Pre-Calculus 11
3.1

1. a.) $x^2 - 16$
$= x^2 - 4^2$
$= (x+4)(x-4)$

b.) $25m^2 - 1$
$= (5m)^2 - (1)^2$
$= (5m+1)(5m-1)$

c.) $9 - n^2$
$= (3)^2 - n^2$
$= (3+n)(3-n)$

d.) $64x^4 - 81y^2$
$= (8x^2)^2 - (9y)^2$
$= (8x^2 - 9y)(8x^2 + 9y)$

e.) $x^2 - 8x + 12$          mn=12
$= (x-6)(x-2)$          -6 -2

f.) $m^2 + 5m - 14$          mn=-14
$= (m+7)(m-2)$          +7 -2

g.) $d^2 + 10d + 16$          mn= 16
$= (d+8)(d+2)$          8 2

h.) $a^2 - 4a + 4$          mn= 4
$= (a-2)(a-2)$          -2 -2
$= (a-2)^2$

i.) $2x^2 + 7x + 5$          mn=2(5)
$= 2x^2 + 5x + 2x + 5$          =10
$= x(2x+5) + 1(2x+5)$          5 2
$= (2x+5)(x+1)$

j.) $16m^2 - 40m + 25$          mn=16(25)
$= 16m^2 - 20m - 20m + 25$          =400
$= 4m(4m-5) - 5(4m-5)$          -20 -20
$= (4m-5)(4m-5)$
$= (4m-5)^2$

k.) $5n^2 + 14n - 3$          mn=5(-3)
$= 5n^2 + 15n - 1n - 3$          = -15
$= 5n(n+3) - 1(n+3)$          15 -1
$= (n+3)(5n-1)$

l.) $4x^4 - 12x^2 - 7$          mn=4(-7)
$= 4x^4 - 14x^2 + 2x^2 - 7$          = -28
$= 2x^2(2x^2-7) + 1(2x^2-7)$          -14 +2
$= (2x^2-7)(2x^2+1)$

m.) $6c^4 + 10c^2 + 4$          mn=6(4)
$= 6c^4 + 6c^2 + 4c^2 + 4$          = 24
$= 6c^2(c^2+1) + 4(c^2+1)$          6 4
$= (c^2+1)(6c^2+4)$
$= 2(c^2+1)(3c^2+2)$

n.) $15a^2 - 39a + 18$          mn=270
$= 15a^2 - 30a - 9a + 18$          -30 -9
$= 5a(3a-6) - 3(3a-6)$
$= (3a-6)(5a-3)$
$= 3(a-2)(5a-3)$

1. a.) $6x^2 + 9x - 6$ $\quad$ $mn = 6(-6)$
$\quad = 6x^2 + 12x - 3x - 6$ $\quad = -36$
$\quad = 6x(x+2) - 3(x+2)$ $\quad 12 \smallfrown -3$
$\quad = (x+2)(6x-3)$

2. a.) $x^2 + xy - 12y^2$ $\quad$ $mn = 1(-12)$
$\quad = x^2 + 4xy - 3xy - 12y^2$ $\quad = -12$
$\quad = x(x+4y) - 3y(x+4y)$ $\quad 4 \smallfrown -3$
$\quad = (x+4y)(x-3y)$

b.) $m^2 - 12mn + 27n^2$ $\quad$ $mn = 27$
$\quad = (m-9n)(m-3n)$ $\quad -9 \smallfrown -3$

c.) $-4a^2 + 12ab + 16b^2$ $\quad mn = -4(16)$ $\quad$ or $\quad -4a^2 + 12ab + 16b^2$
$\quad = -4a^2 - 4ab - 16ab + 16b^2$ $\quad = -64$ $\quad = -4(a^2 - 3ab - 4b^2)$ $\quad mn = -4$
$\quad = -4a(a+b) - 16b(a+b)$ $\quad 16 \smallfrown -4$ $\quad = -4(a-4b)(a+b)$ $\quad -4 \smallfrown +1$
$\quad = (-4a - 16b)(a+b)$
$\quad = -4(a-4b)(a+b)$

d.) $-x^2 + 3xy$
$\quad = -x(x - 3y)$

f.) $-u^4 - 5uv^2 + 6v^4$
$\quad = -(u^4 + 5uv^2 - 6v^4)$ $\quad mn = -6$
$\quad = -(u^2 + 6v^2)(u^2 - v^2)$ $\quad 6 \smallfrown -1$

e.) $-5x^2 + 180y^2$
$\quad = -5(x^2 - 36y^2)$
$\quad = -5(x+6y)(x-6y)$

3. a.) $25m^2 - 16n^2$
$= (5m)^2 - (4n)^2$
$= (5m + 4n)(5m - 4n)$

b.) $25(x+2)^2 - 16(3x-1)^2$
$= (5(x+2))^2 - (4(3x-1))^2$
$= [(5(x+2)) + (4(3x-1))][(5(x+2)) - (4(3x-1))]$
$= [5x+10+12x-4][5x+10-12x+4]$
$= (17x+6)(-7x+14)$
$= -7(17x+6)(x-2)$

c.) $a^2 + 3a + 2$
$= (a+2)(a+1)$

d.) $(x-1)^2 + 3(x-1) + 2$     let "a" = $(x-1)$
$= a^2 + 3a + 2$
$= (a+2)(a+1)$     Subs $a = x-1$
$= ((x-1)+2)((x-1)+1)$
$= (x+1)(x)$
$= x(x+1)$

3e + f → at end of assignment

4 a.) $3(2a+1)^2 + (2a+1) - 4$
    let "x" = $2a+1$
$3x^2 + x - 4$      $mn = -12$
$= 3x^2 + 4x - 3x - 4$      $4 \frown -3$
$= x(3x+4) - 1(3x+4)$
$= (3x+4)(x-1)$      sub $x = 2a+1$
$= (3(2a+1)+4)((2a+1)-1)$
$= (6a+3+4)(2a+1-1)$
$= (6a+7)(2a)$
$= 2a(6a+7)$

b.) $5(3x-2)^2 + 7(3x-2)$
    let "a" = $3x-2$
$5a^2 + 7a$
$= a(5a+7)$      Subs $a = 3x-2$
$= (3x-2)(5(3x-2)+7)$
$= (3x-2)(15x-10+7)$
$= (3x-2)(15x-3)$
$= (3x-2)(3)(5x-1)$
$= 3(3x-2)(5x-1)$

c.) $4(m+3)^2 - 9(2m)^2$
$= [2(m+3)]^2 - [3(2m)]^2$
$= [2(m+3) + 3(2m)][2(m+3) - 3(2m)]$
$= (2m+6+6m)(2m+6-6m)$
$= (8m+6)(-4m+6)$
$= 2(4m+3)(-2)(2m-3)$
$= -4(4m+3)(2m-3)$

d.) $-\frac{1}{4}(2x)^2 + 25(2y^3)^2$
$= 25(2y^3)^2 - \frac{1}{4}(2x)^2$
$= [5(2y^3)]^2 - [\frac{1}{2}(2x)]^2$
$= [5(2y^3) + \frac{1}{2}(2x)][5(2y^3) - \frac{1}{2}(2x)]$
$= (10y^3 + x)(10y^3 - x)$

5. a) $12a^2 - 27b^2$
$= 3(4a^2 - 9b^2)$
$= 3[(2a)^2 - (3b)^2]$
$= 3(2a+3b)(2a-3b)$

b) $25m^4n^2 - 36q^2$
$= (5m^2n)^2 - (6q)^2$
$= (5m^2n + 6q)(5m^2n - 6q)$

c) $2x^3y + 7x^2y - 15xy$
$= xy(2x^2 + 7x - 15)$
$= xy(2x^2 + 10x - 3x - 15)$
$= xy[2x(x+5) - 3(x+5)]$
$= xy(x+5)(2x-3)$

$mn = -30$
$10 \quad -3$

6. a) $\frac{1}{2}x^2 - 2x - 6$
$= \frac{1}{2}x^2 - \frac{4}{2}x - \frac{12}{2}$
$= \frac{1}{2}(x^2 - 4x - 12)$
$= \frac{1}{2}(x-6)(x+2)$

$mn = -12$
$-6 \quad +2$

b) $\frac{1}{4}c^2 + \frac{1}{2}c - 6$
$= \frac{1}{4}c^2 + \frac{2}{4}c - \frac{24}{4}$
$= \frac{1}{4}(c^2 + 2c - 24)$
$= \frac{1}{4}(c+6)(c-4)$

$mn = -24$
$6 \quad -4$

c) $\frac{3}{2}x^2 - 5x + \frac{7}{2}$
$= \frac{3}{2}x^2 - \frac{10}{2}x + \frac{7}{2}$
$= \frac{1}{2}(3x^2 - 10x + 7)$
$= \frac{1}{2}(3x^2 - 7x - 3x + 7)$
$= \frac{1}{2}[x(3x-7) - 1(3x-7)]$
$= \frac{1}{2}(3x-7)(x-1)$

$mn = 21$
$-7 \quad -3$

d) $\frac{1}{7}m^2 + \frac{11}{14}m + 1$
$= \frac{2}{14}m^2 + \frac{11}{14}m + \frac{14}{14}$
$= \frac{1}{14}(2m^2 + 11m + 14)$
$= \frac{1}{14}[2m^2 + 7m + 4m + 14]$
$= \frac{1}{14}[m(2m+7) + 2(2m+7)]$
$= \frac{1}{14}(2m+7)(m+2)$

$mn = 28$
$7 \quad 4$

e) $\frac{1}{64}a^2 - \frac{1}{36}b^2$
$= (\frac{1}{8}a)^2 - (\frac{1}{6}b)^2$
$= (\frac{1}{8}a + \frac{1}{6}b)(\frac{1}{8}a - \frac{1}{6}b)$
$= \frac{1}{2}(\frac{1}{4}a + \frac{1}{3}b)(\frac{1}{2})(\frac{1}{4}a - \frac{1}{3}b)$
$= \frac{1}{4}(\frac{1}{4}a + \frac{1}{3}b)(\frac{1}{4}a - \frac{1}{3}b)$

or factor
out $\frac{1}{4}$
to start
with

$\rightarrow$ or $\frac{1}{4}(\frac{1}{16}a^2 - \frac{1}{9}b^2)$
$= \frac{1}{4}(\frac{1}{4}a^2 + \frac{1}{3}b)(\frac{1}{4}a - \frac{1}{3}b)$

f) $\frac{1}{25}d^2 - \frac{1}{100}f^2$
$= (\frac{1}{5}d)^2 - (\frac{1}{10}f)^2$
$= (\frac{1}{5}d + \frac{1}{10}f)(\frac{1}{5}d - \frac{1}{10}f)$
$= \frac{1}{5}(d + \frac{1}{2}f)(\frac{1}{5})(d - \frac{1}{2}f)$
$= \frac{1}{25}(d + \frac{1}{2}f)(d - \frac{1}{2}f)$

$\frac{4}{100}d^2 - \frac{1}{100}f^2$
$= \frac{1}{100}(4d^2 - f^2) = \frac{1}{100}(2d+f)(2d-f)$

7. a) $0.1a^2 - 0.1a - 3$
   $= 0.1(a^2 - a - 30)$    $mn = -30$
   $= 0.1(a-6)(a+5)$    $-6 \; 5$

b) $0.5x^2 - 5.4x + 4$    $mn = 200$
   $= 0.1(5x^2 - 54x + 40)$    $-50 \; -4$
   $= 0.1(5x^2 - 50x - 4x + 40)$
   $= 0.1[5x(x-10) - 4(x-10)]$
   $= 0.1(5x-4)(x-10)$

c) $0.5m^2 - 1.3m - 0.6$    $mn = -30$
   $= 0.1(5m^2 - 13m - 6)$    $-15 + 2$
   $= 0.1(5m^2 - 15m + 2m - 6)$
   $= 0.1[5m(m-3) + 2(m-3)]$
   $= 0.1(m-3)(5m+2)$

d) $0.81x^2 - 0.25y^2$
   $= 0.01(81x^2 - 25y^2)$
   $= 0.01[(9x)^2 - (5y)^2]$
   $= 0.01(9x+5y)(9x-5y)$

e) $1.21k^2 - 0.25n^2$
   $= 0.01(121k^2 - 25n^2)$
   $= 0.01[(11k)^2 - (5n)^2]$
   $= 0.01(11k+5n)(11k-5n)$

f) $0.001c^2 + 0.008c + 0.012$
   $= 0.001(c^2 + 8c + 12)$    $mn = 12$
   $= 0.001(c+6)(c+2)$    $6 \; 2$

8. a) $16m^4 - 1$
   $= (4m^2)^2 - (1)^2$
   $= (4m^2 + 1)(4m^2 - 1)$
   $= (4m^2 + 1)[(2m)^2 - (1)^2]$
   $= (4m^2 + 1)(2m+1)(2m-1)$

b) $4x^4 - 17x^2 + 4$    $mn = 16$
   $= 4x^4 - 16x^2 - 1x^2 + 4$    $-16 \; -1$
   $= 4x^2(x^2 - 4) - 1(x^2 - 4)$
   $= (4x^2 - 1)(x^2 - 4)$
   $= [(2x)^2 - (1)^2][(x)^2 - (2)^2]$
   $= (2x+1)(2x-1)(x+2)(x-2)$

c) $(x^2 + 4x)^2 - (x^2 + 4x) - 20$
   let "a" $= x^2 + 4x$
   $a^2 - a - 20$    $mn = -20$
   $= (a-5)(a+4)$    $-5 \; 4$
   $= [(x^2+4x) - 5][(x^2+4x) + 4]$
   $= (x^2 + 4x - 5)(x^2 + 4x + 4)$
   $= (x+5)(x-1)(x+2)(x+2)$
   $= (x+5)(x-1)(x+2)^2$

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3 e.) $2c^2 + 9c + 4$    $mn = 8$
   $= 2c^2 + 8c + 1c + 4$    $8 \nearrow 1$
   $= 2c(c+4) + 1(c+4)$
   $= (c+4)(2c+1)$

f.) $2(x-3)^2 + 9(x-3) + 4$       let "a" = $x - 3$
   $2a^2 + 9a + 4$
   $= (a+4)(2a+1)$
   $= ((x-3)+4)(2(x-3)+1)$
   $= (x-3+4)(2x-6+1)$
   $= (x+1)(2x-5)$

Pre-Calculus 11
3.2

1. a.) $2x+1 = 3x-7$ ← not quadratic – no "$x^2$"

   b.) $(x+3)(x+2)=0$
   $= x^2+5x+6 = 0$ ← quadratic – "$x^2$" – degree 2

   c.) $x^2 = -3x-2$ ← quadratic – "$x^2$" – degree 2

   d.) $x^2+2 = x^3+1$ ← not quadratic – has degree higher than 2

2. a.) $(x+3)(x+4)=0$
   $x+3=0 \quad x+4=0$
   $x=-3 \quad x=-4$

   b.) $(x+5)(3x-4)=0$
   $x+5=0 \quad 3x-4=0$
   $x=-5 \quad x=\frac{4}{3}$

   c.) $2(x+4)=0$
   $x+4=0$
   $x=-4$

   d.) $3x(x-8)=0$
   $3x=0 \quad x-8=0$
   $x=0 \quad x=8$

3. a.) $m^2-12m+35=0 \qquad mn=35$
   $(m-7)(m-5)=0 \qquad -7 \; -5$
   $m-7=0 \quad m-5=0$
   $m=7 \quad m=5$

   b.) $4c^2-49=0$
   $(2c)^2-(7)^2=0$
   $(2c+7)(2c-7)=0$
   $2c+7=0 \quad 2c-7=0$
   $2c=-7 \qquad 2c=7$
   $c=-\frac{7}{2} \qquad c=\frac{7}{2}$

   c.) $y^2-11y=0$
   $y(y-11)=0$
   $y=0 \quad y-11=0$
   $\qquad y=11$

   d.) $2x^2-7x-15=0 \qquad mn=-30$
   $2x^2-10x+3x-15=0 \qquad -10 \; 3$
   $2x(x-5)+3(x-5)=0$
   $(2x+3)(x-5)=0$
   $2x+3=0 \qquad x-5=0$
   $x=\frac{-3}{2} \qquad x=5$

4.a) $m^2 + 8m = 3m + 24$

$m^2 + 8m - 3m - 24 = 0$

$m^2 + 5m - 24 = 0$      $mn = -24$

$m^2 + 8m - 3m - 24 = 0$      $8 \quad -3$

$m(m+8) - 3(m+8) = 0$

$(m+8)(m-3) = 0$

$m+8 = 0 \quad m-3 = 0$

$m = -8 \quad\quad m = 3$

b) $x(x-6) = 2(x-8)$

$x^2 - 6x = 2x - 16$

$x^2 - 6x - 2x + 16 = 0$

$x^2 - 8x + 16 = 0$      $mn = 16$

$(x-4)(x-4) = 0$      $-4 \quad -4$

$x-4 = 0 \quad x-4 = 0$

$x = 4 \quad\quad x = 4$

c.) $(2x-1)(x-3) = (x+1)(x-2)$

$2x^2 - 6x - x + 3 = x^2 - 2x + x - 2$

$2x^2 - 7x + 3 - x^2 + 2x - x + 2 = 0$

$x^2 - 6x + 5 = 0$      $mn = 5$

$(x-5)(x-1) = 0$      $-5 \quad -1$

$x-5 = 0 \quad x-1 = 0$

$x = 5 \quad\quad x = 1$

d) $(2p-1)^2 - 3 = (p-2)(p-1)$

$(2p-1)(2p-1) - 3 = p^2 - p - 2p + 2$

$4p^2 - 2p - 2p + 1 - 3 = p^2 - 3p + 2$

$4p^2 - 4p - 2 - p^2 + 3p - 2 = 0$

$3p^2 - p - 4 = 0$      $mn = -12$

$3p^2 - 4p + 3p - 4 = 0$      $-4 \quad +3$

$p(3p-4) + 1(3p-4) = 0$

$(3p-4)(p+1) = 0$

$p = \frac{4}{3} \quad\quad p = -1$

5. a) $x^4 - 13x^2 + 36 = 0$      $mn = 36$

$(x^2-9)(x^2-4) = 0$      $-9 \quad -4$

$(x+3)(x-3)(x+2)(x-2) = 0$

$x+3 = 0 \mid x-3 = 0 \mid x+2 = 0 \mid x-2 = 0$

$x = -3 \mid x = 3 \mid x = -2 \mid x = 2$

b) $p^4 + 5p^2 - 6 = 0$      $mn = -6$

$(p^2+6)(p^2-1) = 0$      $6 \quad -1$

$p^2 + 6 = 0 \quad p^2 - 1 = 0$

$p^2 = -6 \quad\quad p^2 = 1$

$p = \pm\sqrt{-6} \quad p = \pm\sqrt{1} = \pm 1$

not permissible

c.) $2x^4 - 5x^2 + 3 = 0$      $mn = 6$

$2x^4 - 2x^2 - 3x^2 + 3 = 0$      $-2 \quad -3$

$2x^2(x^2-1) - 3(x^2-1) = 0$

$(2x^2-3)(x^2-1) = 0$

$2x^2 - 3 = 0 \quad x^2 - 1 = 0$

$x^2 = \frac{3}{2} \quad\quad x^2 = 1$

$x = \pm\sqrt{\frac{3}{2}} \quad x = \pm 1$

d) $3m^4 - 14m^2 - 5 = 0$      $mn = -15$

$3m^4 - 15m^2 + 1m^2 - 5 = 0$      $-15 \quad +1$

$3m^2(m^2-5) + 1(m^2-5) = 0$

$(3m^2+1)(m^2-5) = 0$

$3m^2 + 1 = 0 \quad m^2 - 5 = 0$

$m^2 = -\frac{1}{3} \quad\quad m^2 = 5$

not possible    $m = \pm\sqrt{5}$

6. a.) $x = -5$   $x = -3$
$x + 5 = 0$   $x + 3 = 0$
$(x+5)(x+3) = 0$
$x^2 + 3x + 5x + 15 = 0$
$x^2 + 8x + 15 = 0$

c.) $x = 4$   $x = -\frac{1}{2}$
$x - 4 = 0$   $2x = -1$
$2x + 1 = 0$
$(x-4)(2x+1) = 0$
$2x^2 + x - 8x - 4 = 0$
$2x^2 - 7x - 4 = 0$

b) $x = \frac{1}{3}$,   $x = \frac{2}{3}$
$3x = 1$   $3x = 2$
$3x - 1 = 0$   $3x - 2 = 0$
$(3x-1)(3x-2) = 0$
$9x^2 - 6x - 3x + 2 = 0$
$9x^2 - 9x + 2 = 0$

d.) $x = 0$   $x = \frac{4}{3}$
$3x = 4$
$3x - 4 = 0$
$x(3x - 4) = 0$
$3x^2 - 4x = 0$

7. a.) $x = \sqrt{3x-2}$
$x^2 = (\sqrt{3x-2})^2$
$x^2 = 3x - 2$
$x^2 - 3x + 2 = 0$
$(x-2)(x-1) = 0$
$x - 2 = 0$   $x - 1 = 0$
$x = 2$     $x = 1$

b.) $\sqrt{31-x} = x - 1$
$(\sqrt{31-x})^2 = (x-1)^2$
$31 - x = (x-1)(x-1)$
$31 - x = x^2 - x - x + 1$
$0 = x^2 - 2x + 1 - 31 + x$
$0 = x^2 - x - 30$     $mn = -30$
$0 = (x-6)(x+5)$       $-6$ $5$
$x = 6$   $x = -5$

Check: $x = 2$   |   $x = 1$
$LS = x$  $RS = \sqrt{3x-2}$ | $LS = x$  $RS = \sqrt{3x-2}$
$= 2$    $= \sqrt{3(2)-2}$  | $= 1$    $= \sqrt{3(1)-2}$
         $= \sqrt{6-2}$     |          $= \sqrt{3-2}$
         $= \sqrt{4}$       |          $= \sqrt{1}$
         $= 2$              |          $= 1$
$LS = RS$              |      $LS = RS$

check: $x = 6$
$LS = \sqrt{31-x}$  $RS = x - 1$
$= \sqrt{31-6}$    $= 6 - 1$
$= \sqrt{25}$      $= 5$
$= 5$
$LS = RS$

$x = -5$
$LS = \sqrt{31-x}$  $RS = x-1$
$= \sqrt{31--5}$   $= -5-1$
$= \sqrt{31+5}$    $= -6$
$= \sqrt{36}$
$= 6$
$LS \neq RS$

$\therefore x = 2, \ x = 1$        $\therefore x = 6$

7. c) $\sqrt{4x^2-7} + 2 = 3x-1$

$\sqrt{4x^2-7} = 3x-1-2$

$\sqrt{4x^2-7} = 3x-3$

$(\sqrt{4x^2-7})^2 = (3x-3)^2$

$4x^2-7 = (3x-3)(3x-3)$

$4x^2-7 = 9x^2-9x-9x+9$

$-4x^2+7 \qquad -4x^2 \qquad +7$

$0 = 5x^2-18x+16$

$5x^2-18x+16=0$

$5x^2-10x-8x+16=0$

$5x(x-2)-8(x-2)=0$

$(5x-8)(x-2)=0$

$5x-8=0 \qquad x-2=0$

$x=\dfrac{8}{5} \qquad x=2$

$mn=80$

$-10 \quad -8$

Check $x=\dfrac{8}{5}$ $\qquad$ RS$=3x-1$

$LS = \sqrt{4x^2-7} + 2 \qquad = 3\left(\dfrac{8}{5}\right)-1$

$= \sqrt{4\left(\dfrac{8}{5}\right)^2 - 7} + 2 \qquad = \dfrac{24}{5} - \dfrac{5}{5}$

$= \sqrt{4\left(\dfrac{64}{25}\right) - 7} + 2$

$= \sqrt{\dfrac{256}{25} - \dfrac{175}{25}} + 2 \qquad = \dfrac{19}{5}$

$= \sqrt{\dfrac{81}{25}} + 2$

$= \dfrac{9}{5} + \dfrac{10}{5}$

$= \dfrac{19}{5} \qquad LS=RS.$

Check $x=2$

$LS = \sqrt{4x^2-7} + 2 \qquad$ RS$=3x-1$

$= \sqrt{4(2)^2-7} + 2 \qquad = 3(2)-1$

$= \sqrt{4(4)-7} + 2 \qquad = 6-1$

$= \sqrt{16-7} + 2 \qquad = 5$

$= \sqrt{9} + 2$

$= 3+2$

$= 5 \qquad LS=RS$

$\therefore x=\dfrac{8}{5}$ and $x=2$.

8. Let $x$ and $(x+1)$ be two consecutive integers.

$x(x+1) = 156$

$x^2 + x - 156 = 0 \qquad mn=-156$

$(x-12)(x+13)=0 \qquad -12 \quad 13$

$x=12 \qquad x=-13$

If $x=12$, then $x+1 = 12+1 = 13$.

If $x=-13$, then $x+1 = -13+1 = -12$

$\therefore$ The two consecutive numbers are $12, 13$

or $-12, -13$.

9. Let $n$ be a number

then $n^2$ is its square

$n^2 - n = 30$

$n^2 - n - 30 = 0$

$(n-6)(n+5) = 0$

$n = 6 \quad n = -5$

∴ The number is 6 or $-5$.

Check: $n = 6$

$LS = n^2 - n$

$= 6^2 - 6$

$= 36 - 6$

$= 36$

$= RS$

$n = -5$

$LS = n^2 - n$
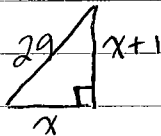
$= (-5)^2 - (-5)$

$= 25 + 5$

$= 36$

$= RS$.

10.



$x^2 + (x+1)^2 = 29^2$

$x^2 + (x+1)(x+1) = 841$

$x^2 + x^2 + 2x + 1 = 841$

$2x^2 + 2x + 1 - 841 = 0$

$2x^2 + 2x - 840 = 0$

$2(x^2 + x - 420) = 0$

$2(x+21)(x-20) = 0$

$x = -21 \quad x = 20$

can't have negative length

$x = 20$

$x + 1 = 20 + 1 = 21$

∴ The lengths of the other two sides are 20cm and 21cm

11. $h = 20t - 5t^2 \rightarrow h = 15$

$15 = 20t - 5t^2$

$5t^2 - 20t + 15 = 0$

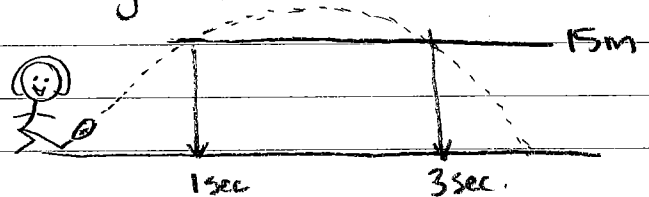$5(t^2 - 4t + 3) = 0$
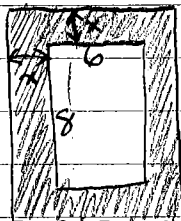
$5(t-3)(t-1) = 0$

$t = 3 \quad t = 1$

The football is at a height of 15m at a time of 1 second and again at 3 seconds



12.



Total area of just path = 120 m²

Path Area = $(6+2x)(8+2x) - 6(8)$

$120 = 48 + 12x + 16x + 4x^2 - 48$

$0 = 4x^2 + 28x - 120$

$0 = 4(x^2 + 7x - 30)$

$0 = 4(x+10)(x-3)$

cannot have negative width

$x = -10 \quad x = 3$

∴ The pathway is 3m wide.

13 a.) $(3x+5)^2 = (9x-1)^2$ $\qquad$ OR// $(3x+5)(3x+5) = (9x-1)(9x-1)$

$(3x+5)^2 - (9x-1)^2 = 0$ $\qquad\qquad$ $9x^2+30x+25 = 81x^2-18x+1$

$[(3x+5)+(9x-1)][(3x+5)-(9x-1)]=0$ $\qquad$ $-9x^2-30x-25 \qquad -9x^2-30x-25$

$(3x+5+9x-1)(3x+5-9x+1)=0$ $\qquad$ $0 = 72x^2 - 48x - 24$

$(12x+4)(-6x+6)=0$ $\qquad\qquad$ $0 = 24(3x^2-2x-1)$ $\qquad$ mn = -3

$12x+4=0 \qquad -6x+6=0$ $\qquad\qquad$ $0 = 24(3x^2-3x+1x-1)$ $\qquad$ -3 +1

$12x = -4 \qquad -6x = -6$ $\qquad\qquad$ $0 = 24[3x(x-1)+1(x-1)]$

$x = -\frac{4}{12} \qquad x = \frac{-6}{-6}$ $\qquad\qquad$ $0 = 24(3x+1)(x-1)$

$x = -\frac{1}{3} \qquad x = 1$ $\qquad\qquad\qquad$ $x = -\frac{1}{3} \quad x = 1$

b.) $(2x+1)^2 - 3(2x+1) - 4 = 0$ $\qquad$ OR// $(2x+1)(2x+1) - 3(2x+1) - 4 = 0$

$\quad$ let $a = 2x+1$ $\qquad\qquad\qquad$ $4x^2+4x+1-6x-3-4=0$

$a^2 - 3a - 4 = 0$ $\qquad\qquad\qquad$ $4x^2 - 2x - 6 = 0$

$(a-4)(a+1)=0$ $\qquad\qquad\qquad$ $2(2x^2-x-3)=0$ $\qquad$ mn = -6

$[(2x+1)-4][(2x+1)+1]=0$ $\qquad\quad$ $2(2x^2-3x+2x-3)=0$ $\qquad$ -3 +2

$(2x+1-4)(2x+1+1)=0$ $\qquad\qquad$ $2[x(2x-3)+1(2x-3)]=0$

$(2x-3)(2x+2)=0$ $\qquad\qquad\qquad$ $2(x+1)(2x-3)=0$

$x = \frac{3}{2} \qquad x = -1$ $\qquad\qquad\qquad$ $x = -1 \quad x = \frac{3}{2}$

c.) $3(x-1)^2 - 8(x-1) + 5 = 0$ $\qquad$ OR// $3(x-1)(x-1) - 8(x-1) + 5 = 0$

$\quad$ let $a = x-1$ $\qquad\qquad\qquad$ $(3x-3)(x-1) - 8x + 8 + 5 = 0$

$3a^2 - 8a + 5 = 0$ $\quad$ mn = 15 $\qquad\qquad$ $3x^2-3x-3x+3-8x+13=0$

$3a^2 - 3a - 5a + 5 = 0$ $\qquad$ -3 -5 $\qquad$ $3x^2 - 14x + 16 = 0$ $\qquad$ mn = 48

$3a(a-1) - 5(a-1) = 0$ $\qquad\qquad\qquad$ $3x^2 - 8x - 6x + 16 = 0$ $\qquad$ -8 -6

$(3a-5)(a-1)=0$ $\qquad\qquad\qquad$ $x(3x-8) - 2(3x-8)=0$

$[3(x-1)-5][(x-1)-1]=0$ $\qquad\qquad$ $(3x-8)(x-2)=0$

$(3x-3-5)(x-1-1)=0$ $\qquad\qquad\quad$ $x = \frac{8}{3} \qquad x = 2$

$(3x-8)(x-2)=0$

$x = \frac{8}{3} \quad x = 2$

14. If one root is 5, then $x=5$ so $x-5=0$

    ∴ $(x-5)$ is a factor.

a.) $x^2 - 3x + k = 0$     $-5 \times \square = k$    $\Rightarrow -5 \times 2 = -10$

    $(x-5)(x+\square) = 0$    $-5 + \square = -3$    $-5 + \boxed{2} = -3$

    $(x-5)(x+\boxed{2}) = 0$

        $x = -2$     ∴ $K = -10, \ x = -2$

b.) $x^2 + kx + 40 = 0$    $-5 \times \square = 40$   $\Rightarrow -5 \times \boxed{-8} = 40$

    $(x-5)(x+\square) = 0$    $-5 + \square = k$   $\Rightarrow -5 + -8 = -13$

    $(x-5)(x+\boxed{-8}) = 0$

        $x = 8$     ∴ $K = -13, \ x = +8$

c.) $x^2 + kx + 25 = 0$    $-5 \times \square = 25$   $\Rightarrow -5 \times \boxed{-5} = 25$

    $(x-5)(x+\square) = 0$    $-5 + \square = k$   $\Rightarrow -5 + -5 = -16$

    $(x-5)(x+\boxed{-5}) = 0$

        $x = 5$     ∴ $K = -10, \ x = +5$

d.) $x^2 - 2x + k = 0$    $-5 \times \square = k$    $\Rightarrow -5 \times \boxed{+3} = -15$

    $(x-5)(x+\square) = 0$    $-5 + \square = -2$   $\Rightarrow -5 + \boxed{3} = -12$

    $(x-5)(x+\boxed{3}) = 0$

        $x = -3$     ∴ $K = -15, \ x = -3$

15 a) $x^2 - ax = 0$
   $x(x-a) = 0$
   $x = 0 \quad x - a = 0$
   $\qquad\qquad x = a$

b) $x^2 - (a+b)x + ab = 0$
   $(x-a)(x-b)$
   $x - a = 0 \quad x - b = 0$
   $x = a \qquad x = b$

$\underline{-a} \times \underline{b} = ab$
$\underline{-a} + \underline{b} = -(a+b)$

c) $x^2 + ax = ab + bx$
   $x^2 + ax - ab - bx = 0$
   $x^2 + ax - bx - ab = 0$
   $x^2 + (a-b)x - ab = 0$
   $(x+a)(x-b) = 0$
   $x = -a \quad x = b$

$\underline{a} \times \underline{-b} = -ab$
$\underline{a} + \underline{-b} = (a-b)$

d) $x^2 - 2pq = p^2 + q^2$
   $x^2 = p^2 + 2pq + q^2 \quad \Leftarrow \text{perfect square trinomial}$
   $x^2 = (p+q)^2$
   $x^2 - (p+q)^2 = 0$
   $(x + (p+q))(x - (p+q)) = 0$
   $(x + p + q)(x - p - q) = 0$
   $x + p + q = 0 \qquad x - p - q = 0$
   $x = -p - q \qquad x = p + q$

PC₁₁ 3.3

1. a.) $x^2 + 12 = 48$
$\quad\quad -12 \quad -12$
$\quad x^2 = 36$
$\quad x = \pm\sqrt{36}$
$\quad x = \pm 6$

b.) $2x^2 - 32 = 0$
$\quad \dfrac{2x^2}{2} = \dfrac{32}{2}$
$\quad x^2 = 16$
$\quad x = \pm\sqrt{16}$
$\quad x = \pm 4$

c.) $m^2 - 7.5 = 92.5$
$\quad\quad +7.5 \quad +7.5$
$\quad m^2 = 100$
$\quad m = \pm\sqrt{100}$
$\quad m = \pm 10$

d.) $3x^2 + 2 = 29$
$\quad\quad -2 \quad -2$
$\quad \dfrac{3x^2}{3} = \dfrac{27}{3}$
$\quad x^2 = 9$
$\quad x = \pm\sqrt{9}$
$\quad x = \pm 3$

e.) $10 - z^2 = 4$
$\quad -10 \quad\quad -10$
$\quad \dfrac{-z^2}{-1} = \dfrac{-6}{-1}$
$\quad z^2 = 6$
$\quad z = \pm\sqrt{6}$

f.) $-3 = 12 - 5d^2$
$\quad -12 \quad -12$
$\quad \dfrac{-15}{-5} = \dfrac{-5d^2}{-5}$
$\quad 3 = d^2$
$\quad d = \pm\sqrt{3}$

g.) $(y+2)^2 = 9$
$\quad y + 2 = \pm\sqrt{9}$
$\quad y + 2 = \pm 3$
$\quad y = -2 \pm 3$
$\quad y = -2+3 \quad\quad y = -2-3$
$\quad y = 1 \quad\quad\quad\quad y = -5$

h.) $(3-c)^2 - 16 = 0$
$\quad (3-c)^2 = 16$
$\quad 3 - c = \pm\sqrt{16}$
$\quad 3 \pm\sqrt{16} = c$
$\quad 3 \pm 4 = c$
$\quad c = 3+4 \quad\quad c = 3-4$
$\quad c = 7 \quad\quad\quad c = -1$

i.) $\left(m - \dfrac{1}{3}\right)^2 = \dfrac{4}{9}$
$\quad m - \dfrac{1}{3} = \pm\sqrt{\dfrac{4}{9}}$
$\quad m - \dfrac{1}{3} = \pm\dfrac{2}{3}$
$\quad m = \pm\dfrac{2}{3} + \dfrac{1}{3}$
$\quad m = \dfrac{2}{3} + \dfrac{1}{3} \quad\quad m = -\dfrac{2}{3} + \dfrac{1}{3}$
$\quad m = \dfrac{3}{3} \quad\quad\quad\quad m = -\dfrac{1}{3}$
$\quad m = 1$

j.) $(1 - 2g)^2 = 49$
$\quad 1 - 2g = \pm\sqrt{49}$
$\quad 1 - 2g = \pm 7$
$\quad -2g = -1 \pm 7$
$\quad -2g = -1 + 7 \quad\quad -2g = -1 - 7$
$\quad \dfrac{-2g}{-2} = \dfrac{6}{-2} \quad\quad \dfrac{-2g}{-2} = \dfrac{-8}{-2}$
$\quad g = -3 \quad\quad\quad\quad g = 4$

1. k.) $1.21 = (10a+1)^2$

$\pm\sqrt{1.21} = 10a+1$

$\pm 1.1 - 1 = 10a$

$10a = 1.1-1 \qquad 10a = -1.1-1$

$\dfrac{10a}{10} = \dfrac{0.1}{10} \qquad \dfrac{10a}{10} = \dfrac{-2.1}{10}$

$a = 0.01 \qquad\qquad a = -0.21$

l.) $\frac{1}{3}(6x-1)^2 - 3 = 0$

$\frac{3}{1} \times \frac{1}{3}(6x-1)^2 = 3 \times \frac{3}{1}$

$(6x-1)^2 = 9$

$6x-1 = \pm\sqrt{9}$

$6x-1 = \pm 3$

$6x = \pm 3+1 \qquad 6x = -3+1$

$6x = 3+1 \qquad\qquad \dfrac{6x}{6} = \dfrac{-2}{6}$

$\dfrac{6x}{6} = \dfrac{4}{6} \qquad\qquad x = -\dfrac{1}{3}$

$x = \dfrac{2}{3}$

m.) $\dfrac{(a+2)^2}{4} = 16$

$\dfrac{4 \cdot (a+2)^2}{4} = 16 \cdot 4$

$(a+2)^2 = 64$

$a+2 = \pm\sqrt{64}$

$a+2 = \pm 8$

$a = \pm 8 - 2$

$a = 8-2 \qquad a = -8-2$

$a = 6 \qquad\quad a = -10$

n.) $\left(a+\frac{3}{4}\right)^2 = \dfrac{3}{16}$

$a + \dfrac{3}{4} = \pm\sqrt{\dfrac{3}{16}}$

$a = \pm\dfrac{\sqrt{3}}{\sqrt{16}} - \dfrac{3}{4}$

$a = \dfrac{\sqrt{3}}{4} - \dfrac{3}{4} \qquad a = -\dfrac{\sqrt{3}}{4} - \dfrac{3}{4}$

$a = \dfrac{\sqrt{3}-3}{4} \qquad a = \dfrac{-\sqrt{3}-3}{4}$

o.) $3(2x^2+5) = 5(3x^2-2)$

$6x^2 + 15 = 15x^2 - 10$

$\underline{-6x^2 \qquad\qquad -6x^2}$

$15 = 9x^2 - 10$

$\underline{+10 \qquad\quad +10}$

$\dfrac{25}{9} = \dfrac{9x^2}{9}$

$\dfrac{25}{9} = x^2$

$x = \pm\sqrt{\dfrac{25}{9}} = \pm\dfrac{5}{3}$

p.) $(2n+3)^2 = (3n+2)^2$

$\pm\sqrt{(2n+3)^2} = \pm\sqrt{(3n+2)^2}$

$\pm(2n+3) = \pm(3n+2)$

$2n+3 = 3n+2 \qquad 2n+3 = -(3n+2)$

$\underline{-2n \qquad -2n} \qquad 2n+3 = -3n-2$

$3 = n+2 \qquad\qquad \underline{+3n \qquad +3n}$

$\underline{-2 \qquad -2} \qquad\quad 5n+3 = -2$

$1 = n \qquad\qquad\qquad \underline{-3 \quad -3}$

$5n = -5$

$n = -1$

2. a) $x^2 + 10x + k$

$\hookrightarrow \frac{1}{2}(10) = 5$

$\hookrightarrow (5)^2 = 25$

$x^2 + 10x + 25$

$= (x+5)^2$

b) $w^2 - 14w + k$

$\hookrightarrow \frac{1}{2}(+14) = -7$

$\hookrightarrow (-7)^2 = 49$

$= w^2 - 14w + 49$

$= (w-7)^2$

c) $m^2 + 3m + k$

$\hookrightarrow \frac{1}{2}\left(\frac{3}{1}\right) = \frac{3}{2}$

$\hookrightarrow \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$= m^2 + 3m + \frac{9}{4}$

$= \left(m + \frac{3}{2}\right)^2$

d) $x^2 + \frac{4}{3}x + k$

$\hookrightarrow \frac{1}{2}\left(\frac{4}{3}\right) = \frac{4}{6}$

$\hookrightarrow \left(\frac{4}{6}\right)^2 = \frac{16}{36}$

$= x^2 + \frac{4}{3}x + \frac{16}{36}$

$= \left(x + \frac{4}{6}\right)^2 = \left(x + \frac{2}{3}\right)^2$

e) $d^2 - \frac{2}{3}d + k$

$\hookrightarrow \frac{1}{2}\left(-\frac{2}{3}\right) = -\frac{2}{6}$

$\hookrightarrow \left(-\frac{2}{6}\right)^2 = \frac{4}{36}$

$= d^2 - \frac{2}{3}d + \frac{4}{36}$

$= \left(d - \frac{2}{6}\right)^2 = \left(d - \frac{1}{3}\right)^2$

f) $x^2 - 0.06x + k$

$\hookrightarrow \frac{1}{2}(-0.06) = -0.03$

$\hookrightarrow (-0.03)^2 = 0.0009$

$= x^2 - 0.06x + 0.0009$

$= (x - 0.03)^2$

3. a) $x^2 + 6x + 4 = 0$

$\hookrightarrow \frac{1}{2}(6) = 3$

$\hookrightarrow 3^2 = 9$

$x^2 + 6x + 9 - 9 + 4 = 0$

$(x^2 + 6x + 9) - 9 + 4 = 0$

$(x+3)^2 - 5 = 0$

$(x+3)^2 = 5$

$x + 3 = \pm\sqrt{5}$

$x = -3 \pm \sqrt{5}$

b) $w^2 - 4w - 11 = 0$

$\hookrightarrow \frac{1}{2}(-4) = -2$

$\hookrightarrow (-2)^2 = 4$

$w^2 - 4w + 4 - 4 - 11 = 0$

$(w^2 - 4w + 4) - 4 - 11 = 0$

$(w-2)^2 - 15 = 0$

$(w-2)^2 = 15$

$(w-2) = \pm\sqrt{15}$

$w = 2 \pm \sqrt{15}$

3. c.) $x^2 + 4x - 12 = 0$

$\hookrightarrow \frac{1}{2}(4) = 2$

$\hookrightarrow 2^2 = 4$

$x^2 + 4x + 4 - 4 - 12 = 0$

$(x^2 + 4x + 4) - 4 - 12 = 0$

$(x+2)^2 - 16 = 0$

$(x+2)^2 = 16$

$x + 2 = \pm\sqrt{16}$

$x = -2 \pm 4$

$x = -2 + 4 \quad x = -2 - 4$

$x = 2 \qquad x = -6$

d.) $x^2 - 18x + 20 = 0$

$\hookrightarrow \frac{1}{2}(-18) = -9$

$\hookrightarrow (-9)^2 = 81$

$x^2 - 18x + 81 - 81 + 20 = 0$

$(x^2 - 18x + 81) - 81 + 20 = 0$

$(x-9)^2 - 61 = 0$

$(x-9)^2 = 61$

$x - 9 = \pm\sqrt{61}$

$x = 9 \pm\sqrt{61}$

e.) $m^2 + 7m + 10 = 0$

$\hookrightarrow \frac{1}{2}(7) = \frac{7}{2}$

$\hookrightarrow (\frac{7}{2})^2 = \frac{49}{4}$

$m^2 + 7m + \frac{49}{4} - \frac{49}{4} + 10 = 0$

$(m^2 + 7m + \frac{49}{4}) - \frac{49}{4} + 10 = 0$

$(m + \frac{7}{2})^2 - \frac{49}{4} + \frac{40}{4} = 0$

$(m + \frac{7}{2})^2 - \frac{9}{4} = 0$

$(m + \frac{7}{2})^2 = \frac{9}{4}$

$m + \frac{7}{2} = \pm\sqrt{\frac{9}{4}}$

$m + \frac{7}{2} = \pm\frac{3}{2}$

$m = -\frac{7}{2} + \frac{3}{2} \quad m = -\frac{7}{2} - \frac{3}{2}$

$m = \frac{-4}{2} \qquad m = \frac{-10}{2}$

$m = -2 \qquad m = -5$

f.) $n^2 - 5n - 6 = 0$

$\hookrightarrow \frac{1}{2}(-5) = -\frac{5}{2}$

$\hookrightarrow (-\frac{5}{2})^2 = \frac{25}{4}$

$n^2 - 5n + \frac{25}{4} - \frac{25}{4} - 6 = 0$

$(n^2 - 5n + \frac{25}{4}) - \frac{25}{4} - 6 = 0$

$(n - \frac{5}{2})^2 - \frac{25}{4} - \frac{24}{4} = 0$

$(n - \frac{5}{2})^2 - \frac{49}{4} = 0$

$(n - \frac{5}{2})^2 = \frac{49}{4}$

$n - \frac{5}{2} = \pm\sqrt{\frac{49}{4}}$

$n - \frac{5}{2} = \pm\frac{7}{2}$

$n = \frac{5}{2} \pm \frac{7}{2}$

$n = \frac{5}{2} + \frac{7}{2} \quad n = \frac{5}{2} - \frac{7}{2}$

$n = \frac{12}{2} \qquad n = \frac{-2}{2}$

$n = 6 \qquad n = -1$

4 a) $2x^2 + 8x + 5 = 0$

$(2x^2 + 8x) + 5 = 0$

$2(x^2 + 4x) + 5 = 0$

↳ $\frac{1}{2}(4) = 2$

↳ $2^2 = 4$

$2(x^2 + 4x + 4 - 4) + 5 = 0$

$2(x^2 + 4x + 4) - 8 + 5 = 0$

$2(x + 2)^2 - 3 = 0$

$\dfrac{2(x+2)^2}{2} = \dfrac{3}{2}$

$(x + 2)^2 = \dfrac{3}{2}$

$x + 2 = \pm\sqrt{\dfrac{3}{2}}$

$x = -2 \pm \sqrt{\dfrac{3}{2}}$

$x = -2 \pm \dfrac{\sqrt{3}}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}$

$x = -2 \pm \dfrac{\sqrt{6}}{2}$

b) $3d^2 + 6d + 2 = 0$

$(3d^2 + 6d) + 2 = 0$

$3(d^2 + 2d) + 2 = 0$

↳ $\frac{1}{2}(2) = 1$

↳ $1^2 = 1$

$3(d^2 + 2d + 1 - 1) + 2 = 0$

$3(d^2 + 2d + 1) - 3 + 2 = 0$

$3(d + 1)^2 - 1 = 0$

$\dfrac{3(d+1)^2}{3} = \dfrac{1}{3}$

$(d + 1)^2 = \dfrac{1}{3}$

$d + 1 = \pm\sqrt{\dfrac{1}{3}}$

$d = -1 \pm \sqrt{\dfrac{1}{3}}$

$d = -1 \pm \sqrt{\dfrac{1}{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$

$d = -1 \pm \dfrac{\sqrt{3}}{3}$

c) $2a^2 - 12a + 3 = 0$

$(2a^2 - 12a) + 3 = 0$

$2(a^2 - 6a) + 3 = 0$

↳ $\frac{1}{2}(-6) = -3$

↳ $(-3)^2 = 9$

$2(a^2 - 6a + 9 - 9) + 3 = 0$

$2(a^2 - 6a + 9) - 18 + 3 = 0$

$2(a - 3)^2 - 15 = 0$

$\dfrac{2(a-3)^2}{2} = \dfrac{15}{2}$

$(a - 3)^2 = \dfrac{15}{2}$

$a - 3 = \pm\sqrt{\dfrac{15}{2}}$

$a = 3 \pm \sqrt{\dfrac{15}{2}}$

$a = 3 \pm \sqrt{\dfrac{15}{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}$

$a = 3 \pm \dfrac{\sqrt{30}}{2}$

d) $5c^2 - 20c + 3 = 0$

$(5c^2 - 20c) + 3 = 0$

$5(c^2 - 4c) + 3 = 0$

↳ $\frac{1}{2}(-4) = -2$

↳ $(-2)^2 = 4$

$5(c^2 - 4c + 4 - 4) + 3 = 0$

$5(c^2 - 4c + 4) - 20 + 3 = 0$

$5(c - 2)^2 - 17 = 0$

$\dfrac{5(c-2)^2}{5} = \dfrac{17}{5}$

$(c - 2)^2 = \dfrac{17}{5}$

$c - 2 = \pm\sqrt{\dfrac{17}{5}}$

$c = 2 \pm \sqrt{\dfrac{17}{5}}$

$c = 2 \pm \sqrt{\dfrac{17}{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}}$

$c = 2 \pm \dfrac{\sqrt{85}}{5}$

5. a) $2x^2 + x - 5 = 0$

$(2x^2 + x) - 5 = 0$

$2(x^2 + \frac{1}{2}x) - 5 = 0$

$\hookrightarrow \frac{1}{2}(\frac{1}{2}) = \frac{1}{4}$

$\hookrightarrow (\frac{1}{4})^2 = \frac{1}{16}$

$2(x^2 + \frac{1}{2}x + \frac{1}{16} - \frac{1}{16}) - 5 = 0$

$2(x^2 + \frac{1}{2}x + \frac{1}{16}) - \frac{2}{16} - 5 = 0$

$2(x + \frac{1}{4})^2 - \frac{1}{8} - \frac{40}{8} = 0$

$\frac{1}{2} \cdot 2(x + \frac{1}{4})^2 = \frac{41}{8} \cdot \frac{1}{2}$

$(x + \frac{1}{4})^2 = \frac{41}{16}$

$x + \frac{1}{4} = \pm\sqrt{\frac{41}{16}}$

$x = -\frac{1}{4} \pm \frac{\sqrt{41}}{4}$

$x = \frac{-1 \pm \sqrt{41}}{4}$

b) $2r^2 - 7r + 1 = 0$

$(2r^2 - 7r) + 1 = 0$

$2(r^2 - \frac{7}{2}r) + 1 = 0$

$\hookrightarrow \frac{1}{2}(-\frac{7}{2}) = -\frac{7}{4}$

$\hookrightarrow (-\frac{7}{4})^2 = \frac{49}{16}$

$2(r^2 - \frac{7}{2}r + \frac{49}{16} - \frac{49}{16}) + 1 = 0$

$2(r^2 - \frac{7}{2}r + \frac{49}{16}) - \frac{98}{16} + 1 = 0$

$2(r - \frac{7}{4})^2 - \frac{49}{8} + \frac{8}{8} = 0$

$\frac{1}{2} \cdot 2(r - \frac{7}{4})^2 = \frac{41}{8} \cdot \frac{1}{2}$

$(r - \frac{7}{4})^2 = \frac{41}{16}$

$r - \frac{7}{4} = \pm\sqrt{\frac{41}{16}}$

$r = \frac{7}{4} \pm \frac{\sqrt{41}}{4}$

$r = \frac{7 \pm \sqrt{41}}{4}$

c) $5c^2 - 2c - 6 = 0$

$(5c^2 - 2c) - 6 = 0$

$5(c^2 - \frac{2}{5}c) - 6 = 0$

$\hookrightarrow \frac{1}{2}(-\frac{2}{5}) = -\frac{2}{10}$

$\hookrightarrow (-\frac{2}{10})^2 = \frac{4}{100}$

$5(c^2 - \frac{2}{5}c + \frac{4}{100} - \frac{4}{100}) - 6 = 0$

$5(c^2 - \frac{2}{5}c + \frac{4}{100}) - \frac{20}{100} - 6 = 0$

$5(c - \frac{2}{10})^2 - \frac{1}{5} - \frac{30}{5} = 0$

$5(c - \frac{2}{10})^2 - \frac{31}{5} = 0$

$\frac{1}{5} \cdot 5(c - \frac{1}{5})^2 = \frac{31}{5} \cdot \frac{1}{5}$

$(c - \frac{1}{5})^2 = \frac{31}{25}$

$c - \frac{1}{5} = \pm\sqrt{\frac{31}{25}}$

$c = \frac{1}{5} \pm \frac{\sqrt{31}}{5}$

$c = \frac{1 \pm \sqrt{31}}{5}$

d) $3m^2 + 5m - 11 = 0$

$(3m^2 + 5m) - 11 = 0$

$3(m^2 + \frac{5}{3}m) - 11 = 0$

$\hookrightarrow \frac{1}{2}(\frac{5}{3}) = \frac{5}{6}$

$\hookrightarrow (\frac{5}{6})^2 = \frac{25}{36}$

$3(m^2 + \frac{5}{3}m + \frac{25}{36} - \frac{25}{36}) - 11 = 0$

$3(m^2 + \frac{5}{3}m + \frac{25}{36}) - \frac{75}{36} - 11 = 0$

$3(m + \frac{5}{6})^2 - \frac{25}{12} - \frac{132}{12} = 0$

$3(m + \frac{5}{6})^2 - \frac{157}{12} = 0$

$\frac{1}{3} \cdot 3(m + \frac{5}{6})^2 = \frac{157}{12} \cdot \frac{1}{3}$

$(m + \frac{5}{6})^2 = \frac{157}{36}$

$m + \frac{5}{6} = \pm\sqrt{\frac{157}{36}}$

$m = -\frac{5}{6} \pm \frac{\sqrt{157}}{6}$

$m = \frac{-5 \pm \sqrt{157}}{6}$

6. a) $\frac{1}{2}x^2 + x - 13 = 0$

$(\frac{1}{2}x^2 + x) - 13 = 0$

$\frac{1}{2}(x^2 + 2x) - 13 = 0$

$\quad\quad \hookrightarrow \frac{1}{2}(2) = 1$

$\quad\quad \hookrightarrow (1)^2 = 1$

$\frac{1}{2}(x^2 + 2x + 1 - 1) - 13 = 0$

$\frac{1}{2}(x^2 + 2x + 1) - \frac{1}{2} - 13 = 0$

$\frac{1}{2}(x+1)^2 - \frac{1}{2} - \frac{26}{2} = 0$

$2 \cdot \frac{1}{2}(x+1)^2 = \frac{27}{2} \cdot 2$

$(x+1)^2 = 27$

$x + 1 = \pm\sqrt{27}$

$x = -1 \pm \sqrt{27}$

$x = -1 + \sqrt{27} \quad\quad x = -1 - \sqrt{27}$

$x = 4.20 \quad\quad\quad x = -6.20$

b) $-3m^2 - 4m + 2 = 0$

$(-3m^2 - 4m) + 2 = 0$

$-3(m^2 + \frac{4}{3}m) + 2 = 0$

$\quad\quad \hookrightarrow \frac{1}{2}(\frac{4}{3}) = \frac{4}{6}$

$\quad\quad \hookrightarrow (\frac{4}{6})^2 = \frac{16}{36}$

$-3(m^2 + \frac{4}{3}m + \frac{16}{36} - \frac{16}{36}) + 2 = 0$

$-3(m^2 + \frac{4}{3}m + \frac{16}{36}) + \frac{48}{36} + 2 = 0$

$-3(m + \frac{4}{6})^2 + \frac{4}{3} + \frac{6}{3} = 0$

$-\frac{1}{3} \cdot -3(m + \frac{4}{6})^2 = -\frac{10}{3} \cdot -\frac{1}{3}$

$(m + \frac{4}{6})^2 = \frac{10}{9}$

$m + \frac{4}{6} = \pm\sqrt{\frac{10}{9}}$

$m = -\frac{4}{6} \pm \frac{\sqrt{10}}{3}$

$m = -\frac{4}{6} + \frac{\sqrt{10}}{3} \quad\quad m = -\frac{4}{6} - \frac{\sqrt{10}}{3}$

$m = 0.39 \quad\quad\quad m = -1.72$

c) $-\frac{1}{4}n^2 - n - \frac{1}{8} = 0$

$(-\frac{1}{4}n^2 - n) - \frac{1}{8} = 0$

$-\frac{1}{4}(n^2 + 4n) - \frac{1}{8} = 0$

$\quad\quad \hookrightarrow \frac{1}{2}(4) = 2$

$\quad\quad \hookrightarrow 2^2 = 4$

$-\frac{1}{4}(n^2 + 4n + 4 - 4) - \frac{1}{8} = 0$

$-\frac{1}{4}(n^2 + 4n + 4) + 1 - \frac{1}{8} = 0$

$-\frac{1}{4}(n+2)^2 + \frac{8}{8} - \frac{1}{8} = 0$

$-\frac{4}{1} \cdot -\frac{1}{4}(n+2)^2 = -\frac{7}{8} \cdot -\frac{4}{1}$

$(n+2)^2 = \frac{28}{8}$

$n + 2 = \pm\sqrt{\frac{28}{8}}$

$n = -2 \pm \sqrt{\frac{28}{8}}$

$n = -2 + \sqrt{\frac{28}{8}} \quad\quad n = -2 - \sqrt{\frac{28}{8}}$

$n = -0.13 \quad\quad\quad n = -3.87$

d) $1.2x^2 - 3x - 6 = 0$

$(1.2x^2 - 3x) - 6 = 0$

$1.2(x^2 - 2.5x) - 6 = 0$

$\quad\quad \hookrightarrow \frac{1}{2}(-2.5) = -1.25$

$\quad\quad \hookrightarrow (-1.25)^2 = 1.5625$

$1.2(x^2 - 2.5x + 1.5625 - 1.5625) - 6 = 0$

$1.2(x^2 - 2.5x + 1.5625) - 1.875 - 6 = 0$

$1.2(x - 1.25)^2 - 7.875 = 0$

$\frac{1.2(x - 1.25)^2}{1.2} = \frac{7.875}{1.2}$

$(x - 1.25)^2 = 6.5625$

$x - 1.25 = \pm\sqrt{6.5625}$

$x = 1.25 \pm \sqrt{6.5625}$

$x = 1.25 + \sqrt{6.5625}$

$x = 3.81$

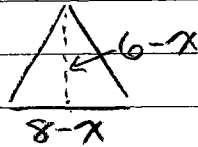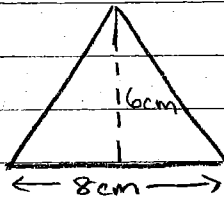$x = 1.25 - \sqrt{6.5625}$

$x = -1.31$

7. 
$$h = -4.9t^2 + d$$
$$0 = -4.9t^2 + 196$$
$$\frac{4.9t^2}{4.9} = \frac{196}{4.9}$$
$$t^2 = 40$$
$$t = \pm\sqrt{40}$$
a) $t = \sqrt{40}$ seconds
b) $t = 6.3$ seconds

8.) 
$$v = \frac{1}{3}hr^2h$$
$$168 = \frac{1}{3}\pi r^2 (9)$$
$$168 = \frac{1}{3}(9)\pi r^2$$
$$\frac{168}{3} = \frac{3\pi r^2}{3}$$
$$\frac{56}{\pi} = \frac{\pi r^2}{\pi}$$

$$r^2 = \frac{56}{\pi}$$
$$r = \sqrt{\frac{56}{\pi}}$$
$$r = 4.2 \text{ cm}$$

9.



$6-x$   Area = $20 \text{ cm}^2$
$8-x$

$$A = \frac{1}{2}bh$$
$$20 = \frac{1}{2}(8-x)(6-x)$$
$$2 \cdot 20 = 2 \cdot \frac{1}{2}(48 - 8x - 6x + x^2)$$
$$40 = 48 - 14x + x^2$$
$$x^2 - 14x + 48 - 40 = 0$$
$$x^2 - 14x + 8 = 0$$

$\hookrightarrow \frac{1}{2}(-14) = -7$
$\hookrightarrow (-7)^2 = 49$
$$x^2 - 14x + 49 - 49 + 8 = 0$$
$$(x + 7)^2 - 41 = 0$$
$$(x - 7)^2 = 41$$
$$x - 7 = \pm\sqrt{41}$$
$$x = +7 \pm\sqrt{41}$$
$x = 7 + \sqrt{41}$     $x = 7 - \sqrt{41}$
$x = 13.4$              $= 0.597$
reject

Base = $8 - 0.6 = 7.4$ cm
Height = $6 - 0.6 = 5.4$ cm

10. $h(t) = -5t^2 + 20t + 2$
a) $0 = -5t^2 + 20t + 2$
$$0 = -5(t^2 - 4t) + 2$$
$\hookrightarrow \frac{1}{2}(-4) = -2$
$\hookrightarrow (-2)^2 = 4$
$$0 = -5(t^2 - 4t + 4 - 4) + 2$$
$$0 = -5(t^2 - 4t + 4) + 20 + 2$$
⊛ $0 = -5(t-2)^2 + 22$
$$\frac{-22}{-5} = \frac{-5(t-2)^2}{-5}$$

$4.4 = (t-2)^2$
$$t - 2 = \pm\sqrt{4.4}$$
$$t = 2 \pm\sqrt{4.4}$$
$$t = 2 \pm\sqrt{4.4}$$
$$t = 2 \pm 2.1 \quad \text{(can't be negative)}$$
$$t = 4.1 \text{ sec}$$

Take it from this step h=17

b) ⊛ $17 = -5(t-2)^2 + 22$
$$17 - 22 = -5(t-2)^2$$
$$\frac{-5}{5} = \frac{-5(t-2)^2}{-5}$$
$$1 = (t-2)^2$$
$$t - 2 = \pm 1$$
$$t = 2 \pm 1$$
$t = 2 + 1$     $t = 2 - 1$
$t = 3$        $t = 1$

From 1sec to 3 sec, the ball is above 17m so in total, the ball is above 17m for 2 seconds

11. a.) $x^2 + 2x - k = 0$

$\rightarrow \frac{1}{2}(2) = 1$

$\rightarrow (1)^2 = 1$

$(x^2 + 2x + 1 - 1) - k = 0$

$(x+1)^2 - 1 - k = 0$

$(x+1)^2 = 1 + k$

$x + 1 = \pm\sqrt{1+k}$

$x = -1 \pm\sqrt{1+k}$

b.) $kx^2 - 2x = k$

$k(x^2 - \frac{2}{k}x) - k = 0$

$\rightarrow \frac{1}{2}(-\frac{2}{k}) = -\frac{2}{2k} = -\frac{1}{k}$

$\rightarrow (-\frac{1}{k})^2 = \frac{1}{k^2}$

$k(x^2 - \frac{2}{k}x + \frac{1}{k^2} - \frac{1}{k^2}) - k = 0$

$k(x^2 - \frac{2}{k}x + \frac{1}{k^2}) - \frac{1}{k} - k = 0$

$k(x - \frac{1}{k})^2 - \frac{1}{k} - \frac{k^2}{k} = 0$

$k(x - \frac{1}{k})^2 = \frac{1}{k} - k$

$\frac{k(x - \frac{1}{k})^2}{k} = \frac{\frac{1}{k} - k}{k}$

$(x - \frac{1}{k})^2 = (\frac{1}{k} - k) \times \frac{1}{k}$

$(x - \frac{1}{k})^2 = \frac{1}{k^2} - \frac{k}{k}$

$x - \frac{1}{k} = \pm\sqrt{\frac{1 - k^2}{k^2}}$

$x = \frac{1}{k} \pm \frac{\sqrt{1 - k^2}}{k}$

$x = \frac{1 \pm \sqrt{1 - k^2}}{k}$

c.) $x^2 = kx + 1$

$x^2 - kx - 1 = 0$

$\rightarrow \frac{1}{2}(-k) = -\frac{k}{2}$

$\rightarrow (-\frac{k}{2})^2 = \frac{k^2}{4}$

$x^2 - kx + \frac{k^2}{4} - \frac{k^2}{4} - 1 = 0$

$(x - \frac{k}{2})^2 = \frac{k^2}{4} + \frac{4}{4}$

$x - \frac{k}{2} = \pm\sqrt{\frac{k^2+4}{4}}$

$x = \frac{k}{2} \pm \frac{\sqrt{k^2+4}}{2} = \frac{k \pm \sqrt{k^2+4}}{2}$

12. $x^2 + (k+7)x + (7k+1)$

If this is a perfect square trinomial, then

$[\frac{1}{2}(k+7)]^2 = 7k+1$

$\frac{1}{4}(k+7)(k+7) = 7k+1$

$\frac{1}{4}(k^2 + 14k + 49) = 7k+1$

$\frac{1}{4}k^2 + \frac{14}{4}k + \frac{49}{4} = 7k+1$

$\frac{1}{4}k^2 + \frac{14}{4}k + \frac{49}{4} - 7k - 1 = 0$

$\frac{1}{4}k^2 + \frac{14}{4}k - \frac{28}{4}k + \frac{49}{4} - \frac{4}{4} = 0$

$\frac{1}{4}k^2 - \frac{14}{4}k + \frac{45}{4} = 0$

$\frac{1}{4}(k^2 - 14k) + \frac{45}{4} = 0$

$\rightarrow \frac{1}{2}(-14) = -7$

$\rightarrow (-7)^2 = 49$

$\frac{1}{4}(k^2 - 14k + 49 - 49) + \frac{45}{4} = 0$

$\frac{1}{4}(k^2 - 14k + 49) - \frac{49}{4} + \frac{45}{4} = 0$

$\Rightarrow \frac{1}{4}(k-7)^2 - \frac{4}{4} = 0$

$\frac{1}{4}(k-7)^2 = 1$

$4 \cdot \frac{1}{4}(k-7)^2 = 1 \cdot 4$

$(k-7)^2 = 4$

$k - 7 = \pm\sqrt{4}$

$k = 7 \pm 2$

$k = 7 + 2 \qquad k = 7 - 2$

$k = 9 \qquad\qquad k = 5$

13. $x^2 + bx + c = 0$

$$\rightarrow \tfrac{1}{2}(b) = \tfrac{b}{2}$$
$$\rightarrow \left(\tfrac{b}{2}\right)^2 = \tfrac{b^2}{4}$$

$x^2 + bx + \tfrac{b^2}{4} - \tfrac{b^2}{4} + c = 0$

$\left(x + \tfrac{b}{2}\right)^2 = \tfrac{b^2}{4} - c$

$x + \tfrac{b}{2} = \pm \sqrt{\tfrac{b^2}{4} - c}$

$x = -\tfrac{b}{2} \pm \sqrt{\tfrac{b^2 - 4c}{4}}$

$= -\tfrac{b}{2} \pm \tfrac{\sqrt{b^2 - 4c}}{2}$

$x = \dfrac{-b \pm \sqrt{b^2 - 4c}}{2}$

14. $ax^2 + bx + c = 0$

$a\left(x^2 + \tfrac{b}{a}x\right) + c = 0$

$$\rightarrow \tfrac{1}{2}\left(\tfrac{b}{a}\right) = \tfrac{b}{2a}$$
$$\rightarrow \left(\tfrac{b}{2a}\right)^2 = \tfrac{b^2}{4a^2}$$

$a\left(x^2 + \tfrac{b}{a}x + \tfrac{b^2}{4a^2} - \tfrac{b^2}{4a^2}\right) + c = 0$

$a\left(x^2 + \tfrac{b}{a}x + \tfrac{b^2}{4a^2}\right) - \tfrac{b^2}{4a} + c = 0$

$a\left(x + \tfrac{b}{2a}\right)^2 = \tfrac{b^2}{4a} + c$

$\tfrac{1}{a} \cdot a\left(x + \tfrac{b}{2a}\right)^2 = \left(\tfrac{b^2 + 4ac}{4a}\right) \cdot \tfrac{1}{a}$

$\left(x + \tfrac{b}{2a}\right)^2 = \tfrac{b^2 + 4ac}{4a^2}$

$x + \tfrac{b}{2a} = \pm \sqrt{\tfrac{b^2 + 4ac}{4a^2}}$

$x = -\tfrac{b}{2a} \pm \tfrac{\sqrt{b^2 + 4ac}}{2a}$ $\qquad = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

PCII 3.4

1. a.) $2x^2-5x+2=0$

$a=2 \quad b=-5 \quad c=2$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-(-5)\pm\sqrt{(-5)^2-4(2)(2)}}{2(2)}$

$x=\dfrac{5\pm\sqrt{25-16}}{4}$

$x=\dfrac{5\pm\sqrt{9}}{4}$

$x=\dfrac{5\pm 3}{4}$

$x=\dfrac{5+3}{4} \qquad x=\dfrac{5-3}{4}$

$x=\dfrac{8}{4} \qquad x=\dfrac{2}{4}$

$x=2 \qquad x=\dfrac{1}{2}$

b.) $3n^2-11n-14=0$

$a=3 \quad b=-11 \quad c=-14$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$n=\dfrac{-(-11)\pm\sqrt{(-11)^2-4(3)(-14)}}{2(3)}$

$=\dfrac{11\pm\sqrt{121+168}}{6}$

$=\dfrac{11\pm\sqrt{289}}{6}$

$=\dfrac{11\pm 17}{6}$

$n=\dfrac{11+17}{6} \qquad n=\dfrac{11-17}{6}$

$n=\dfrac{28}{6} \qquad n=\dfrac{-6}{6}$

$n=\dfrac{14}{3} \qquad n=-1$

c.) $f^2-6f+4=0$

$a=1 \quad b=-6 \quad c=4$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$f=\dfrac{-(-6)\pm\sqrt{(-6)^2-4(1)(4)}}{2(1)}$

$=\dfrac{6\pm\sqrt{20}}{2}$

$=\dfrac{6\pm\sqrt{4\cdot 5}}{2}$

$=\dfrac{6\pm 2\sqrt{5}}{2}$

$f=3\pm\sqrt{5}$

d.) $x^2+7x+3=0$

$a=1 \quad b=7 \quad c=3$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-7\pm\sqrt{(7)^2-4(1)(3)}}{2(1)}$

$=\dfrac{-7\pm\sqrt{49-12}}{2}$

$x=\dfrac{-7\pm\sqrt{37}}{2}$

1. e.) $6t^2 - t - 1 = 0$

$a = 6 \quad b = -1 \quad c = -1$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(6)(-1)}}{2(6)}$

$= \dfrac{1 \pm \sqrt{1 + 24}}{12}$

$= \dfrac{1 \pm \sqrt{25}}{12}$

$t = \dfrac{1 \pm 5}{12} \qquad t = \dfrac{1 - 5}{12}$

$t = \dfrac{1 + 5}{12} \qquad t = \dfrac{-4}{12}$

$t = \dfrac{6}{12} = \dfrac{1}{2} \qquad t = \dfrac{-1}{3}$

f.) $5h^2 + 7h + 2 = 0$

$a = 5 \quad b = 7 \quad c = 2$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$h = \dfrac{-(7) \pm \sqrt{(7)^2 - 4(5)(2)}}{2(5)}$

$= \dfrac{-7 \pm \sqrt{49 - 40}}{10}$

$= \dfrac{-7 \pm \sqrt{9}}{10}$

$h = \dfrac{-7 + 3}{10} \qquad h = \dfrac{-7 - 3}{10}$

$h = \dfrac{-4}{10} \qquad h = \dfrac{-10}{10}$

$h = \dfrac{-2}{5} \qquad h = -1$

g.) $6m^2 - 7m + 2 = 0$

$a = 6 \quad b = -7 \quad c = 2$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(2)}}{2(6)}$

$= \dfrac{7 \pm \sqrt{49 - 48}}{12}$

$= \dfrac{7 \pm \sqrt{1}}{12}$

$m = \dfrac{7 + 1}{12} \qquad m = \dfrac{7 - 1}{12}$

$m = \dfrac{8}{12} \qquad m = \dfrac{6}{12}$

$m = \dfrac{2}{3} \qquad m = \dfrac{1}{2}$

h.) $a^2 - a + 5 = 0$

$a = 1 \quad b = -1 \quad c = 5$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = \dfrac{-(1) \pm \sqrt{(-1)^2 - 4(1)(5)}}{2(1)}$

$= \dfrac{-1 \pm \sqrt{1 - 20}}{2}$

$a = \dfrac{-1 \pm \sqrt{-19}}{2} \quad \uparrow$ no real roots.

1. i.) $w^2 + 3w - 1 = 0$

$a = 1 \quad b = 3 \quad c = -1$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$w = \dfrac{-3 \pm \sqrt{3^2 - 4(1)(-1)}}{2(1)}$

$w = \dfrac{-3 \pm \sqrt{9 + 4}}{2}$

$w = \dfrac{-3 \pm \sqrt{13}}{2}$

2. a.) $25q^2 + 70q + 49 = 0$

$a = 25 \quad b = 70 \quad c = 49$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$q = \dfrac{-70 \pm \sqrt{(70)^2 - 4(25)(49)}}{2(25)}$

$= \dfrac{-70 \pm \sqrt{4900 - 4900}}{50}$

$= \dfrac{-70 \pm \sqrt{0}}{50}$

$q = -\dfrac{7}{5}$

b.) $12v^2 - 192 = 0$

$a = 12 \quad b = 0 \quad c = -192$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$v = \dfrac{-0 \pm \sqrt{0^2 - 4(12)(-192)}}{2(12)}$

$= \dfrac{\pm \sqrt{9216}}{24}$

$= \dfrac{\pm 96}{24}$

$v = \pm 4$

c.) $3r^2 - 4r = 0$

$a = 3 \quad b = -4 \quad c = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$r = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(0)}}{2(3)}$

$= \dfrac{4 \pm \sqrt{16}}{6}$

$r = \dfrac{4 \pm 4}{6} \qquad r = \dfrac{4 - 4}{6}$

$r = \dfrac{4 + 4}{6} \qquad r = \dfrac{0}{6}$

$r = \dfrac{8}{6} = \dfrac{4}{3} \qquad r = 0$

d.) $5x^2 + 12x + 9 = 0$

$a = 5 \quad b = 12 \quad c = 9$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-12 \pm \sqrt{12^2 - 4(5)(9)}}{2(4)}$

$= \dfrac{-12 \pm \sqrt{144 - 180}}{8}$

$= \dfrac{-12 \pm \sqrt{-36}}{8}$

no real roots

2. e) $16d^2 + 40d + 25 = 0$

$a = 16 \quad b = 40 \quad c = 25$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$d = \dfrac{-40 \pm \sqrt{40^2 - 4(16)(25)}}{2(16)}$

$= \dfrac{-40 \pm \sqrt{1600 - 1600}}{32}$

$= \dfrac{-40 \pm \sqrt{0}}{32}$

$d = \dfrac{-40}{32} = \dfrac{-5}{4}$

f.) $y^2 - 4 = 0$

$a = 1 \quad b = 0 \quad c = -4$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \dfrac{-0 \pm \sqrt{0^2 - 4(1)(-4)}}{2(1)}$

$y = \dfrac{\pm \sqrt{16}}{2}$

$y = \dfrac{\pm 4}{2}$

$y = \pm 2$

3. a) $\dfrac{5}{2}x^2 - \dfrac{3}{2}x - \dfrac{1}{4} = 0$

$a = \dfrac{5}{2} \quad b = -\dfrac{3}{2} \quad c = -\dfrac{1}{4}$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-\left(-\frac{3}{2}\right) \pm \sqrt{\left(-\frac{3}{2}\right)^2 - 4\left(\frac{5}{2}\right)\left(-\frac{1}{4}\right)}}{2\left(\frac{5}{2}\right)}$

$x = \dfrac{\frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{20}{8}}}{\frac{10}{2}}$

$= \dfrac{\frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{10}{4}}}{5}$

$= \dfrac{\frac{3}{2} \pm \sqrt{\frac{19}{4}}}{5}$

$= \dfrac{\frac{3}{2} \pm \frac{\sqrt{19}}{2}}{5}$

$= \dfrac{3 \pm \sqrt{19}}{2} \cdot \dfrac{1}{5}$

$= \dfrac{3 \pm \sqrt{19}}{10}$

b) $\dfrac{2n^2}{3} - \dfrac{n}{3} = 2$

$\dfrac{2}{3}n^2 - \dfrac{1}{3}n - 2 = 0$

$a = \dfrac{2}{3} \quad b = -\dfrac{1}{3} \quad c = -2$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$n = \dfrac{-\left(-\frac{1}{3}\right) \pm \sqrt{\left(-\frac{1}{3}\right)^2 - 4\left(\frac{2}{3}\right)(-2)}}{2\left(\frac{2}{3}\right)}$

$= \dfrac{\frac{1}{3} \pm \sqrt{\frac{1}{9} + \frac{16}{3}}}{\frac{4}{3}}$

$= \dfrac{\frac{1}{3} \pm \sqrt{\frac{1}{9} + \frac{48}{9}}}{\frac{4}{3}}$

$= \dfrac{\frac{1}{3} \pm \sqrt{\frac{49}{9}}}{\frac{4}{3}}$

$= \dfrac{\frac{1}{3} \pm \frac{7}{3}}{\frac{4}{3}}$

$n = \dfrac{\frac{1}{3} + \frac{7}{3}}{\frac{4}{3}} \qquad n = \dfrac{\frac{1}{3} - \frac{7}{3}}{\frac{4}{3}}$

$n = \dfrac{8}{3} \times \dfrac{3}{4} \qquad = \dfrac{-6}{3} \times \dfrac{3}{4}$

$n = \dfrac{8}{4} \qquad = \dfrac{-6}{4}$

$n = 2 \qquad = -\dfrac{3}{2}$

3. c) $\dfrac{x^2}{2} = x + \dfrac{5}{2}$

$\dfrac{1}{2}x^2 - x - \dfrac{5}{2} = 0$

$a = \dfrac{1}{2}$   $b = -1$   $c = -\dfrac{5}{2}$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4\left(\frac{1}{2}\right)\left(-\frac{5}{2}\right)}}{2\left(\frac{1}{2}\right)}$

$= \dfrac{1 \pm \sqrt{1 + \frac{20}{4}}}{1}$

$= \dfrac{1 \pm \sqrt{\frac{4}{4} + \frac{20}{4}}}{1}$

$= 1 \pm \sqrt{\dfrac{24}{4}}$

$x = 1 \pm \sqrt{6}$

4a) $0.2z^2 - z - 3.2 = 0$

$a = 0.2$   $b = -1$   $c = -3.2$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$z = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(0.2)(-3.2)}}{2(0.2)}$

$= \dfrac{1 \pm \sqrt{1 + 2.56}}{0.4}$

$= \dfrac{1 \pm \sqrt{3.56}}{0.4}$

$z = \dfrac{1 + \sqrt{3.56}}{0.4}$      $z = \dfrac{1 - \sqrt{3.56}}{0.4}$

$z = 7.22$            $z = -2.12$

4. b) $1.2x^2 = 1.4x - 1$

$1.2x^2 - 1.4x + 1 = 0$

$a = 1.2$   $b = -1.4$   $c = 1$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(-1.4) \pm \sqrt{(-1.4)^2 - 4(1.2)(1)}}{2(1.2)}$

$= \dfrac{1.4 \pm \sqrt{1.96 + 4.8}}{2.4}$

$= \dfrac{1.4 \pm \sqrt{-2.84}}{2.4}$

no real roots

c) $0.1 = -2.2y^2 - 2.4y$

$0 = -2.2y^2 - 2.4y - 0.1$

$a = -2.2$   $b = -2.4$   $c = -0.1$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \dfrac{-(-2.4) \pm \sqrt{(-2.4)^2 - 4(-2.2)(-0.1)}}{2(-2.2)}$

$= \dfrac{2.4 \pm \sqrt{5.76 - 0.88}}{-4.4}$

$= \dfrac{2.4 \pm \sqrt{4.88}}{-4.4}$

$y = \dfrac{2.4 + \sqrt{4.88}}{-4.4}$      $y = \dfrac{2.4 - \sqrt{4.88}}{-4.4}$

$y = -1.05$            $y = -0.04$

5. a.) $2x^2 + 6x + 2 = 0$

$a = 2 \quad b = 6 \quad c = 2$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-6 \pm \sqrt{(6)^2 - 4(2)(2)}}{2(2)}$

$x = \dfrac{-6 \pm \sqrt{20}}{4}$

$x = \dfrac{-6 \pm \sqrt{4 \cdot 5}}{4}$

$x = \dfrac{-6 \pm 2\sqrt{5}}{4}$

$x = \dfrac{-3 \pm \sqrt{5}}{2}$

b.) $z^2 - 6z + 7 = 0$

$a = 1 \quad b = -6 \quad c = 7$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$z = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)}$

$= \dfrac{6 \pm \sqrt{36 - 28}}{2}$

$= \dfrac{6 \pm \sqrt{8}}{2}$

$= \dfrac{6 \pm \sqrt{4 \cdot 2}}{2} \qquad = \dfrac{6 \pm 2\sqrt{2}}{2}$

$z = 3 \pm \sqrt{2}$

c.) $9m^2 - 6m - 1 = 0$

$a = 9 \quad b = -6 \quad c = -1$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(-1)}}{2(9)}$

$= \dfrac{6 \pm \sqrt{36 + 36}}{18}$

$= \dfrac{6 \pm \sqrt{72}}{18}$

$= \dfrac{6 \pm \sqrt{36 \cdot 2}}{18}$

$= \dfrac{6 \pm 6\sqrt{2}}{18}$

$m = \dfrac{1 \pm \sqrt{2}}{3}$

6. a) $(n-4)(n-2)=12$

$n^2-2n-4n+8-12=0$

$n^2-6n-4=0$

$a=1 \quad b=-6 \quad c=-4$

$n=\dfrac{-(-6)\pm\sqrt{(-6)^2-4(1)(-4)}}{2(1)}$

$=\dfrac{6\pm\sqrt{36+16}}{2}$

$=\dfrac{6\pm\sqrt{52}}{2}$

$=\dfrac{6\pm\sqrt{4\cdot13}}{2}$

$=\dfrac{6\pm2\sqrt{13}}{2} = 3\pm\sqrt{13}$

b) $(2x-1)(3x+5)=(x+2)(2x-1)$

$6x^2+10x-3x-5=2x^2-x+4x-2$

$6x^2+7x-5-2x^2-3x+2=0$

$4x^2+4x-3=0$

$a=4 \quad b=4 \quad c=-3$

$x=\dfrac{-4\pm\sqrt{(4)^2-4(4)(-3)}}{2(4)}$

$=\dfrac{-4\pm\sqrt{16+48}}{8}$

$=\dfrac{-4\pm\sqrt{64}}{8} \quad = \dfrac{-4\pm8}{8}$

$x=\dfrac{-4+8}{8} \qquad x=\dfrac{-4-8}{8}$

$x=\dfrac{4}{8} \qquad\qquad x=\dfrac{-12}{8}$

$x=\dfrac{1}{2} \qquad\qquad x=-\dfrac{3}{2}$

c) $3m^2-(5m+1)(2m-3)=3$

$3m^2-(10m^2-15m+2m-3)=3$

$3m^2-10m^2+15m-2m+3-3=0$

$-7m^2+13m=0$

$a=-7 \quad b=13 \quad c=0$

$x=\dfrac{-(13)\pm\sqrt{(13)^2-4(-7)(0)}}{2(-7)}$

$=\dfrac{-13\pm\sqrt{169}}{-14}$

$=\dfrac{-13\pm13}{-14}$

$x=\dfrac{-13+13}{-14} \qquad x=\dfrac{-13-13}{-14}$

$x=0 \qquad\qquad =\dfrac{-26}{-14}$

$\qquad\qquad\qquad x=\dfrac{13}{7}$

d) $2(w-2)(w+1)-(w+3)=0$

$(2w-4)(w+1)-w-3=0$

$2w^2+2w-4w-4-w-3=0$

$2w^2-3w-7=0$

$a=2 \quad b=-3 \quad c=-7$

$w=\dfrac{-(-3)\pm\sqrt{(-3)^2-4(2)(-7)}}{2(2)}$
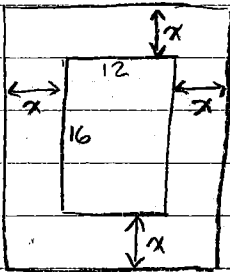
$=\dfrac{3\pm\sqrt{9+56}}{4}$

$=\dfrac{3\pm\sqrt{65}}{4}$

7.

$(12+2x)(16+2x) - 12(16) = 12(16)$

$192 + 24x + 32x + 4x^2 - 192 = 192$

$4x^2 + 56x - 192 = 0$

$a=4 \quad b=56 \quad c=-192$

$x = \dfrac{-56 \pm \sqrt{(56)^2 - 4(4)(-192)}}{2(4)}$

$= \dfrac{-56 \pm \sqrt{6208}}{8}$

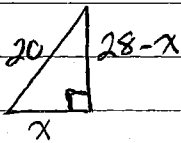$x = \dfrac{-56 + \sqrt{6208}}{8} \qquad\qquad x = \dfrac{-56 - \sqrt{6208}}{8}$

$x = 2.85 \qquad\qquad\qquad x = -16.85 \quad \leftarrow$ Can't be negative

The width of the mat is 2.85cm.

8.

$x^2 + (28-x)^2 = 20^2$

$x^2 + (28-x)(28-x) = 400$

$x^2 + 784 - 28x - 28x + x^2 = 400$

$2x^2 - 56x + 384 = 0$

$a=2 \quad b=-56 \quad c=384$

$x = \dfrac{-(-56) \pm \sqrt{(-56)^2 - 4(2)(384)}}{2(2)}$

$= \dfrac{56 \pm \sqrt{3136 - 3072}}{4}$

$x = \dfrac{56 \pm \sqrt{64}}{4}$

$x = \dfrac{56 \pm 8}{4}$

$x = \dfrac{56+8}{4} \qquad x = \dfrac{56-8}{4} \qquad$ The lengths

$x = 16 \qquad\qquad x = 12 \qquad$ are 16cm & 12cm

then $28-x$

$28 - 16 = 12$

9. a.) $px^2 + qx + r = 0$

$a = p \quad b = q \quad c = r$

$x = \dfrac{-q \pm \sqrt{q^2 - 4pr}}{2p}$

b.) $3x^2 + nx - 5 = 0$

$a = 3 \quad b = n \quad c = -5$

$x = \dfrac{-n \pm \sqrt{n^2 - 4(3)(5)}}{2(3)}$

$x = \dfrac{-n \pm \sqrt{n^2 + 60}}{6}$

c.) $x^2 + (3m - 2n)x = 6mn$

$x^2 + (3m - 2n)x - 6mn = 0$

$a = 1 \quad b = 3m - 2n \quad c = -6mn$

$x = \dfrac{-(3m - 2n) \pm \sqrt{(3m - 2n)^2 - 4(1)(-6mn)}}{2(1)}$

$= \dfrac{-3m + 2n \pm \sqrt{9m^2 - 12mn + 4n^2 + 24mn}}{2}$

$= \dfrac{-3m + 2n \pm \sqrt{9m^2 + 12mn + 4n^2}}{2}$

$= \dfrac{-3m + 2n \pm \sqrt{(3m + 2n)^2}}{2}$

$= \dfrac{-3m + 2n \pm (3m + 2n)}{2}$

$x = \dfrac{-3m + 2n + 3m + 2n}{2}$          $x = \dfrac{-3n + 2n - 3m - 2n}{2}$

$x = \dfrac{4n}{2}$          $x = \dfrac{-6m}{2}$

$x = 2n$          $x = -3m$

PCII 3.5

1. a) $x^2 + 11x + 24 = 0$
$a = 1 \quad b = 11 \quad c = 24$
$b^2 - 4ac$
$= 11^2 - 4(1)(24)$
$= 25$
∴ There are 2 real roots

b) $n^2 - 4n + 2 = 0$
$a = 1 \quad b = -4 \quad c = 2$
$b^2 - 4ac$
$= (-4)^2 - 4(1)(2)$
$= 8$
∴ There are 2 real roots

c) $4m^2 - 20m + 25 = 0$
$a = 4 \quad b = -20 \quad c = 25$
$b^2 - 4ac$
$= (-20)^2 - 4(4)(25)$
$= 0$
∴ There is one real root

d) $2x^2 - 5x + 8 = 0$
$a = 2 \quad b = -5 \quad c = 8$
$b^2 - 4ac$
$= (-5)^2 - 4(2)(8)$
$= -39$
∴ There are No real roots

e) $3q^2 + 13q - 10 = 0$
$a = 3 \quad b = 13 \quad c = -10$
$b^2 - 4ac$
$= (13)^2 - 4(3)(-10)$
$= 289$
∴ There are 2 real roots

f) $7p^2 + 12p + 6 = 0$
$a = 7 \quad b = 12 \quad c = 6$
$b^2 - 4ac$
$= (12)^2 - 4(7)(6)$
$= -24$
∴ There are No real roots

2. a) $w^2 + 5 = 3w$
$w^2 - 3w + 5 = 0$
$a = 1 \quad b = -3 \quad c = 5$
$b^2 - 4ac$
$= (-3)^2 - 4(1)(5)$
$= 9 - 20$
$= -11$
∴ There are No real roots

b) $3 - 5t = t^2$
$-t^2 - 5t + 3 = 0$
$a = -1 \quad b = -5 \quad c = 3$
$b^2 - 4ac$
$= (-5)^2 - 4(-1)(3)$
$= 25 + 12$
$= 37$
∴ There are 2 real roots

2. c.) $25 + x^2 = -10x$

$x^2 + 10x + 25 = 0$

$a = 1 \quad b = 10 \quad c = 25$

$b^2 - 4ac$

$= (10)^2 - 4(1)(25)$

$= 100 - 100$

$= 0$

∴ There is ONE real root

d.) $9n^2 = 5n$

$9n^2 - 5n = 0$

$a = 9 \quad b = -5 \quad c = 0$

$b^2 - 4ac$

$= (-5)^2 - 4(9)(0)$

$= 25 - 0$

$= 25$

∴ There are 2 real roots.

e.) $0.5x^2 + 4x + 4 = 0$

$a = 0.5 \quad b = 4 \quad c = 4$

$b^2 - 4ac$

$= (4)^2 - 4(0.5)(4)$

$= 16 - 8$

$= 8$

∴ There are 2 real roots.

f.) $(x+1)(x-2) = 4$

$x^2 - 2x + x - 2 - 4 = 0$

$x^2 - x - 6 = 0$

$a = 1 \quad b = -1 \quad c = -6$

$b^2 - 4ac$

$= (-1)^2 - 4(1)(-6)$

$= 25$

∴ There are 2 real roots.

g.) $4(x^2 - 5x + 5) = -5$

$4x^2 - 20x + 20 + 5 = 0$

$4x^2 - 20x + 25 = 0$

$a = 4 \quad b = -20 \quad c = 25$

$b^2 - 4ac$

$= (-20)^2 - 4(4)(25)$

$= 400 - 400$

$= 0$

∴ There is ONE real root

h.) $2(y^2 + 3) = 4y$

$2y^2 + 6 = 4y$

$2y^2 - 4y + 6 = 0$

$a = 2 \quad b = -4 \quad c = 6$

$b^2 - 4ac$

$= (-4)^2 - 4(2)(6)$

$= 16 - 48$

$= -32$

∴ There are NO real roots

2. i.) $\sqrt{5}x^2 + 7x + 2\sqrt{5} = 0$

$a = \sqrt{5}$  $b = 7$  $c = 2\sqrt{5}$

$b^2 - 4ac$

$= (7)^2 - 4(\sqrt{5})(2\sqrt{5})$

$= 49 - 8(5)$

$= 49 - 40$

$= 9$

∴ There are 2 real roots

j.) $\frac{1}{3}x^2 + \frac{1}{2}x - 1 = 0$

$a = \frac{1}{3}$  $b = \frac{1}{2}$  $c = -1$

$b^2 - 4ac$

$= (\frac{1}{2})^2 - 4(\frac{1}{3})(-1)$

$= \frac{1}{4} + \frac{4}{3}$

$= \frac{3}{12} + \frac{16}{12}$

$= \frac{19}{12}$

∴ There are 2 real roots

k.) $\frac{x-1}{2} - x^2 = 3$

$\frac{2(x-1) - 2x^2}{2} = 2(3)$

$x - 1 - 2x^2 = 6$

$-2x^2 + x - 1 - 6 = 0$

$-2x^2 + x - 7 = 0$

$a = -2$  $b = 1$  $c = -7$

$b^2 - 4ac$

$= (1)^2 - 4(-2)(-7)$

$= 1 - 56$

$= -55$

∴ There are No real roots

l.) $\frac{3x-1}{3} - \frac{2x+1}{2} = x^2$

$6(\frac{3x-1}{3}) - 6(\frac{2x+1}{2}) = 6x^2$

$2(3x-1) - 3(2x+1) = 6x^2$

$6x - 2 - 6x - 3 = 6x^2$

$-6x^2 - 5 = 0$

$a = -6$  $b = 0$  $c = -5$

$b^2 - 4ac$

$= (0)^2 - 4(-6)(-5)$

$= -120$

∴ There are No real roots

3. a.) $x^2 - 6x + k = 0$

$a = 1$  $b = -6$  $c = k$

$b^2 - 4ac = 0$

$(-6)^2 - 4(1)(k) = 0$

$36 - 4k = 0$

$36 = 4k$

$\frac{36}{4} = \frac{4k}{4}$

$k = 9$

b.) $kx^2 - 2x + 1 = 0$

$a = k$  $b = -2$  $c = 1$

$b^2 - 4ac > 0$

$(-2)^2 - 4(k)(1) > 0$

$4 - 4k > 0$

$-4k > -4$

$\frac{k < -4}{-4}$

$k < 1$

3. c) $x^2 + 4x - 2k = 0$

$a = 1$ $b = 4$ $c = -2k$

$b^2 - 4ac < 0$

$(4)^2 - 4(1)(-2k) < 0$

$16 + 8k < 0$

$8k < -16$

$k < \dfrac{-16}{8}$

$k < -2$

d) $2kx^2 - 4x + 3 = 0$

$a = 2k$ $b = -4$ $c = 3$

$b^2 - 4ac > 0$

$(-4)^2 - 4(2k)(3) > 0$

$16 - 24k > 0$

$16 > 24k$

$\dfrac{16}{24} > k$

$k < \dfrac{2}{3}$

e) $m^2 + 4km + 1 = 0$

$a = 1$ $b = 4k$ $c = 1$

$b^2 - 4ac = 0$

$(4k)^2 - 4(1)(1) = 0$

$16k^2 - 4 = 0$

$16k^2 = 4$

$k^2 = \dfrac{4}{16}$

$k = \pm\sqrt{\dfrac{4}{16}}$

$k = \pm\dfrac{2}{4}$

$k = \pm\dfrac{1}{2}$

f) $(k+1)x^2 - 2x - 3 = 0$

$a = k+1$ $b = -2$ $c = -3$

$b^2 - 4ac < 0$

$(-2)^2 - 4(k+1)(-3) < 0$

$4 + 12(k+1) < 0$

$4 + 12k + 12 < 0$

$16 + 12k < 0$

$12k < -16$

$k < \dfrac{-16}{12}$

$k < -\dfrac{4}{3}$

g) $y^2 + (k+2)y + 2k = 0$

$a = 1$ $b = k+2$ $c = 2k$

$b^2 - 4ac = 0$

$(k+2)^2 - 4(1)(2k) = 0$

$k^2 + 4k + 4 - 8k = 0$

$k^2 - 4k + 4 = 0$

$(k-2)(k-2) = 0$

$k - 2 = 0$  $k - 2 = 0$

$k = 2$  $k = 2$

h) $2x^2 + 5x - 2(k-1) = 0$

$a = 2$ $b = 5$ $c = -2(k-1)$

$b^2 - 4ac < 0$

$5^2 - 4(2)(-2(k-1)) < 0$

$25 + 16(k-1) < 0$

$25 + 16k - 16 < 0$

$16k + 9 < 0$

$16k < -9$

$k < -\dfrac{9}{16}$

4. a) $kx^2 + (3k+2)x + (2k+3) = 0$

     $a = k \quad b = 3k+2 \quad c = 2k+3$

     $b^2 - 4ac$

     $= (3k+2)^2 - 4k(2k+3)$

     $= (3k+2)(3k+2) - 8k^2 - 12k$

     $= 9k^2 + 6k + 6k + 4 - 8k^2 - 12k$

     $= \underline{k^2 + 4}$

         when you square a number and add a positive
         number, you will always get a positive number
         ∴ There will always be two real roots.

b) $(k+1)x^2 + 2kx + (k-1) = 0$

     $a = k+1 \quad b = 2k \quad c = k-1$

     $b^2 - 4ac$

     $= (2k)^2 - 4(k+1)(k-1)$

     $= 4k^2 - 4(k^2 - 1)$

     $= 4k^2 - 4k^2 + 4$

     $= 4$

       ↳ This is a positive number, therefore, you will
         always get two real roots