

Pre-Calculus II 2.1

$$\begin{aligned} \text{1. a.) } & |3-7| \\ & = |-4| \\ & = 4 \end{aligned}$$

$$\begin{aligned} \text{b.) } & |4(2)-9| \\ & = |8-9| \\ & = |-1| \\ & = 1 \end{aligned}$$

$$\begin{aligned} \text{c.) } & -3|-1-7(2)| \\ & = -3|-1-14| \\ & = -3|-15| \\ & = -3(15) = -45 \end{aligned}$$

$$\begin{aligned} \text{d.) } & |4+(-5)-2(3)| \\ & = |-1-6| \\ & = |-7| \\ & = 7 \end{aligned}$$

$$\begin{aligned} \text{e.) } & |6-9| + |5+(-11)| \\ & = |-3| + |-6| \\ & = 3+6 \\ & = 9 \end{aligned}$$

$$\begin{aligned} \text{f.) } & 5|-2| + 2|7-1| \\ & = 5(2) + 2(6) \\ & = 10 + 12 \\ & = 22 \end{aligned}$$

$$\begin{aligned} \text{g.) } & \left| \frac{1}{3} - \frac{5}{8} \right| \\ & = \left| \frac{8}{24} - \frac{15}{24} \right| \\ & = \left| \frac{-7}{24} \right| \\ & = \frac{7}{24} \end{aligned}$$

$$\begin{aligned} \text{h.) } & \left| \frac{1}{2} - \frac{2}{5} - \frac{3}{2} \right| \\ & = \left| \frac{1}{2} - \frac{4}{10} - \frac{15}{10} \right| \\ & = \left| \frac{1}{2} - \frac{19}{10} \right| \\ & = \frac{1}{2} \left(\frac{19}{10} \right) \\ & = \frac{19}{20} \end{aligned}$$

$$\begin{aligned} \text{i.) } & \left| -\frac{7}{3} + \frac{1}{2} \right| - \left| \frac{3}{4} - \frac{1}{3} \right| \\ & = \left| \frac{-28}{12} + \frac{6}{12} \right| - \left| \frac{9}{12} - \frac{4}{12} \right| \\ & = \left| \frac{-22}{12} \right| - \left| \frac{5}{12} \right| \\ & = \frac{22}{12} - \frac{5}{12} \\ & = \frac{17}{12} \end{aligned}$$

$$\begin{aligned} \text{j.) } & \frac{|3+(-8)|}{|-2| + |2-3|} \\ & = \frac{|-5|}{2 + |-1|} \\ & = \frac{5}{2+1} \\ & = \frac{5}{3} \end{aligned}$$

$$\begin{aligned} \text{k.) } & \frac{\sqrt{(7-9)^2}}{|-4-2+1|} \\ & = \frac{\sqrt{(-2)^2}}{|-5|} \\ & = \frac{\sqrt{4}}{|-5|} \\ & = \frac{2}{5} \end{aligned}$$

$$\begin{aligned} \text{l.) } & \frac{|-3|\sqrt{(-3-2)^2}}{2(|-10+1| - \sqrt{(-5)^2})} \\ & = \frac{3\sqrt{(-5)^2}}{2(|-9| - \sqrt{25})} \\ & = \frac{3\sqrt{25}}{2(9-5)} \\ & = \frac{3(5)}{2(4)} \\ & = \frac{15}{8} \end{aligned}$$

PC 11 2.1 con't. 2

$$\begin{aligned} 2. a) \quad & |3x-15|, x=2 \\ & = |3(2)-15| \\ & = |6-15| \\ & = |-9| \\ & = 9 \end{aligned}$$

$$\begin{aligned} b) \quad & |x^2+2x-9|, x=-3 \\ & = |(-3)^2+2(-3)-9| \\ & = |9-6-9| \\ & = |-6| \\ & = 6 \end{aligned}$$

$$\begin{aligned} c) \quad & |x^3-x-1|, x=-2 \\ & = |(-2)^3-(-2)-1| \\ & = |-8+2-1| \\ & = |-7| \\ & = 7 \end{aligned}$$

$$3. \quad |5+x| = 9$$

$$\begin{aligned} 5+x &= 9 \\ -5 \quad -5 & \\ \hline x &= 4 \end{aligned}$$

$$\begin{aligned} -(5+x) &= 9 \\ -5-x &= 9 \\ +5 \quad +5 & \\ \hline -x &= 14 \\ \frac{-1}{-1} \quad \frac{-1}{-1} & \\ \hline x &= -14 \end{aligned}$$

Pre-Calculus II 2.2.

$$1. a.) 3 = \sqrt{9} \quad -\sqrt{1}, \sqrt{4}, \sqrt{9}, \sqrt{16}$$

$$\frac{\sqrt{16}}{\sqrt{4}} \rightarrow -1, \sqrt{4}, 3, \sqrt{16}$$

$$-1 = -\sqrt{1}$$

$$\sqrt{4}$$

$$b.) 3\sqrt[3]{5} \rightarrow -2\sqrt[3]{5}, 0, \sqrt[3]{5}, 3\sqrt[3]{5}$$

$$-2\sqrt[3]{5}$$

$$\sqrt[3]{5}$$

$$0$$

$$c.) 3\sqrt{5} = \sqrt{9 \cdot 5} = \sqrt{45} \rightarrow \sqrt{20}, \sqrt{45}, \sqrt{125}$$

$$\frac{\sqrt{125}}{\sqrt{20}} \rightarrow \sqrt{20}, 3\sqrt{5}, \sqrt{125}$$

$$\sqrt{20}$$

$$d.) 7\sqrt{2} = \sqrt{49 \cdot 2} = \sqrt{98} \quad \sqrt{45}, \sqrt{75}, \sqrt{98}$$

$$3\sqrt{5} = \sqrt{9 \cdot 5} = \sqrt{45} \quad 3\sqrt{5}, 5\sqrt{3}, 7\sqrt{2}$$

$$5\sqrt{3} = \sqrt{25 \cdot 3} = \sqrt{75}$$

$$2. a.) \frac{\sqrt{52}}{\sqrt{4 \cdot 13}}$$

$$= 2\sqrt{13}$$

$$b.) \frac{\sqrt[3]{-54}}{\sqrt[3]{-27 \cdot 2}}$$

$$= -3\sqrt[3]{2}$$

$$c.) \frac{\sqrt[4]{48}}{\sqrt[4]{16 \cdot 3}}$$

$$= 2\sqrt[4]{3}$$

$$d.) \frac{2\sqrt{147}}{2\sqrt{49 \cdot 3}}$$

$$= 2(7)\sqrt{3}$$

$$= 14\sqrt{3}$$

$$e.) \frac{\sqrt{20}}{\sqrt{27}}$$

$$= \frac{\sqrt{4 \cdot 5}}{\sqrt{9 \cdot 3}}$$

$$= \frac{2\sqrt{5}}{3\sqrt{3}}$$

$$f.) \frac{\sqrt{48}}{\sqrt{32}}$$

$$= \frac{\sqrt{16 \cdot 3}}{\sqrt{16 \cdot 2}}$$

$$= \frac{4\sqrt{3}}{4\sqrt{2}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}}$$

$$g.) \frac{\sqrt{-24}}{\sqrt[3]{54}}$$

$$= \frac{\sqrt{-8 \cdot 3}}{\sqrt[3]{27 \cdot 2}}$$

$$= \frac{-2\sqrt{3}}{3\sqrt[3]{2}}$$

$$h.) \frac{\sqrt{250}}{\sqrt[3]{56}}$$

$$= \frac{\sqrt{125 \cdot 2}}{\sqrt[3]{8 \cdot 7}}$$

$$= \frac{5\sqrt{2}}{2\sqrt[3]{7}}$$

Pre-Calculus 11 2.2 cont...2

3. a) $2\sqrt{\frac{2}{3}}$
 $= \frac{\sqrt{4} \cdot \sqrt{2}}{\sqrt{3}}$
 $= \frac{\sqrt{8}}{\sqrt{3}}$

b) $\frac{3\sqrt{2}}{2\sqrt{5}}$
 $= \frac{\sqrt{9} \cdot \sqrt{2}}{\sqrt{4} \sqrt{5}}$
 $= \frac{\sqrt{18}}{\sqrt{20}}$
 $= \frac{\sqrt{9}}{\sqrt{10}}$

c) $\frac{1}{2} \sqrt{\frac{-4}{9}}$
 $= \frac{\sqrt{1} \cdot \sqrt{-4}}{\sqrt{4} \sqrt{9}}$
 $= \frac{\sqrt{-4}}{\sqrt{36}}$
 $= \frac{\sqrt{-1}}{\sqrt{9}}$

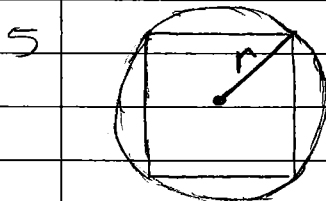
d) $\frac{4}{3} \sqrt{\frac{2}{3}}$
 $= \frac{\sqrt{16} \cdot \sqrt{2}}{\sqrt{9} \sqrt{3}}$
 $= \frac{\sqrt{32}}{\sqrt{27}}$
 $= \frac{\sqrt{128}}{\sqrt{81}}$

4. a) $\sqrt{12m}$, $12m \geq 0$
 $= \sqrt{4 \cdot 3m}$, $m \geq 0$
 $= 2\sqrt{3m}$

b) $\sqrt{5x^2}$, $5x^2 \geq 0$
 $= \sqrt{x^2} \sqrt{5}$, $x^2 \geq 0$ Always.
 $= x\sqrt{5}$, $\therefore x \in \mathbb{R}$

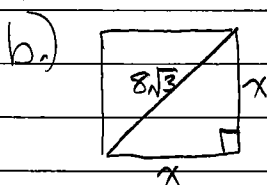
c) $\sqrt[3]{16x^5}$, $16x^5$ can be
 $= \sqrt[3]{8 \cdot 2x^3 \cdot x^2}$, pos or neg
 $= 2x\sqrt[3]{2x^2}$, $x \in \mathbb{R}$

d) $\sqrt[4]{-7a^{17}}$, $-7a^{17} \geq 0$
 $= \sqrt[4]{(-1)a^4 \cdot a^4 \cdot a^4 \cdot a^4 \cdot a}$, $-a^{17} \geq 0$
 $= aaaa\sqrt[4]{-7a}$, $a^{17} \leq 0$
 $= a^4 \sqrt[4]{-7a}$, $a \leq 0$



Circle $A = \pi r^2$
 $\frac{48\pi}{\pi} = \frac{\pi r^2}{\pi}$
 $48 = r^2$
 $\sqrt{48} = r$
 $r = \sqrt{48}$
 $= \sqrt{16 \cdot 3}$
 $= 4\sqrt{3} \text{ cm}$

a) diagonal of square = diameter of circle
 $d = 2(4\sqrt{3})$
 $d = 8\sqrt{3}$



$x^2 + x^2 = (8\sqrt{3})^2$
 $2x^2 = 64(3)$
 $\frac{2x^2}{2} = \frac{192}{2}$
 $x^2 = 96$
 $x = \sqrt{96} = \sqrt{16 \cdot 6} = 4\sqrt{6}$

$P = 4(4\sqrt{6})$
 $= 16\sqrt{6} \text{ cm}$

PC II cont... 3

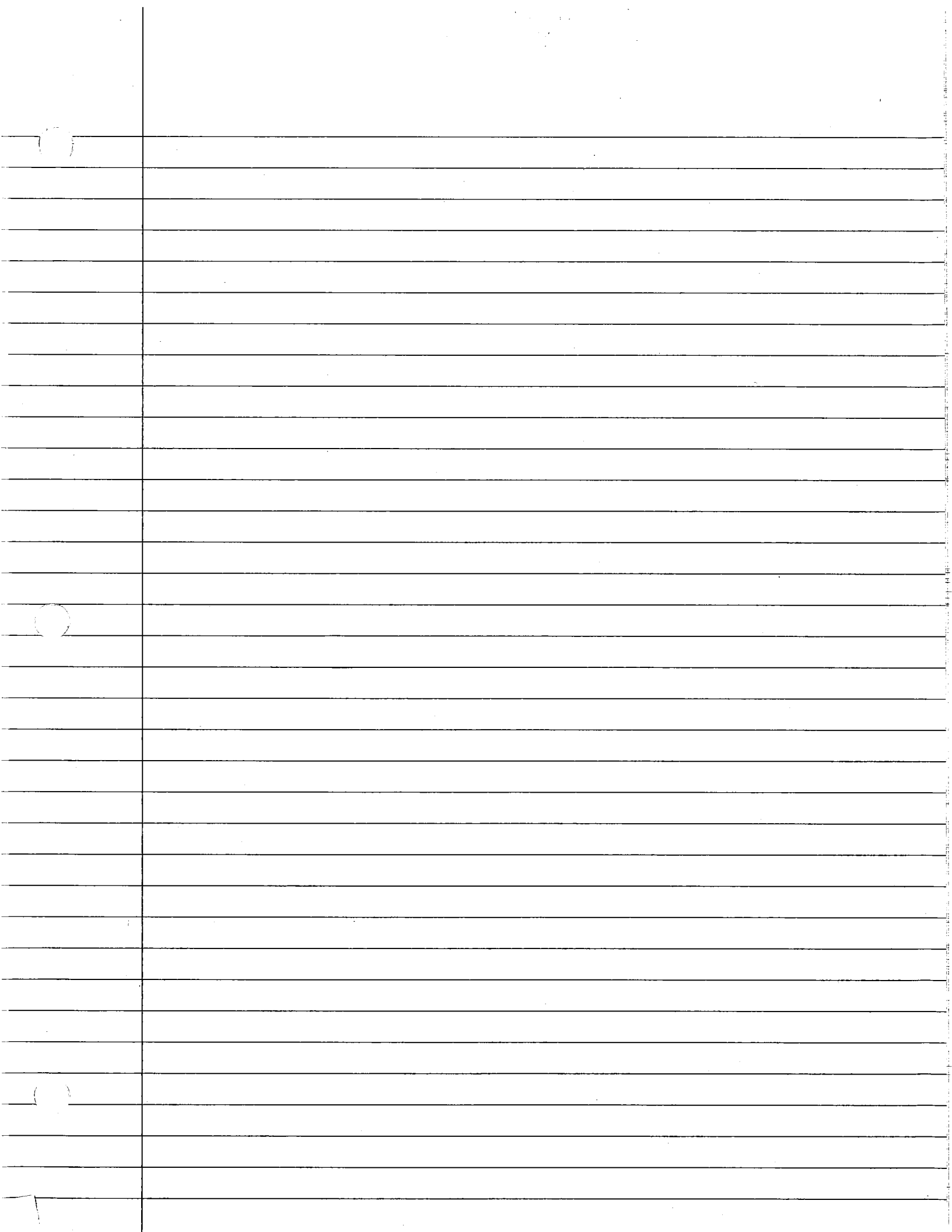
$$6. a) \sqrt{9ab} \rightarrow 9ab \geq 0$$
$$= 3\sqrt{ab} \quad \text{so either } a, b \geq 0 \text{ or } a, b \leq 0$$

$$b) \sqrt{\frac{1}{4} a^2 b} \rightarrow \frac{1}{4} a^2 b \geq 0$$
$$= \frac{1}{2} a \sqrt{b}$$
$$= \frac{a \sqrt{b}}{2}$$

\downarrow
 a^2 will always be positive so $a \in \mathbb{R}$
 $\therefore b \geq 0$

$$c) \sqrt[4]{a^3 b^4} \rightarrow a^3 b^4 \geq 0$$
$$= b \sqrt[4]{a^3}$$

\downarrow
 b^4 will always be positive so $b \in \mathbb{R}$
 $\therefore a \geq 0$



Pre-Calculus II 2.3

1. a) $\sqrt{3} - 3\sqrt{3} + 4\sqrt{3} + 7\sqrt{3}$
 $= 9\sqrt{3}$

b) $\sqrt{2} + 5\sqrt{2} - \sqrt{8}$
 $= 6\sqrt{2} - \sqrt{4 \cdot 2}$
 $= 6\sqrt{2} - 2\sqrt{2}$
 $= 4\sqrt{2}$

c) $-\sqrt{12} + \sqrt{3} + \sqrt{75}$
 $= -\sqrt{4(3)} + \sqrt{3} + \sqrt{25 \cdot 3}$
 $= -2\sqrt{3} + \sqrt{3} + 5\sqrt{3}$
 $= 4\sqrt{3}$

d) $3\sqrt{2} - \sqrt{50} + 4\sqrt{75} + 2\sqrt{8}$
 $= 3\sqrt{2} - \sqrt{25 \cdot 2} + 4\sqrt{25 \cdot 3} + 2\sqrt{4 \cdot 2}$
 $= 3\sqrt{2} - 5\sqrt{2} + 4(5)\sqrt{3} + 2(2)\sqrt{2}$
 $= 3\sqrt{2} - 5\sqrt{2} + 20\sqrt{3} + 4\sqrt{2}$
 $= 2\sqrt{2} + 20\sqrt{3}$

2. a) $3\sqrt{x} + 4\sqrt{x} - 2\sqrt{x}$
 $= 5\sqrt{x}$

b) $-\sqrt{a} + \sqrt{9a} - \sqrt{16a}$
 $= -\sqrt{a} + 3\sqrt{a} - 4\sqrt{a}$
 $= -8\sqrt{a}$

c) $\sqrt[3]{-8b} + 5\sqrt[3]{b} + \sqrt[3]{64b}$
 $= -2\sqrt[3]{b} + 5\sqrt[3]{b} + 4\sqrt[3]{b}$
 $= 7\sqrt[3]{b}$

d) $\sqrt{12xy^2} + 2\sqrt{49xy^2} - 3\sqrt{25xy^2}$
 $= 11y\sqrt{x} + 2(7)y\sqrt{x} - 3(5)y\sqrt{x}$
 $= 11y\sqrt{x} + 14y\sqrt{x} - 15y\sqrt{x}$
 $= 10y\sqrt{x}$

3. a) $3\sqrt{-a} + \sqrt{-16a} + 7\sqrt{a}$
 $-a \geq 0$
 $a \leq 0$
 $= 3\sqrt{-a} + 4\sqrt{-a} + 7\sqrt{a}$
 $= 14\sqrt{-a}$

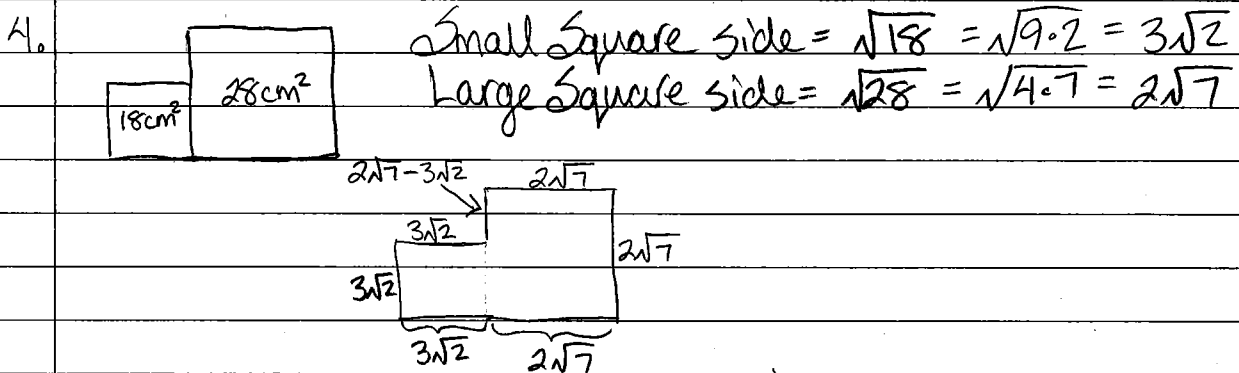
b) $\sqrt[3]{8x^5y} + 5\sqrt[3]{x^5y} - x\sqrt[3]{27x^2y}$
 cubed root \rightarrow radicand
 can be positive or
 negative
 $\therefore \forall x \in \mathbb{R}, y \in \mathbb{R}$

c) $2\sqrt{3b} + \sqrt[3]{24b^2} + \sqrt{27b} - \sqrt{12b}$
 $b \geq 0$
 $= 2\sqrt{3b} + \sqrt[3]{8 \cdot 3b^2} + \sqrt{9 \cdot 3b} - \sqrt{4 \cdot 3b}$
 $= 2\sqrt{3b} + 2\sqrt[3]{3b^2} + 3\sqrt{3b} - 2\sqrt{3b}$
 $= 2\sqrt[3]{3b^2} + 3\sqrt{3b}$

$\left. \begin{aligned} &\sqrt[3]{8x^3x^2y} + 5\sqrt[3]{x^3x^2y} - x\sqrt[3]{27x^2y} \\ &= 2x\sqrt{x^2y} + 5x\sqrt[3]{x^2y} - x(3)\sqrt{x^2y} \\ &= 7x\sqrt{x^2y} - 3x\sqrt{x^2y} \\ &= 4x\sqrt{x^2y} \end{aligned} \right\}$

PC 11 2.3 con't... 2.

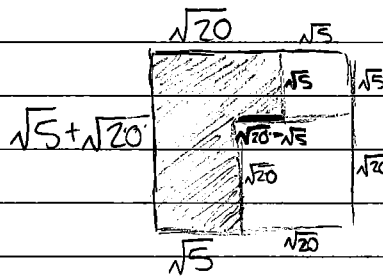
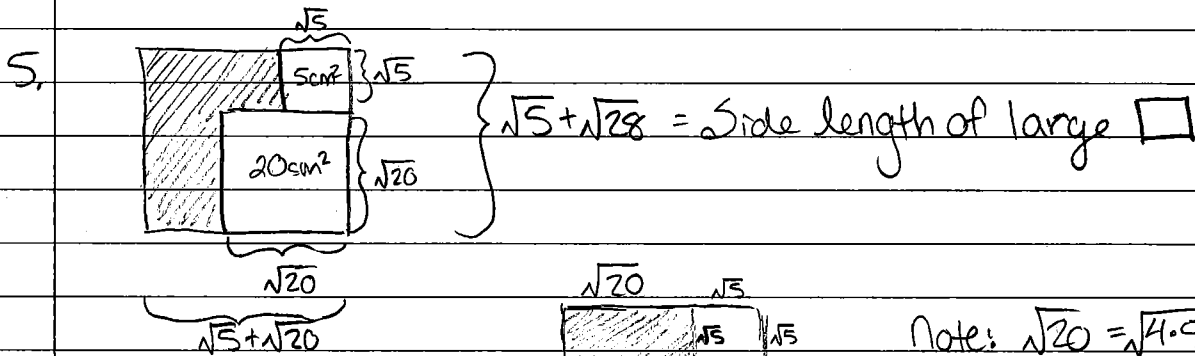
3. d) $5\sqrt{a^3} + 3\sqrt{2a} + \sqrt{4a^3}$ $a \geq 0$
 $= 5\sqrt{a^3} + 3\sqrt{2a} + 2\sqrt{a^3}$
 $= 7\sqrt{a^3} + 3\sqrt{2a}$



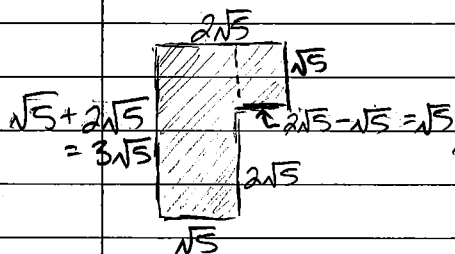
$$P = 2\sqrt{7} - 3\sqrt{2} + 3(2\sqrt{7}) + 3(3\sqrt{2})$$

$$= 2\sqrt{7} - 3\sqrt{2} + 6\sqrt{7} + 9\sqrt{2}$$

$$P = (8\sqrt{7} + 6\sqrt{2}) \text{ cm}$$



Note: $\sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}$



$$P = 3\sqrt{5} + 2\sqrt{5} + \sqrt{5} + \sqrt{5} + 2\sqrt{5} + \sqrt{5}$$

$$= 10\sqrt{5} \text{ cm}$$

Area = $\frac{3\sqrt{5}}{3\sqrt{5}} A + \frac{\sqrt{5}}{\sqrt{5}} \sqrt{5}$ $= 3\sqrt{5}(\sqrt{5}) + \sqrt{5}\sqrt{5}$
 $= 3(5) + 5$
 $= 15 + 5$
 $= 20 \text{ cm}^2$

Pre-Calculus II 2.4

$$1. a) \sqrt{3}(\sqrt{5}+4) \\ = \sqrt{15} + 4\sqrt{3}$$

$$b) \sqrt{6}(2-\sqrt{7}) \\ = 2\sqrt{6} - \sqrt{42}$$

$$c) \sqrt{7}(-3+\sqrt{2}) \\ = -3\sqrt{7} + \sqrt{14}$$

$$d) -\sqrt{11}(\sqrt{3}-7) \\ = -\sqrt{33} + 7\sqrt{11}$$

$$e) \sqrt{8}(\sqrt{2}+1) \\ = \sqrt{16} + \sqrt{8} \\ = 4 + \sqrt{4 \cdot 2} \\ = 4 + 2\sqrt{2}$$

$$f) -2\sqrt{3}(\sqrt{3}+\sqrt{6}) \\ = -2\sqrt{9} - 2\sqrt{18} \\ = -2(3) - 2\sqrt{9 \cdot 2} \\ = -6 - 2(3)\sqrt{2} \\ = -6 - 6\sqrt{2}$$

$$g) (\sqrt{2}-3)(\sqrt{2}+3) \\ = \sqrt{4} + 3\sqrt{2} - 3\sqrt{2} - 9 \\ = 2 + 0 - 9 \\ = -7$$

$$h) (\sqrt{3}-5)^2 \\ = (\sqrt{3}-5)(\sqrt{3}-5) \\ = \sqrt{9} - 5\sqrt{3} - 5\sqrt{3} + 25 \\ = 3 - 10\sqrt{3} + 25 \\ = 28 - 10\sqrt{3}$$

$$i) (\sqrt{7}+\sqrt{3})^2 \\ = (\sqrt{7}+\sqrt{3})(\sqrt{7}+\sqrt{3}) \\ = \sqrt{49} + \sqrt{21} + \sqrt{21} + \sqrt{9} \\ = 7 + 2\sqrt{21} + 3 \\ = 10 + 2\sqrt{21}$$

$$j) (2\sqrt{5}-\sqrt{3})(\sqrt{5}+3\sqrt{3}) \\ = 2(5) + 6\sqrt{15} - \sqrt{15} - 3(3) \\ = 10 + 5\sqrt{15} - 9 \\ = 1 + 5\sqrt{15}$$

$$k) (\sqrt{2}+3)(3\sqrt{2}-1) - \sqrt{2}(4\sqrt{2}-5) \\ = 3(2) - \sqrt{2} + 9\sqrt{2} - 3 - 4(2) + 5\sqrt{2} \\ = 6 + 8\sqrt{2} - 3 - 8 + 5\sqrt{2} \\ = -5 + 13\sqrt{2}$$

PC II 2.4 con't...2

$$\begin{aligned}
 1. \quad l.) & (3\sqrt{5} + 2\sqrt{3})^2 + (2\sqrt{5} - \sqrt{3})(2\sqrt{5} + \sqrt{3}) \\
 & = (3\sqrt{5} + 2\sqrt{3})(3\sqrt{5} + 2\sqrt{3}) + (4(5) + 2\sqrt{15} - 2\sqrt{15} - 3) \\
 & = 9(5) + 6\sqrt{15} + 6\sqrt{15} + 4(3) + 20 + 0 - 3 \\
 & = 45 + 12\sqrt{15} + 12 + 17 \\
 & = 74 + 12\sqrt{15}
 \end{aligned}$$

$$\begin{aligned}
 m.) & -(3\sqrt{3} - \sqrt{2})(\sqrt{3} + 4) - (2\sqrt{3} - \sqrt{2})^2 + 5 \\
 & = -(3(3) + 12\sqrt{3} - \sqrt{6} - 4\sqrt{2}) - (2\sqrt{3} - \sqrt{2})(2\sqrt{3} - \sqrt{2}) + 5 \\
 & = -(9 + 12\sqrt{3} - \sqrt{6} - 4\sqrt{2}) - (4(3) - 2\sqrt{6} - 2\sqrt{6} + 2) + 5 \\
 & = -9 - 12\sqrt{3} + \sqrt{6} + 4\sqrt{2} - 12 + 2\sqrt{6} + 2\sqrt{6} - 2 + 5 \\
 & = -18 - 12\sqrt{3} + 5\sqrt{6} + 4\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad a.) & \sqrt{a}(5\sqrt{a} - 3) \quad a \geq 0 & b.) & (2\sqrt{m} + 7)(3\sqrt{m} - 1) \quad m \geq 0 \\
 & = 5(a) - 3\sqrt{a} & & = 6m - 2\sqrt{m} + 21\sqrt{m} - 7 \\
 & = 5a - 3\sqrt{a} & & = 6m + 19\sqrt{m} - 7
 \end{aligned}$$

$$\begin{aligned}
 c.) & (\sqrt{x} - \sqrt{y})(\sqrt{x} + \sqrt{y}) \quad \begin{cases} x \geq 0 \\ y \geq 0 \end{cases} & d.) & (2\sqrt{a} - \sqrt{b})^2 \quad a \geq 0, b \geq 0 \\
 & = x + \sqrt{xy} - \sqrt{xy} - y & & = (2\sqrt{a} - \sqrt{b})(2\sqrt{a} - \sqrt{b}) \\
 & = x - y & & = 4a - 2\sqrt{ab} - 2\sqrt{ab} + b \\
 & & & = 4a - 4\sqrt{ab} + b
 \end{aligned}$$

$$\begin{aligned}
 e.) & (\sqrt{x} - 3\sqrt{y})(3\sqrt{x} - 5\sqrt{y}) - (2\sqrt{x} - \sqrt{y})^2 \quad x \geq 0, y \geq 0 \\
 & = 3x - 5\sqrt{xy} - 9\sqrt{xy} + 15y - (2\sqrt{x} - \sqrt{y})(2\sqrt{x} - \sqrt{y}) \\
 & = 3x - 14\sqrt{xy} + 15y - (4x - 2\sqrt{xy} - 2\sqrt{xy} + y) \\
 & = 3x - 14\sqrt{xy} + 15y - 4x + 2\sqrt{xy} + 2\sqrt{xy} - y \\
 & = -x - 10\sqrt{xy} + 14y
 \end{aligned}$$

$$\begin{aligned}
 3. \quad a.) & \frac{7\sqrt{3} + 2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} & b.) & \frac{2\sqrt{5} - 5\sqrt{3}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} \rightarrow = \frac{2\sqrt{30} - 5(3)\sqrt{2}}{6} \\
 & = \frac{7(3) + 2\sqrt{3}}{3} & & = \frac{2\sqrt{30} - 5\sqrt{18}}{6} = \frac{2\sqrt{30} - 15\sqrt{2}}{6} \\
 & = \frac{21 + 2\sqrt{3}}{3} & & = \frac{2\sqrt{30} - 5\sqrt{9 \cdot 2}}{6}
 \end{aligned}$$

PC II 2.4 cont. ... 3.

$$3. c.) \frac{-2\sqrt{7} + 3\sqrt{3}}{2\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}}$$

$$= \frac{-2\sqrt{70} + 3\sqrt{30}}{2(10)}$$

$$= \frac{-2\sqrt{70} + 3\sqrt{30}}{20}$$

$$d.) \frac{-2\sqrt{3} - 3\sqrt{2}}{5\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$$

$$= \frac{-2\sqrt{18} - 3\sqrt{12}}{5(6)}$$

$$= \frac{-2(9 \cdot 2) - 3(4 \cdot 3)}{30}$$

$$= \frac{-2(3)\sqrt{2} - 3(2)\sqrt{3}}{30}$$

$$= \frac{-6\sqrt{2} - 6\sqrt{3}}{30}$$

6 goes into each term

$$= \frac{-\sqrt{2} - \sqrt{3}}{5}$$

$$e.) \frac{-3}{5 + \sqrt{7}} \cdot \frac{(5 - \sqrt{7})}{(5 - \sqrt{7})}$$

$$= \frac{-15 + 3\sqrt{7}}{25 - 7}$$

$$= \frac{-15 + 3\sqrt{7}}{18} \quad \div 3$$

$$= \frac{-5 + \sqrt{7}}{6} \quad \div 3$$

$$f.) \frac{\sqrt{6}}{-\sqrt{8} - \sqrt{10}} \cdot \frac{(-\sqrt{8} + \sqrt{10})}{(-\sqrt{8} + \sqrt{10})}$$

$$= \frac{-\sqrt{48} + \sqrt{60}}{+8 - 10}$$

$$= \frac{-\sqrt{16 \cdot 3} + \sqrt{4 \cdot 15}}{-2}$$

$$= \frac{-4\sqrt{3} + 2\sqrt{15}}{-2} = 2\sqrt{3} - \sqrt{15}$$

$$g.) \frac{(7 + \sqrt{3})(1 + \sqrt{3})}{(1 - \sqrt{3})(1 + \sqrt{3})}$$

$$= \frac{7 + 7\sqrt{3} + \sqrt{3} + 3}{1 - 3}$$

$$= \frac{10 + 8\sqrt{3}}{-2}$$

$$= -5 - 4\sqrt{3}$$

$$h.) \frac{(\sqrt{2} + \sqrt{5})(-\sqrt{7} - \sqrt{2})}{(\sqrt{7} + \sqrt{2})(-\sqrt{7} - \sqrt{2})}$$

$$= \frac{-\sqrt{14} - 2 - \sqrt{35} - \sqrt{10}}{+7 - 2}$$

$$= \frac{-\sqrt{14} - 2 - \sqrt{35} - \sqrt{10}}{5}$$

$$\text{or} = \frac{\sqrt{14} + 2 + \sqrt{35} + \sqrt{10}}{-5}$$

$$i.) \frac{(-\sqrt{7} + 2\sqrt{6})(2\sqrt{3} - \sqrt{2})}{(2\sqrt{3} + \sqrt{2})(2\sqrt{3} - \sqrt{2})}$$

$$= \frac{-2\sqrt{21} + \sqrt{14} + 4\sqrt{18} - 2\sqrt{12}}{4(3) - 2}$$

$$= \frac{-2\sqrt{21} + \sqrt{14} + 4\sqrt{9 \cdot 2} - 2\sqrt{4 \cdot 3}}{12 - 2}$$

$$\rightarrow = \frac{-2\sqrt{21} + \sqrt{14} + 4(3)\sqrt{2} - 2(2)\sqrt{3}}{10}$$

$$= \frac{-2\sqrt{21} + \sqrt{14} + 12\sqrt{2} - 4\sqrt{3}}{10}$$

PC 11 2.4 con't...4

$$\begin{aligned}
 4. a) & 2 + \frac{1}{\sqrt{11}} \\
 &= 2 + \frac{1 \cdot \sqrt{11}}{\sqrt{11} \cdot \sqrt{11}} \\
 &= \frac{2}{1} + \frac{\sqrt{11}}{11} \\
 &= \frac{22}{11} + \frac{\sqrt{11}}{11} \\
 &= \frac{22 + \sqrt{11}}{11}
 \end{aligned}$$

$$\begin{aligned}
 b) & \frac{1}{\sqrt{3}} - \frac{2}{\sqrt{5}} \\
 &= \frac{1 \left(\frac{\sqrt{3}}{\sqrt{3}} \right) - 2 \left(\frac{\sqrt{5}}{\sqrt{5}} \right)}{\sqrt{3} \sqrt{3} \quad \sqrt{5} \sqrt{5}} \\
 &= \frac{\sqrt{3}}{3} - \frac{2\sqrt{5}}{5} \\
 &= \frac{\sqrt{3} \left(\frac{5}{5} \right) - 2\sqrt{5} \left(\frac{3}{3} \right)}{3 \left(\frac{5}{5} \right) \quad 5 \left(\frac{3}{3} \right)} \\
 &= \frac{5\sqrt{3} - 6\sqrt{5}}{15}
 \end{aligned}$$

Note: you can rationalize the denominators before finding the LCD or after

$$\begin{aligned}
 c) & \frac{3}{\sqrt{8}} + \frac{\sqrt{5}}{\sqrt{12}} \\
 &= \frac{3 \left(\frac{\sqrt{3}}{\sqrt{3}} \right) + \sqrt{5} \left(\frac{\sqrt{2}}{\sqrt{2}} \right)}{\sqrt{8} \sqrt{3} \quad \sqrt{12} \sqrt{2}} \\
 &= \frac{3\sqrt{3} + \sqrt{10}}{\sqrt{24}} \\
 &= \frac{(3\sqrt{3} + \sqrt{10}) \sqrt{24}}{\sqrt{24} \cdot \sqrt{24}} \\
 &= \frac{3\sqrt{72} + \sqrt{240}}{24} \\
 &= \frac{3\sqrt{36 \cdot 2} + \sqrt{16 \cdot 15}}{24} \\
 &= \frac{3(6\sqrt{2}) + 4\sqrt{15}}{24} \\
 &= \frac{18\sqrt{2} + 4\sqrt{15}}{24} \quad \div 2 \\
 &= \frac{9\sqrt{2} + 2\sqrt{15}}{12} \quad \div 2
 \end{aligned}$$

$$\begin{aligned}
 d) & \frac{5}{\sqrt{2}-\sqrt{3}} + \frac{1}{2} \\
 &= \frac{5 \left(\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}} \right) + \frac{1}{2}}{(\sqrt{2}-\sqrt{3})(\sqrt{2}+\sqrt{3})} \\
 &= \frac{5\sqrt{2} + 5\sqrt{3} + \frac{1}{2}}{2-3} \\
 &= \frac{5\sqrt{2} + 5\sqrt{3} + \frac{1}{2}}{-1} \\
 &= \frac{(-5\sqrt{2} - 5\sqrt{3}) \left(\frac{2}{2} \right) + \frac{1}{2}}{1} \\
 &= \frac{-10\sqrt{2} - 10\sqrt{3} + \frac{1}{2}}{2} \\
 &= \frac{-10\sqrt{2} - 10\sqrt{3} + 1}{2}
 \end{aligned}$$

PC11 2.4 con't...5

$$\begin{aligned} 4. e) & \frac{2}{\sqrt{5}+\sqrt{2}} + \frac{3}{\sqrt{5}-\sqrt{2}} \\ &= \frac{2(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})} + \frac{3(\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})} \\ &= \frac{2\sqrt{5}-2\sqrt{2}}{5-2} + \frac{3\sqrt{5}+3\sqrt{2}}{5-2} \\ &= \frac{5\sqrt{5}+\sqrt{2}}{3} \end{aligned}$$

$$\begin{aligned} f.) & \frac{\sqrt{2}}{\sqrt{3}+2\sqrt{6}} + \frac{4}{\sqrt{3}-2\sqrt{6}} \\ &= \frac{\sqrt{2}(\sqrt{3}-2\sqrt{6})}{(\sqrt{3}+2\sqrt{6})(\sqrt{3}-2\sqrt{6})} + \frac{4(\sqrt{3}+2\sqrt{6})}{(\sqrt{3}-2\sqrt{6})(\sqrt{3}+2\sqrt{6})} \\ &= \frac{\sqrt{6}-2\sqrt{12}}{3-4(6)} + \frac{4\sqrt{3}+8\sqrt{6}}{3-4(6)} \\ &= \frac{\sqrt{6}-2\sqrt{4 \cdot 3}}{3-24} + \frac{4\sqrt{3}+8\sqrt{6}}{3-24} \\ &= \frac{\sqrt{6}-2(2)\sqrt{3}+4\sqrt{3}+8\sqrt{6}}{-21} \\ &= \frac{9\sqrt{6}-4\sqrt{3}+4\sqrt{3}}{-21} \\ &= \frac{9\sqrt{6}}{-21} = -\frac{3\sqrt{6}}{7} \end{aligned}$$

$$\begin{aligned} 5. a.) & \frac{2}{\sqrt[3]{5}} \left(\frac{\sqrt[3]{5}}{\sqrt[3]{5}} \right) \left(\frac{\sqrt[3]{5}}{\sqrt[3]{5}} \right) \\ &= \frac{2\sqrt[3]{25}}{\sqrt[3]{5^3}} \\ &= \frac{2\sqrt[3]{25}}{5} \end{aligned}$$

$$\begin{aligned} b.) & \frac{21}{\sqrt[4]{3}} \left(\frac{\sqrt[4]{3^3}}{\sqrt[4]{3^3}} \right) \\ &= \frac{21\sqrt[4]{3^3}}{\sqrt[4]{3^4}} \\ &= \frac{21\sqrt[4]{3^3}}{3} \\ &= 7\sqrt[4]{3^3} \\ &= 7\sqrt[4]{27} \end{aligned}$$

$$\begin{aligned} c.) & \frac{10+\sqrt[3]{4}}{\sqrt[3]{2}} \left(\frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}} \right) \\ &= \frac{10\sqrt[3]{2^2} + \sqrt[3]{4 \cdot 2^2}}{\sqrt[3]{2^3}} \\ &= \frac{10\sqrt[3]{4} + \sqrt[3]{16}}{2} \\ &= \frac{10\sqrt[3]{4} + \sqrt[3]{8 \cdot 2}}{2} \\ &= \frac{10\sqrt[3]{4} + 2\sqrt[3]{2}}{2} = 5\sqrt[3]{4} + \sqrt[3]{2} \end{aligned}$$

Pre-Calculus II 2.5

1. a) $\sqrt{2x} = 8$

$$(\sqrt{2x})^2 = (8)^2$$

$$\frac{2x}{2} = \frac{64}{2}$$

$$x = 32$$

b) $\sqrt{-x} + 5 = 7$

$$\begin{array}{r} -5 \\ -5 \end{array}$$

$$\sqrt{-x} = 2$$

$$(\sqrt{-x})^2 = (2)^2$$

$$-x = 4$$

$$x = -4$$

c) $\frac{2\sqrt{3x}}{2} = \frac{12}{2}$

$$(\sqrt{3x})^2 = (6)^2$$

$$\frac{3x}{3} = \frac{36}{3}$$

$$x = 12$$

d) $\frac{30}{5} = \frac{5\sqrt{2x}}{5}$

$$6^2 = (\sqrt{2x})^2$$

$$\frac{36}{2} = \frac{2x}{2}$$

$$18 = x$$

e) $7 = 2\sqrt{3x} - 1$

$$\begin{array}{r} +1 \\ +1 \end{array}$$

$$\frac{8}{2} = \frac{2\sqrt{3x}}{2}$$

$$4^2 = (\sqrt{3x})^2$$

$$\frac{16}{3} = \frac{3x}{3}$$

$$\frac{16}{3} = x$$

$$x = \frac{16}{3}$$

$$x = \frac{16}{3}$$

$$x = \frac{16}{3}$$

f) $\sqrt{2x-1} = 3$

$$(\sqrt{2x-1})^2 = 3^2$$

$$2x-1 = 9$$

$$\begin{array}{r} +1 \\ +1 \end{array}$$

$$\frac{2x}{2} = \frac{10}{2}$$

$$x = 5$$

$$x = 5$$

g) $\sqrt{x+3} = 5$

$$(\sqrt{x+3})^2 = 5^2$$

$$x+3 = 25$$

$$\begin{array}{r} -3 \\ -3 \end{array}$$

$$x = 22$$

h) $\sqrt{3-x} + 1 = 4$

$$\begin{array}{r} -1 \\ -1 \end{array}$$

$$\sqrt{3-x} = 3$$

$$(\sqrt{3-x})^2 = 3^2$$

$$3-x = 9$$

$$\begin{array}{r} -3 \\ -3 \end{array}$$

$$-x = 6$$

$$x = -6$$

PC II 2.5 continue 2

$$i.) \begin{array}{r} 2\sqrt{3x} + 5 = 6 \\ -5 \quad -5 \end{array}$$

$$\frac{2\sqrt{3x}}{2} = \frac{1}{2}$$

$$(\sqrt{3x})^2 = \left(\frac{1}{2}\right)^2$$

$$3x = \frac{1}{4}$$

$$x = \frac{1}{4} \div 3$$

$$x = \frac{1}{4} \cdot \frac{1}{3}$$

$$x = \frac{1}{12}$$

$$j.) \begin{array}{r} 2\sqrt{x} + 5 = 3 \\ -5 \quad -5 \end{array}$$

$$\frac{2\sqrt{x}}{2} = \frac{-2}{2}$$

$$\sqrt{x} = -1$$

does not

exist.

→ won't get a negative

$$k.) \begin{array}{r} \sqrt{3x+4} - 5 = -1 \\ +5 \quad +5 \end{array}$$

$$\sqrt{3x+4}^2 = 4^2$$

$$3x+4 = 16$$

$$-4 \quad -4$$

$$3x = 12$$

$$\frac{3x}{3} = \frac{12}{3}$$

$$x = 4$$

$$l.) \begin{array}{r} -4 = -\sqrt{-x+2} \\ -1 \quad -1 \end{array}$$

$$4^2 = (\sqrt{-x+2})^2$$

$$16 = -x+2$$

$$-2 \quad -2$$

$$14 = -x$$

$$\frac{-1}{-1} \quad \frac{-1}{-1}$$

$$x = 14$$

$$m.) \begin{array}{r} 3\sqrt{x+1} = 1 \\ 3 \quad 3 \end{array}$$

$$(\sqrt{x+1})^2 = \left(\frac{1}{3}\right)^2$$

$$x+1 = \frac{1}{9}$$

$$-1 \quad -1$$

$$x = \frac{1}{9} - \frac{9}{9}$$

$$x = -\frac{8}{9}$$

$$n.) \begin{array}{r} 2\sqrt{1-x} = 3 \\ 2 \quad 2 \end{array}$$

$$(\sqrt{1-x})^2 = \left(\frac{3}{2}\right)^2$$

$$1-x = \frac{9}{4}$$

$$-1 \quad -1$$

$$-x = \frac{9}{4} - \frac{4}{4}$$

$$-x = \frac{5}{4}$$

$$x = -\frac{5}{4}$$

$$o.) \begin{array}{r} 5 = 2\sqrt{x} + 9 \\ -9 \quad -9 \end{array}$$

$$\frac{-4}{2} = \frac{2\sqrt{x}}{2}$$

$$-2 = \sqrt{x}$$

does not exist.

$$p.) \begin{array}{r} 2\sqrt{3x-4} + 5 = 6 \\ -5 \quad -5 \end{array}$$

$$\frac{2\sqrt{3x-4}}{2} = \frac{1}{2}$$

$$\sqrt{3x-4}^2 = \left(\frac{1}{2}\right)^2$$

$$3x-4 = \frac{1}{4}$$

$$+4 \quad +4$$

$$3x = \frac{1}{4} + \frac{16}{4}$$

$$x = \frac{17}{4} \div 3$$

$$x = \frac{17}{4} \cdot \frac{1}{3}$$

$$x = \frac{17}{12}$$

$$q.) \begin{array}{r} 1 - 3\sqrt{2-x} = -3 \\ -1 \quad -1 \end{array}$$

$$\frac{-3\sqrt{2-x}}{-3} = \frac{-4}{-3}$$

$$\sqrt{2-x}^2 = \left(\frac{4}{3}\right)^2$$

$$2-x = \frac{16}{9}$$

$$-2 \quad -2$$

$$-x = \frac{16}{9} - \frac{18}{9}$$

$$-x = -\frac{2}{9}$$

$$x = \frac{2}{9}$$

PC11 2.5 con't...3

$$1. a) \frac{\sqrt{3x}}{2} + 1 = 5 \quad \rightarrow \quad \sqrt{3x} = 4.2$$

$$\frac{\sqrt{3x}}{2} = 4 \quad \rightarrow \quad (\sqrt{3x})^2 = 8^2$$

$$3x = 64$$

$$x = \frac{64}{3}$$

$$2. a) \sqrt{\frac{1}{2}x} = \sqrt{x-3}$$

$$\frac{1}{2}x = x-3$$

$$\frac{1}{2}x - x = -3$$

$$\frac{1}{2}x - \frac{2}{2}x = -3$$

$$-\frac{1}{2}x = -3$$

$$x = -3 \div -\frac{1}{2}$$

$$x = -\frac{3}{1} \times -\frac{2}{1}$$

$$x = 6$$

$$b) \sqrt{5-x} = \sqrt{x+2}$$

$$5-x = x+2$$

$$5-2 = x+x$$

$$3 = 2x$$

$$\frac{3}{2} = x$$

$$c) 2\sqrt{x-2} = \sqrt{3x+5}$$

$$(2\sqrt{x-2})^2 = (\sqrt{3x+5})^2$$

$$4(x-2) = 3x+5$$

$$4x-8 = 3x+5$$

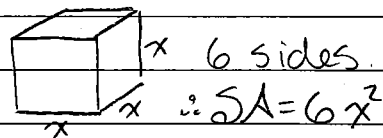
$$4x-3x = 5+8$$

$$x = 13$$

$$3. a) \text{Cub: } SA = 4\pi(695800)^2$$

$$\text{Cube: } 6x^2 = 4\pi(695800)^2$$

$$x^2 = \frac{4\pi(695800)^2}{6}$$



$$x = \sqrt{\frac{4\pi(695800)^2}{6}}$$

$$x = 1006963.506$$

Edge Length
= 1 006 963.506 km

PC II 2.5 con't...5

$$4. j.) 1 + \sqrt[5]{x+7} = 3$$

$$\begin{array}{r} -1 \qquad -1 \\ \hline \end{array}$$

$$\sqrt[5]{x+7} = 2$$

$$(\sqrt[5]{x+7})^5 = 2^5$$

$$x+7 = 32$$

$$\begin{array}{r} -7 \quad -7 \\ \hline \end{array}$$

$$x = 25$$

$$5. a.) \sqrt{x} + 1 = \sqrt{x+2}$$

$$(\sqrt{x} + 1)^2 = (\sqrt{x+2})^2$$

$$(\sqrt{x} + 1)(\sqrt{x} + 1) = x+2$$

$$x + \sqrt{x} + \sqrt{x} + 1 = x+2$$

$$\begin{array}{r} -x \qquad -1 \quad -x-1 \\ \hline \end{array}$$

$$2\sqrt{x} = 1$$

$$\sqrt{x} = \frac{1}{2}$$

$$(\sqrt{x})^2 = \left(\frac{1}{2}\right)^2$$

$$x = \frac{1}{4}$$

$$c.) 2\sqrt[4]{x} = \sqrt[4]{x+3}$$

$$(2\sqrt[4]{x})^4 = (\sqrt[4]{x+3})^4$$

$$2^4 \cdot x = x+3$$

$$16x = x+3$$

$$\begin{array}{r} -x \quad -x \\ \hline \end{array}$$

$$15x = 3$$

$$\frac{15x}{15} = \frac{3}{15}$$

$$x = \frac{1}{5}$$

$$b.) 3 - \sqrt{x-8} = \sqrt{x-5}$$

$$(3 - \sqrt{x-8})^2 = (\sqrt{x-5})^2$$

$$(3 - \sqrt{x-8})(3 - \sqrt{x-8}) = x-5$$

$$9 - 3\sqrt{x-8} - 3\sqrt{x-8} + (x-8) = x-5$$

$$9 - 6\sqrt{x-8} + x - 8 = x-5$$

$$1 - 6\sqrt{x-8} + x = x-5$$

$$\begin{array}{r} -1 \qquad -x \quad -x \quad -1 \\ \hline \end{array}$$

$$-6\sqrt{x-8} = -6$$

$$\begin{array}{r} -6 \qquad -6 \\ \hline \end{array}$$

$$\sqrt{x-8} = 1$$

$$(\sqrt{x-8})^2 = 1^2$$

$$x-8 = 1$$

$$\begin{array}{r} +8 \quad +8 \\ \hline \end{array}$$

$$x = 9$$

$$d.) \sqrt[4]{x+1} = \sqrt{3}$$

$$(\sqrt[4]{x+1})^4 = (\sqrt{3})^4$$

$$x+1 = \sqrt{81}$$

$$x+1 = 9$$

$$\begin{array}{r} -1 \quad -1 \\ \hline \end{array}$$

$$x = 8$$