x-7)$

b.) x-intercepts -6 & -2

point $(\underset{x}{-3}, \underset{y}{-6})$

$y = a(x - x_1)(x - x_2)$

$y = a(x - -6)(x - -2)$

$-6 = a(-3+6)(-3+2)$

$-6 = a(3)(-1)$

$\frac{-6}{-3} = \frac{a(-3)}{-3}$

$2 = a$

$\therefore y = 2(x+6)(x+2)$

PC11 4.6 con't...5

8. c) x-ints: 0 & 4

point $(\underset{x}{2},\underset{y}{8})$

$y=a(x-x_1)(x-x_2)$

$y=a(x-0)(x-4)$

$8=a(2-0)(2-4)$

$8=a(2)(-2)$

$\frac{8}{-4}=\frac{a(-4)}{-4}$

$-2=a$

$\therefore y=-2x(x-4)$

d.) x-ints: -4 & 4

point $(\underset{x}{0},\underset{y}{-4})$

$y=a(x-x_1)(x-x_2)$

$y=a(x--4)(x-4)$

$-4=a(0+4)(0-4)$

$-4=a(4)(-4)$

$\frac{-4}{-16}=\frac{a(-16)}{-16}$

$\frac{1}{4}=a$

$\therefore y=\frac{1}{4}(x+4)(x-4)$

9. If an x-intercept is 5, then $x=5$

so $x-5=0 \Rightarrow (x-5)$ is a factor

a) $y=x^2-3x+k$

$=(x-5)(x+\boxed{})$

Rules of factoring:

$-5+\boxed{}=-3 \rightarrow -5+\boxed{2}=-3$

$-5\times\boxed{}=k \rightarrow -5\times\boxed{2}=-10=k$

$y=(x-5)(x+\boxed{2})$

$x-5=0 \qquad x+2=0$

$x=5 \qquad x=-2$

$\therefore$ the other x-intercept $=-2$

and $k=-10$

b.) $y=x^2+kx+40$

$=(x-5)(x+\boxed{})$

Rules of factoring:

$-5+\boxed{}=k \rightarrow -5+\boxed{-8}=-13=k$

$-5\times\boxed{}=40 \rightarrow -5\times\boxed{-8}=40$

$y=(x-5)(x+\boxed{-8})$

$x-5=0 \qquad x+-8=0$

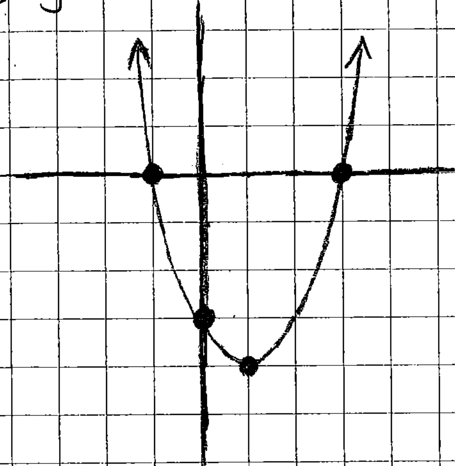
$x=5 \qquad x=8$

$\therefore$ the other x-intercept $=8$

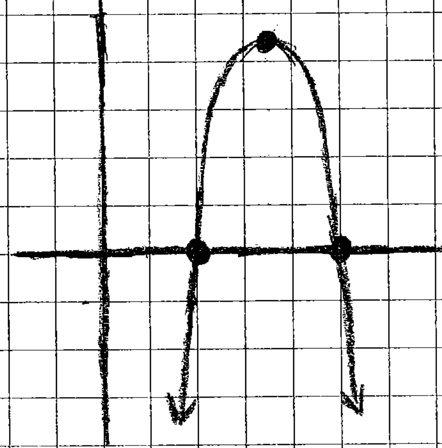
and $k=-13$

PC 11 4.6 con't... 6

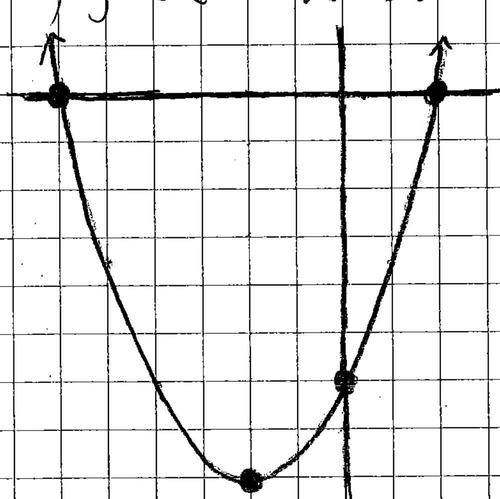
4. a) $y=(x+1)(x-3)$



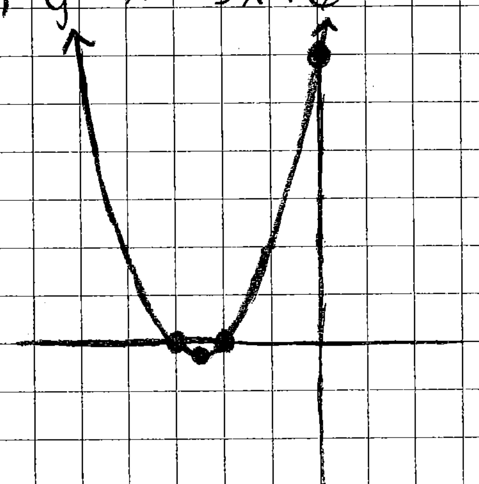
b.) $y=-2(x-2)(x-5)$



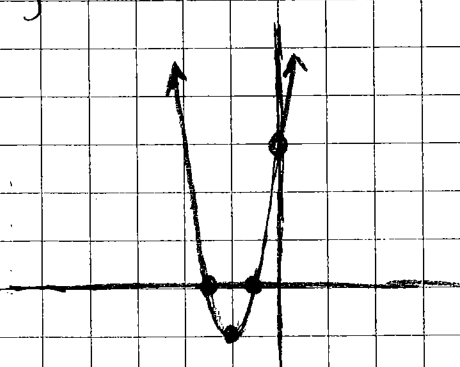
c) $y=\frac{1}{2}(x+6)(x-2)$



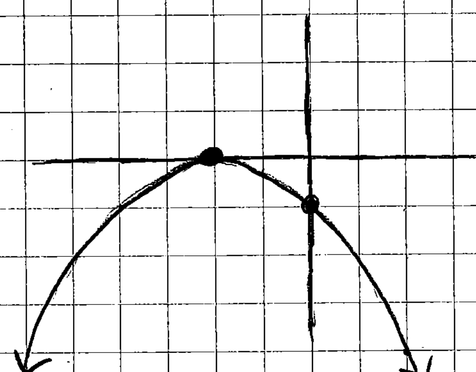
d.) $y=x^2+5x+6$



e) $y=4x^2+8x+3$



f.) $y=-\frac{1}{4}x^2-x-1$



Pre-Calculus 11 4.7

1. Let x be one number
   y be the other number
   ① $x-y=10 \rightarrow x=10+y$
   ② $P=xy$

Solve by substitution
$$P=xy$$
$$=(10+y)(y)$$
$$P=10y+y^2$$
$$P=y^2+10y$$

→ There will be a minimum.
(Complete the square to find it)

$$P=y^2+10y \qquad \tfrac{1}{2}(10)=5$$
$$P=y^2+10y+25-25 \qquad \hookrightarrow 5^2=25$$
$$=(y^2+10y+25)-25$$
$$=(y+5)^2-25$$
$$y=-5 \qquad \text{minimum value}$$

$$x=10+-5$$
$$x=5$$

∴ The two numbers are 5 & −5, and the minimum product is −25.

2. Let x be one number
   y be the other number
   ① $x+y=34 \rightarrow y=34-x$
   ② $P=xy$

Solve by substitution
$$P=xy$$
$$P=x(34-x)$$
$$P=34x-x^2$$
$$P=-x^2+34x$$

→ There will be a maximum

$$P=-x^2+34x \qquad \tfrac{1}{2}(-34)=-17$$
$$P=-(x^2-34x) \qquad \hookrightarrow(-17)^2=289$$
$$P=-(x^2-34x+289-289)$$
$$P=-(x^2-34x+289)+289$$
$$P=-(x-17)^2+289$$
$$x=17 \qquad \text{maximum value}$$

$$y=34-17$$
$$y=17$$

∴ The two numbers are 17 & 17 and the maximum product is 289.

PC 11 4.7 con't...2

3. Let x be one number

y be another number

① $x+y=34 \rightarrow y=34-x$

② $S=x^2+y^2$

Solve by substitution

$S=x^2+y^2$

$S=x^2+(34-x)^2$

$=x^2+(1156-68x+x^2)$

$S=2x^2-68x+1156$

There will be a minimum

$S=2x^2-68x+1156$

$S=2(x^2-34x)+1156$

$\left.\begin{array}{l}\frac{1}{2}(-34)=-17\\ \hookrightarrow(-17)^2=289\end{array}\right.$

$S=2(x^2-34x+289-289)+1156$

$=2(x^2-34x+289)-578+1156$

$=2(x-17)^2+578$

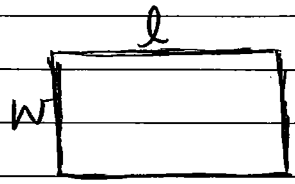
$x=17$ minimum value

$y=34-17$

$y=17$

∴ The two numbers are 17 + 17. and the minimum sum of the squares is 578.

4



Let l = length

w = width

$A=lw$

$2l+2w=400$

$\hookrightarrow 2l=400-2w$

$\hookrightarrow l=200-w$

Substitute

$A=lw$

$A=w(200-w)$

$A=200w-w^2$

$A=-w^2+200w$

There will be a maximum

$A=-w^2+200w$ $\quad \frac{1}{2}(-200)=-100$

$=-(w^2-200w)$ $\quad \hookrightarrow(-100)^2=10\,000$

$A=-(w^2-200w+10\,000-10\,000)$

$=-(w^2-200w+10\,000)+10\,000$

$A=-(w-100)^2+10\,000$

$w=100$ maximum value

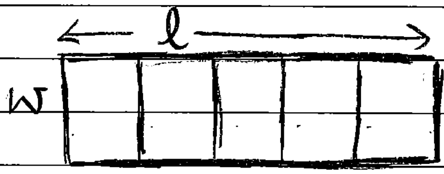
$l=200-100$

$l=100$

∴ The dimensions of the fence are 100m by 100m to create a maximum area of $10\,000\,m^2$

PC11 4.7 con't...3

5.



Let l = length
w = width

$A = lw$
$2l + 6w = 120$
$\hookrightarrow 2l = 120 - 6w$
$\hookrightarrow l = 60 - 3w$

Substitute
$A = lw$
$A = w(60 - 3w)$
$= 60w - 3w^2$
$A = -3w^2 + 60w$

There will be a maximum
$A = -3w^2 + 60w$ $\quad \frac{1}{2}(-20) = -10$
$= -3(w^2 - 20w)$ $\quad \hookrightarrow (-10)^2 = 100$
$A = -3(w^2 - 20w + 100 - 100)$
$= -3(w^2 - 20w + 100) + 300$
$= -3(w - 10)^2 + 300$

$w = 10$ $\quad$ maximum value

$l = 60 - 3(10)$
$l = 60 - 30$
$l = 30$

$\therefore$ The dimensions of the enclosure is 30m by 10m to create a total maximum area of $300m^2$

6. Let x = number of \$1 increases in price
Revenue = (# of tickets sold)(price per ticket)

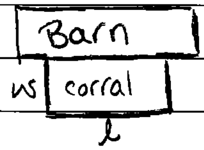
a) $R(x) = (2000 - 100x)(8 + 1x)$
b) $= 16000 + 2000x - 800x - 100x^2$
$= -100x^2 + 1200x + 16\,000$ $\quad \frac{1}{2}(-12) = -6$
$= -100(x^2 - 12x) + 16\,000$ $\quad \hookrightarrow (-6)^2 = 36$
$= -100(x^2 - 12x + 36 - 36) + 16000$
$= -100(x^2 - 12x + 36) + 3600 + 16000$ $\quad$ note: Maximum
$= -100(x - 6)^2 + 19600$ $\quad \leftarrow$ Revenue is \$19 600

b) Maximum Point: (6, 19 600)

c) $x = 6$ so ticket price $= 8 + 1x$
$= 8 + 1(6) = \$14$

d.) # of visitors $= 2000 - 100x$
$= 2000 - 100(6)$
$= 1400$ visitors

PC11 4.7 con't... 4

7. 

Let l = length
w = width

$A = lw$
$2w + l = 60$
$\hookrightarrow l = 60 - 2w$

Substitute
$A = lw$
$A = w(60 - 2w)$
$= 60w - 2w^2$
$A = -2w^2 + 60w$

There will be a maximum

$A = -2w^2 + 60w$ $\qquad \frac{1}{2}(-30) = -15$
$= -2(w^2 - 30w)$ $\qquad \hookrightarrow (-15)^2 = 225$
$= -2(w^2 - 30w + 225 - 225)$
$= -2(w^2 - 30w + 225) + 450$
$= -2(w - 15)^2 + 450$
$w = 15$
$l = 60 - 2(15) = 30$ max area $= 450 m^2$

$\therefore$ A width of 15m & length of 30m will give maximum area of $450 m^2$

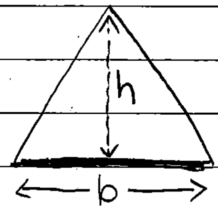
8. Let x = number of \$2 increases in price.
Revenue = (# shirts sold)(price per shirt)

$R(x) = (1200 - 60x)(20 + 2x)$
$= 24000 + 2400x - 1200x - 120x^2$
$= -120x^2 + 1200x + 24000$ $\qquad \frac{1}{2}(-10) = -5$
$= -120(x^2 - 10x) + 24000$ $\qquad \hookrightarrow (-5)^2 = 25$
$= -120(x^2 - 10x + 25 - 25) + 24000$
$= -120(x^2 - 10x + 25) + 3000 + 24000$
$= -120(x - 5)^2 + 27000$

$x = 5$

$\therefore$ Price per shirt $= 20 + 2x$
$= 20 + 2(5)$
$= 20 + 10$
$= \$30$

PC11 4.7 con't...5.

9. 

Let b = base
h = height

① $A = \frac{1}{2}bh$

② $b + h = 10$

↳ $b = 10 - h$

Substitute

$A = \frac{1}{2}bh$

$= \frac{1}{2}(10-h)h$

$= \frac{1}{2}h(10-h)$

$= 5h - \frac{1}{2}h^2$

$A = -\frac{1}{2}h^2 + 5h$

Find maximum

$A = -\frac{1}{2}h^2 + 5h$

$= -\frac{1}{2}(h^2 - 10h)$

$= -\frac{1}{2}(h^2 - 10h + 25 - 25)$

$= -\frac{1}{2}(h^2 - 10h + 25) + \frac{25}{2}$

$A = -\frac{1}{2}(h-5)^2 + 12.5$

$\frac{1}{2}(-10) = -5$

↳ $(-5)^2 = 25$

$h = 5$

maximum

$b = 10 - 5$

$b = 5$

∴ The maximum area of the triangle is $12.5 cm^2$