Pre-Calculus 11 1.1

1\. a) $d = 8 - 5$
$= 3$

b) $d = -10 - -3$
$= -10 + 3 = -7$

c) $d = -2.3 - -4$
$= -2.3 + 4 = 1.7$

d) $d = -1.8 - 2$
$= -3.8$

e) $d = 1.7 - 1.7$
$= 0$

f) $d = 6x - 2x$
$= 4x$

2\. a) $t_1 = 3$, $d = 4$, $n = 7$, $t_7 = x$

$t_n = t_1 + (n-1)d$
$x = 3 + (7-1)(4)$
$= 3 + (6)(4)$
$= 3 + 24$
$= 27$

b) $t_1 = -2$, $d = 2.3$, $n = 9$, $t_9 = x$

$t_n = t_1 + (n-1)d$
$x = -2 + (9-1)(2.3)$
$= -2 + (8)(2.3)$
$= -2 + 18.4$
$= 16.4$

c) $t_1 = -3$, $d = -7$, $t_5 = x$, $n = 5$

$t_n = t_1 + (n-1)d$
$x = -3 + (5-1)(-7)$
$= -3 + (4)(-7)$
$= -3 - 28$
$= -31$

d) $t_1 = 10.1$, $d = -3$, $t_x = -13.9$, $n = x$

$t_n = t_1 + (n-1)d$
$-13.9 = 10.1 + (x-1)(-3)$
$-13.9 - 10.1 = (x-1)(-3)$
$\frac{-24}{-3} = \frac{(x-1)(-3)}{-3}$
$8 = x + 1$
$x = 7$

e) $t_1 = -5$, $d = 6$, $t_x = 115$, $n = x$

$t_n = t_1 + (n-1)d$
$115 = -5 + (x-1)(6)$
$115 + 5 = 6x - 6$ $\quad {+6} \quad {+6}$
$120 + 6 = 6x$
$126 = 6x$
$21 = x$

f) $t_1 = -7$, $d = -12$, $t_x = -127$, $n = x$

$t_n = t_1 + (n-1)d$
$-127 = -7 + (x-1)(-12)$
$-127 + 7 = -12x + 12$ $\quad {-12} \quad {-12}$
$-120 - 12 = -12x$
$-132 = -12x$
$11 = x$

g) $t_1 = a$, $d = 2$, $t_8 = x$, $n = 8$

$t_n = t_1 + (n-1)d$
$x = a + (8-1)(2)$
$x = a + 7(2)$
$x = a + 14$

h) $t_1 = b$, $d = 3n$, $t_{10} = x$, $n = 10$

$t_n = t_1 + (n-1)d$
$x = b + (10-1)(3n)$
$x = b + 9(3n)$
$x = b + 27n$

PC11 1.1 con't...2

2. i.) $t_1 = 3$, $d = x$, $t_7 = 33$, $n = 7$

$t_n = t_1 + (n-1)d$
$33 = 3 + (7-1)x$
$33 - 3 = 6x$
$\frac{30}{6} = \frac{6x}{6}$
$5 = x$

j.) $t_1 = x$, $d = -11$, $t_{12} = -98$, $n = 12$

$t_n = t_1 + (n-1)d$
$-98 = x + (12-1)(-11)$
$-98 = x + (11)(-11)$
$-98 = x - 121$
$+121 \quad +121$
$23 = x$

k.) $t_1 = -13$, $d = x$, $t_9 = -69$, $n = 9$

$t_n = t_1 + (n-1)d$
$-69 = -13 + (9-1)(x)$
$-69 + 13 = 8x$
$\frac{-56}{8} = \frac{8x}{8}$
$-7 = x$

l.) $t_1 = x$, $d = -3.25$, $t_{17} = -25$, $n = 17$

$t_n = t_1 + (n-1)d$
$-25 = x + (17-1)(-3.25)$
$-25 = x + 16(-3.25)$
$-25 = x - 52$
$+52 \quad +52$
$27 = x$

m.) $t_1 = -19$, $d = x$, $t_{17} = 0.52$, $n = 17$

$t_n = t_1 + (n-1)d$
$0.52 = -19 + (17-1)(x)$
$0.52 + 19 = 16x$
$\frac{19.52}{16} = \frac{16x}{16}$
$1.22 = x$

n.) $t_1 = n$, $d = x$, $t_7 = -11n$, $n = 7$

$t_n = t_1 + (n-1)d$
$-11n = n + (7-1)x$
$-11n - n = 6x$
$\frac{-12n}{6} = \frac{6x}{6}$
$-2n = x$

o.) $t_1 = 3m$, $d = x$, $t_{13} = -12.36m$, $n = 13$

$t_n = t_1 + (n-1)d$
$-12.36m = 3m + (13-1)(x)$
$-12.36m - 3m = 12x$
$\frac{-15.36m}{12} = \frac{12x}{12}$
$-1.28m = x$

PC11 1.1 con't...3.

3. $t_1 = 7 \quad t_4 = 16, \quad d = ?$

$\underset{t_1}{7} \xrightarrow{+d} \underset{t_2}{\_\_} \xrightarrow{+d} \underset{t_3}{\_\_} \xrightarrow{+d} \underset{t_4}{16}$

$7 + 3d = 16$

$-7 \qquad -7$

$3d = 9$

$d = 3$

Method 2: $n = 4$

$t_n = t_1 + (n-1)d$

$16 = 7 + (4-1)d$

$16 - 7 = 3d$

$\frac{9}{3} = \frac{3d}{3}$

$3 = d$

4. $t_3 = -6.5, \quad t_5 = -3.9, \quad d = ?$

$\_\_, \_\_, \underset{t_3}{-6.5} \xrightarrow{+d} \_\_ \xrightarrow{+d} \underset{t_5}{-3.9}, \_\_$

$-6.5 + 2d = -3.9$

$+6.5 \qquad +6.5$

$\frac{2d}{2} = \frac{2.6}{2}$

$d = 1.3$

5. $t_3 = 8, \ t_6 = 29, \ t_1 = ?$

$\underset{t_1}{\_\_} \xleftarrow{-d} \_\_ \xleftarrow{-d} \underset{t_3}{8} \xrightarrow{+d} \_\_ \xrightarrow{+d} \_\_ \xrightarrow{+d} \underset{t_6}{29}$

$8 + 3d = 29$

$-8 \qquad -8$

$3d = 21$

$d = 7$

$8 - 2d = t_1$

$8 - 2(7) = t_1$

$8 - 14 = t_1$

$-6 = t_1$

PC 11 con't... 4

6. $t_4 = -11.98, \quad t_8 = 1.1, \quad t_2 = ?$

$-d \quad -d \quad +d \quad +d \quad +d \quad +d$

$\underline{\qquad}_{t_1}, \underline{\qquad}_{t_2}, \underline{\qquad}_{t_3}, \underline{-11.98}_{t_4}, \underline{\qquad}_{t_5}, \underline{\qquad}_{t_6}, \underline{\qquad}_{t_7}, \underline{1.1}_{t_8}$

$$-11.98 + 4d = 1.1$$
$$+11.98 \qquad + 11.98$$
$$\frac{4d}{4} = \frac{13.08}{4}$$
$$d = 3.27$$

$$-11.98 - 2d = t_2$$
$$-11.98 - 2(3.27) = t_2$$
$$-11.98 - 6.54 = t_2$$
$$-18.52 = t_2$$

7. 126 students | +4 students/car | +17 cars

$t_1 = 126 \qquad d = 4 \qquad n = 18$

$$t_n = t_1 + (n-1)d$$
$$t_{17} = 126 + (18-1)(4)$$
$$= 126 + 17(4)$$
$$= 126 + 68$$
$$= 194$$

If we start with 126 students (t_1) and add 17 cars we finish at t_{18}

$t_n = t_1 + (n-1)d$ $\quad S_n = \frac{n(t_1+t_n)}{2}$

Pre-Calculus 11 1.2 $\quad S_n = \frac{n(2t_1+d(n-1))}{2}$

1. a) $5+8+11+14+17+20 = 75$ b) $-3-10-17-24-31-38 = -123$

2. a) $t_1=3$, $t_n=23$, $d=2$, $n=?$

$t_n = t_1+(n-1)d$
$23 = 3+(n-1)(2)$
$-3 \quad -3$
$\frac{20}{2} = \frac{(n-1)(2)}{2}$
$10 = n-1$
$11 = n$

$S_n = \frac{n(t_1+t_n)}{2}$
$S_{11} = \frac{11(3+23)}{2}$
$= \frac{11(26)}{2}$
$= 143$

b) $t_1=5$, $t_n=-35$, $d=-4$, $n=?$

$t_n = t_1+(n-1)d$
$-35 = 5+(n-1)(-4)$
$-5 \quad -5$
$\frac{-40}{-4} = \frac{(n-1)(-4)}{-4}$
$10 = n-1$
$n = 11$

$S_n = \frac{n(t_1+t_n)}{2}$
$S_{11} = \frac{11(5+-35)}{2}$
$= \frac{11(-30)}{2}$
$= -165$

c) $t_1=-4$, $t_n=13$, $d=-2.3--4$, $n=-2.3+4$, $=1.7$, $n=?$

$t_n = t_1+(n-1)d$
$13 = -4+(n-1)(1.7)$
$+4 \quad +4$
$\frac{17}{1.7} = \frac{(n-1)(1.7)}{1.7}$
$10 = n-1$
$n = 11$

$S_n = \frac{n(t_1+t_n)}{2}$
$S_{11} = \frac{11(-4+13)}{2}$
$= \frac{11(9)}{2}$
$= 49.5$

d) $t_1=2$, $t_n=-43.6$, $d=-1.8-2$, $=-3.8$, $n=?$

$t_n = t_1+(n-1)d$
$-43.6 = 2+(n-1)(-3.8)$
$-2 \quad -2$
$\frac{-45.6}{-3.8} = \frac{(n-1)(-3.8)}{-3.8}$
$12 = n-1$
$13 = n$

$S_n = \frac{n(t_1+t_n)}{2}$
$S_{13} = \frac{13(2+-43.6)}{2}$
$= \frac{13(-41.6)}{2}$
$= -270.4$

PC11 1.2 con't...2

2. e) $t_1 = 1.4$
$t_n = 0.1$
$d = 1.3 - 1.4$
$= -0.1$
$n = ?$

$t_n = t_1 + (n-1)d$
$0.1 = 1.4 + (n-1)(-0.1)$
$-1.4 \quad -1.4$
$\frac{-1.3}{-0.1} = \frac{(n-1)(-0.1)}{-0.1}$
$13 = n - 1$
$14 = n$

$S_n = \frac{n(t_1 + t_n)}{2}$
$S_{14} = \frac{14(1.4 + 0.1)}{2}$
$= \frac{14(1.5)}{2}$
$= 10.5$

f.) $t_1 = x$
$t_n = 21x$
$d = 6x - x$
$= 5x$
$n = ?$

$t_n = t_1 + (n-1)d$
$21x = x + (n-1)(5x)$
$-x \quad -x$
$\frac{20x}{5x} = \frac{(n-1)(5x)}{5x}$
$4 = n - 1$
$5 = n$

$S_n = \frac{n(t_1 + t_n)}{2}$
$S_5 = \frac{5(x + 21x)}{2}$
$= \frac{5(22x)}{2}$
$= 55x$

3. a.) $t_1 = 5$
$d = 3$
$S_{10} = x$
$n = 10$

$S_n = \frac{n(2t_1 + d(n-1))}{2}$
$S_{10} = \frac{10(2(5) + 3(10-1))}{2}$
$= \frac{10(10 + 3(9))}{2}$
$= 5(10 + 27)$
$= 5(37)$
$= 185$

b.) $t_1 = -4$
$d = 1.3$
$S_8 = x$
$n = 8$

$S_n = \frac{n(2t_1 + d(n-1))}{2}$
$S_8 = \frac{8(2(-4) + 1.3(8-1))}{2}$
$= \frac{8(-8 + 1.3(7))}{2}$
$= 4(-8 + 9.1)$
$= 4(1.1)$
$= 4.4$

PC11 1.2 con't...3

3\. c.) $t_1=-2$, $d=-6$, $S_7=x$, $n=7$

$S_n = \frac{n(2t_1+d(n-1))}{2}$

$S_7 = \frac{7(2(-2)+(-6)(7-1))}{2}$

$= \frac{7(-4+-6(6))}{2}$

$=3.5(-4+-36)$

$=3.5(-40)$

$x=-140$

d.) $t_1=a$, $d=3$, $S_8=x$, $n=8$

$S_n = \frac{n(2t_1+d(n-1))}{2}$

$x = \frac{8(2a+3(8-1))}{2}$

$= \frac{8(2a+3(7))}{2}$

$=4(2a+21)$

$x=8a+84.$

e.) $t_1=b$, $d=5b$, $S_5=x$, $n=5$

$S_n = \frac{n(2t_1+d(n-1))}{2}$

$x = \frac{5(2(b)+5b(5-1))}{2}$

$= \frac{5(2b+25b-5b)}{2}$

$=\frac{5}{2}(22b)$

$x=55b$

f.) $t_2=5$, $d=2$, $S_7=x$, $n=7$

$S_n = \frac{n(2t_1+d(n-1))}{2}$

$x = \frac{7(2(3)+2(7-1))}{2}$

$x = \frac{7(6+2(6))}{2}$

$=\frac{7}{2}(6+12)$

$=\frac{7}{2}(18)$

$=63$

g.) $t_1=x$, $d=-3$, $S_{12}=-138$, $n=12$

$S_n = \frac{n(2t_1+d(n-1))}{2}$

$-138 = \frac{12(2(x)+(-3)(12-1))}{2}$

$-138\cdot 2 = 12(2x-3(11))$

$\frac{-276}{-12} = \frac{12(2x-33)}{12}$

$-23 = 2x-33$

$+33 \qquad +33$

$10=2x$

$5=x$

PC11 1.2 con't...4

3. h.) $t_1 = x$

$d = 3$

$S_{13} = 78$

$n = 13$

$$S_n = \frac{n(2t_1 + d(n-1))}{2}$$

$$78 = \frac{13(2(x) + 3(13-1))}{2}$$

$$78 \cdot 2 = 13(2x + 3(12))$$

$$\frac{156}{13} = \frac{13(2x+36)}{13}$$

$$12 = 2x + 36$$

$$-36 \qquad -36$$

$$\frac{-24}{2} = \frac{2x}{2}$$

$$x = -12$$

i.) $t_1 = 1.1$

$d = x$

$S_{17} = 73.1$

$n = 17$

$$S_n = \frac{n(2t_1 + d(n-1))}{2}$$

$$73.1 = \frac{17(2(1.1) + x(17-1))}{2}$$

$$73.1(2) = 17(2.2 + x(16))$$

$$\frac{146.2}{17} = \frac{17(2.2 + 16x)}{17}$$

$$8.6 = 2.2 + 16x$$

$$-2.2 \qquad -2.2$$

$$\frac{6.4}{16} = \frac{16x}{16}$$

$$0.4 = x$$

PC 11 1.2 con't...4

4. $t_1 = 13.7$ $\quad S_n = \frac{n(t_1+t_2)}{2}$ $\quad S_3 = \frac{3(26)}{2}$

$t_3 = 12.3$

$S_3 = ?$ $\quad S_3 = \frac{3(13.7+12.3)}{2}$ $\quad S_3 = 39$

$n = 3$

5. $t_1 = 4$ $\quad t_n = t_1 + (n-1)d$ $\quad S_{10} = ?$

$t_3 = 8$ $\quad 8 = 4 + (3-1)d$ $\quad S_n = \frac{n(2t_1 + d(n-1))}{2}$

$d = ?$ $\quad -4 \quad -4$

$4 = 2d$ $\quad S_{10} = \frac{10(2(4)+2(10-1))}{2}$

$2 = d$

$= \frac{10}{2}(8+2(9))$ $\quad = 5(26)$

$= 5(8+18)$ $\quad = 130$

6. $t_2 = -12.5$ $\quad \overset{+d}{\frown}\ \overset{+d}{\frown}$ $\quad S_{15} = ?$

$t_4 = -10.7$ $\quad \underline{\quad}_{t_1}, \underline{-12.5}_{t_2}, \underline{\quad}_{t_3}, \underline{-10.7}_{t_4}$ $\quad S_n = \frac{n(2t_1 + d(n-1))}{2}$

$d = ?$

$-12.5 + 2d = -10.7$ $\quad S_{15} = \frac{15(2(-13.4)+0.9(15-1))}{2}$

$+12.5 \quad +12.5$

$2d = 1.8$ $\quad = \frac{15}{2}(-26.8+0.9(14))$

$d = 0.9$ $\quad = \frac{15}{2}(-14.2)$

$t_1 = -12.5 - 0.9 = -13.4$ $\quad S_{15} = -106.5$

7. First Second Third Thirteenth

\$1 + \$3 + \$5 + ... + ? = Sum?

$t_1 = 1$ $\quad S_n = \frac{n(2t_1 + d(n-1))}{2}$

$d = 2$

$n = 13$ $\quad S_{13} = \frac{13(2(1)+2(13-1))}{2}$

$S_{13} = ?$

$= \frac{13(2+2(12))}{2}$

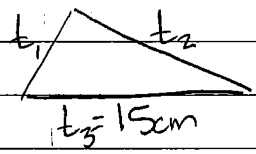
$= \frac{13}{2}(2+24)$

$= \frac{13}{2}(26) = 169$

Helmut would have \$ 169 in total on his thirteeth birthday.

PC11 1.2 con't...5

8.



Perimeter = 33

$\therefore S_3 = 33$

$n = 3$

$t_3 = 15$

$t_1 = ?$

$$S_n = \frac{n(t_1 + t_n)}{2}$$

$$33 = \frac{3(t_1 + 15)}{2}$$

$$33 \cdot 2 = 3(t_1 + 15)$$

$$\frac{66}{3} = \frac{3(t_1 + 15)}{3}$$

$$22 = t_1 + 15$$

$$-15 \qquad -15$$

$$7 = t_1$$

$$\underset{t_1}{\underline{7}} \xrightarrow{+d} + \underset{t_2}{\underline{?}} \xrightarrow{+d} + \underset{t_3}{\underline{15}}$$

$$7 + 2d = 15$$

$$-7 \qquad -7$$

$$\frac{2d}{2} = \frac{8}{2}$$

$$d = 4$$

$t_2 = 7 + 4 = 11$

$\therefore$ The lengths of the other sides are 7cm and 11cm.

9. $S_6 = -15$

$S_7 = -28$

$d = -3$

$S_4 = ?$

$$\underbrace{\underbrace{\underset{t_1}{\_\_} + \underset{t_2}{\_\_} + \underset{t_3}{\_\_} + \underset{t_4}{\_\_} + \underset{t_5}{\_\_} + \underset{t_6}{\_\_}}_{\text{Sum of 6 terms, } S_6 = -15} + \underset{t_7}{\_\_}}_{\text{Add one more term, } t_7 \text{ and now Sum of 7 terms, } S_7 = -28}$$

$$\therefore S_6 + t_7 = S_7$$

$$-15 + t_7 = -28$$

$$+15 \qquad +15$$

$$t_7 = -13$$

Find t_1:

$$t_n = t_1 + (n-1)d$$

$$-13 = t_1 + (7-1)(-3)$$

$$-13 = t_1 + 6(-3)$$

$$-13 = t_1 - 18 \Rightarrow t_1 = 5$$

$$+18 \qquad +18$$

$$S_n = \frac{n(2t_1 + (n-1)d)}{2}$$

$$S_4 = \frac{4(2(5) + (4-1)(-3))}{2}$$

$$= \frac{4}{2}(10 + 3(-3))$$

$$= 2(10 - 9)$$

$$= 2(1)$$

$$= 2.$$

PC11 1.2 con't...6

Ch.3 Fun

10. a.) $t_1 = 7$
$d = 2$
$S_x = 216$
$n = x$?

$S_n = \frac{n(2t_1 + d(n-1))}{2}$

$216 = \frac{x(2(7) + 2(x-1))}{2}$

$216 \cdot 2 = x(14 + 2x - 2)$

$432 = 14x + 2x^2 - 2x$

$-432 \quad -432$

$0 = 2x^2 + 12x - 432$

Factor

$0 = 2(x^2 + 6x - 216)$

$0 = 2(x+18)(x-12)$

$0 = x + 18$ or $0 = x - 12$

$x = -18$ or $x = 12$

can't have a negative number of terms.

b.) $t_1 = -10$
$d = 7$
$S_x = 45$
$n = x$?

$S_n = \frac{n(2t_1 + d(n-1))}{2}$

$45 = \frac{x(2(-10) + 7(x-1))}{2}$

$45 \cdot 2 = x(-20 + 7x - 7)$

$90 = -20x + 7x^2 - 7x$

$-90 \qquad -90$

$0 = 7x^2 - 27x - 90$

Factor

$0 = 7x^2 - 42x + 15x - 90$

$0 = 7x(x-6) + 15(x-6)$

$0 = (x-6)(7x+15)$

$x - 6 = 0 \qquad 7x + 15 = 0$

$x = 6 \qquad 7x = -15$

$x = -\frac{15}{7}$

can't have $-\frac{15}{7}$ number of terms.

c.) $t_1 = 36.1$
$d = -4$
$S_x = -38$
$n = x$?

$S_n = \frac{n(2t_1 + d(n-1))}{2}$

$-38 = \frac{x(2(36.1) + -4(x-1))}{2}$

$-38 \times 2 = x(72.2 - 4x + 4)$

$-76 = 72.2x - 4x^2 + 4x$

$4x^2 - 76.2x - 76 = 0$

$40x^2 - 762x - 760 = 0$

$20x^2 - 381x - 380 = 0$

Quadratic Formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-381) \pm \sqrt{(-381)^2 - 4(20)(-380)}}{2(20)}$

$= \frac{381 \pm 419}{40}$

$x = \frac{381 + 419}{40} \qquad x = \frac{381 - 419}{40}$

$x = 20 \qquad x = -38$

can't be negative

Pre-Calculus 11 1.3

1. a) $r = \frac{-8}{2}$

$r = -4$

b) $r = \frac{4.8}{4}$

$r = 1.2$

c) $r = \frac{27}{-81}$

$r = -\frac{1}{3}$

d) $r = \frac{96}{64}$

$r = \frac{3}{2}$

e) $r = \frac{-2\sqrt{2}}{2}$

$= -\sqrt{2}$

f) $r = \frac{12\sqrt{3}}{4}$

$= 3\sqrt{3}$

g) $r = \frac{m^5}{m^2}$

$= m^3$

h) $r = \frac{27a^4}{3a^6}$

$= \frac{9}{a^2}$

i) $r = \frac{2x+1}{1}$

$r = 2x+1$

j) $6, \_\_, \_\_, 48, \ldots$ ($\times r$, $\times r$, $\times r$)

$\frac{6r^3}{6} = \frac{48}{6}$

$r^3 = 8$

$r = \sqrt[3]{8}$

$r = 2$

k) $-3, \_\_, -48, \ldots$ ($\times r$, $\times r$)

$\frac{-3r^2}{-3} = \frac{-48}{-3}$

$r^2 = 16$

$r = \pm\sqrt{16}$

$r = \pm 4$

l) $\_\_, 2a^4, \_\_, 4a^8$ ($\times r$, $\times r$)

$\frac{2a^4 \cdot r^2}{2a^4} = \frac{4a^8}{2a^4}$

$r^2 = 2a^4$

$r = \pm\sqrt{2a^4}$

$r = \pm\sqrt{2}\,a^2$

2. a) $t_1 = 2$

$r = 3$

$t_7 = x$

$t_n = t_1 r^{n-1}$

$t_7 = 2(3)^{7-1}$

$x = 2(3)^6$

$= 2(729)$

$= 1458$

b) $t_1 = 81$

$r = \frac{1}{3}$

$t_{10} = x$

$t_n = t_1 r^{n-1}$

$t_{10} = 81\left(\frac{1}{3}\right)^{10-1}$

$x = 81\left(\frac{1}{3}\right)^9$

$= 81\left(\frac{1}{19683}\right)$

$= \frac{1}{243}$

c) $t_1 = -4$

$r = -2$

$t_8 = x$

$t_n = t_1 r^{n-1}$

$t_8 = -4(-2)^{8-1}$

$x = -4(-2)^7$

$= -4(-128)$

$= 512$

PC 11 1.3 con't...2

2. d.) $t_2 = 6$
$t_3 = 12$
$t_{10} = x$

$\underline{\quad}, \underline{6}, \underline{12}$
$t_1 \quad t_2 \quad t_3$

$r = \frac{12}{6} = 2$

$t_1 = \frac{6}{2} = 3$

$t_n = t_1 r^{n-1}$
$t_{10} = 3(2)^{10-1}$
$x = 3(2)^9$
$= 3(512)$
$x = 1536$

e.) $t_2 = -8$
$t_4 = -2$
$t_9 = x$

$\underline{\quad}, \underline{-8}, \underline{\quad}, \underline{-2}$
$t_1 \quad t_2 \xrightarrow{\cdot r} t_3 \xrightarrow{\cdot r} t_4$

$\frac{-8 \cdot r^2}{-8} = \frac{-2}{-8}$
$r^2 = \frac{1}{4}$
$r = \pm\sqrt{\frac{1}{4}}$
$r = \pm\frac{1}{2}$

Ⓐ $t_1 = \frac{-8}{\frac{1}{2}}$
$= -16$

$t_n = t_1 r^{n-1}$
$t_9 = -16\left(\frac{1}{2}\right)^{9-1} = -16\left(\frac{1}{2}\right)^8$
$x = -16\left(\frac{1}{256}\right) = -\frac{1}{16}$

OR//

Ⓑ $t_1 = \frac{-8}{-\frac{1}{2}}$
$= 16$

$t_n = t_1 r^{n-1}$
$t_9 = 16\left(-\frac{1}{2}\right)^{9-1} = 16\left(-\frac{1}{2}\right)^8$
$x = 16\left(\frac{1}{256}\right) = \frac{1}{16}$

f.) $t_2 = 8$
$t_5 = 27$
$t_{10} = x$

$\underline{\quad}, \underline{8}, \underline{\quad}, \underline{\quad}, \underline{27}$
$t_1 \quad t_2 \rightarrow t_3 \rightarrow t_4 \rightarrow t_5$

$\frac{8 \cdot r^3}{8} = \frac{27}{8}$
$r^3 = \frac{27}{8}$
$r = \sqrt[3]{\frac{27}{8}} = \frac{3}{2}$

OR. Method 2 to find "r"

$t_n = t_1 r^{n-1}$
$8 = t_1 r^{2-1}$ $\quad$ $27 = t_1 r^{5-1}$
$8 = t_1 r^1$ $\quad$ $27 = t_1 r^4$
$\frac{8}{r} = t_1 \rightarrow \text{subs} \rightarrow$ $27 = \left(\frac{8}{r}\right) r^4$
$27 = 8r^3$
$\frac{27}{8} = r^3$
$r = \sqrt[3]{\frac{27}{8}} = \frac{3}{2}$

$t_1 = 8 \div \frac{3}{2}$
$= 8 \cdot \frac{2}{3}$
$= \frac{16}{3}$

$t_n = t_1 r^{n-1}$
$t_{10} = \frac{16}{3}\left(\frac{3}{2}\right)^{10-1}$
$= \frac{16}{3}\left(\frac{3}{2}\right)^9$
$= \frac{16}{3}\left(\frac{19683}{512}\right)$

$= \frac{314928}{1536}$
$= \frac{6516}{32}$

PC11 1.3 con't...3

2\. g) $t_1 = 10$, $r = 2$, $t_x = 2560$

$t_n = t_1 r^{n-1}$

$\frac{2560}{10} = \frac{10(2)^{x-1}}{10}$

$256 = 2^{x-1}$

$2^8 = 2^{x-1}$

$8 = x-1$

$x = 9$

h) $t_1 = -32$, $r = \frac{3}{2}$, $t_x = -243$

$t_n = t_1 r^{n-1}$

$\frac{-243}{-32} = \frac{-32\left(\frac{3}{2}\right)^{x-1}}{-32}$

$\frac{243}{32} = \left(\frac{3}{2}\right)^{x-1}$

$\left(\frac{3}{2}\right)^5 = \left(\frac{3}{2}\right)^{x-1}$

$5 = x-1$

$x = 6$

i) $t_1 = -32$, $r = -0.72$, $t_x = 1.66395$

$t_n = t_1 r^{n-1}$

$\frac{1.66395}{-32} = \frac{-32(-0.72)^{x-1}}{-32}$

$-0.0519984375 = (-0.72)^{x-1}$ Guess & Check.

$(-0.72)^9 = (-0.72)^{x-1}$

$9 = x-1$

$x = 10$

j) $t_1 = m^2$, $r = 2m$, $t_7 = x$

$t_n = t_1 r^{n-1}$

$t_7 = m^2(2m)^{7-1}$

$= m^2(2m)^6$

$= m^2(2)^6 m^6$

$= 64m^8$

k) $t_1 = \frac{a^2}{b}$, $r = \frac{b}{a}$, $t_{21} = x$

$t_n = t_1 r^{n-1}$

$t_{21} = \left(\frac{a^2}{b}\right)\left(\frac{b}{a}\right)^{21-1}$

$x = \left(\frac{a^2}{b}\right)\left(\frac{b}{a}\right)^{20}$

$= \left(\frac{a^2}{b}\right)\left(\frac{b^{20}}{a^{20}}\right)$

$= \frac{b^{19}}{a^{18}}$

l) $t_3 = 8a^2$, $t_6 = \frac{1}{a}$, $t_{12} = x$

$\underline{\quad}_{t_1}\ \underline{\quad}_{t_2}\ \underline{8a^2}_{t_3}\ \underline{\quad}_{t_4}\ \underline{\quad}_{t_5}\ \underline{\tfrac{1}{a}}_{t_6}$

$\cdot r \quad \cdot r \quad \cdot r$

$8a^2 \cdot r^3 = \frac{1}{a}$

$\frac{1}{8a^2} \cdot 8a^2 \cdot r^3 = \frac{1}{a} \cdot \frac{1}{8a^2}$

$r^3 = \frac{1}{8a^3}$

$r = \sqrt[3]{\frac{1}{8a^3}} = \frac{1}{2a}$

$t_n = t_1 r^{n-1}$

$8a^2 = t_1\left(\frac{1}{2a}\right)^{3-1}$

$8a^2 = t_1\left(\frac{1}{2a}\right)^2$

$4a^2 \cdot 8a^2 = \frac{t_1}{4a^2} \cdot 4a^2$

$32a^4 = t_1$

$t_n = t_1 r^{n-1}$

$t_{12} = 32a^4\left(\frac{1}{2a}\right)^{12-1}$

$x = 32a^4\left(\frac{1}{2a}\right)^{11}$

$x = \frac{32a^4}{2048a^{11}}$

$= \frac{1}{64a^7}$

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

Pre-Calculus 11 1.4

1\. a) $t_1 = 2$, $r = 3$, $S_7 = x$

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

$$S_7 = \frac{2(1-3^7)}{1-3}$$

$$x = \frac{2(1-2187)}{-2}$$

$$x = 2186$$

b) $t_1 = -3$, $r = -2$, $S_{10} = x$

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

$$S_{10} = \frac{-3(1-(-2)^{10})}{1-(-2)}$$

$$= \frac{-3(1-1024)}{3}$$

$$= 1023$$

c) $t_1 = 64$, $r = \frac{3}{2}$, $S_8 = x$

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

$$S_8 = \frac{64\left(1-\left(\frac{3}{2}\right)^8\right)}{1-\frac{3}{2}}$$

$$x = \frac{64\left(1-\frac{6561}{256}\right)}{\frac{2}{2}-\frac{3}{2}}$$

$$= \frac{64\left(\frac{-6305}{256}\right)}{-\frac{1}{2}}$$

$$= 3152.5$$

d) $t_1 = 16807$, $r = -\frac{2}{7}$, $S_6 = x$

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

$$S_6 = \frac{16807\left(1-\left(-\frac{2}{7}\right)^6\right)}{1-\left(-\frac{2}{7}\right)}$$

$$= \frac{16807\left(1-\frac{64}{117649}\right)}{\frac{7}{7}+\frac{2}{7}}$$

$$= \frac{16807\left(\frac{117585}{117649}\right)}{\frac{9}{7}}$$

$$= 13065$$

e) $t_1 = 27$, $r = \frac{1}{3}$, $S_7 = x$

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

$$x = \frac{27\left(1-\left(\frac{1}{3}\right)^7\right)}{1-\left(\frac{1}{3}\right)}$$

$$= \frac{27\left(1-\frac{1}{2187}\right)}{\frac{3}{3}-\frac{1}{3}}$$

$$= \frac{27\left(\frac{2186}{2187}\right)}{\frac{2}{3}}$$

$$= \frac{59022}{2187} \div \frac{2}{3}$$

$$= \frac{59022}{2187} \cdot \frac{3}{2}$$

$$= \frac{1093}{27}$$

$$= 40.\overline{481}$$

f) $t_1 = 81$, $t_4 = 24$, $S_{10} = x$

$$t_n = t_1 r^{n-1}$$

$$24 = 81 r^{4-1}$$

$$\frac{24}{81} = r^3$$

$$r = \sqrt[3]{\frac{24}{81}} = \frac{\sqrt[3]{8\cdot 3}}{\sqrt[3]{27\cdot 3}} = \frac{2\sqrt[3]{3}}{3\sqrt[3]{3}}$$

$$r = \frac{2}{3}$$

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

$$S_{10} = \frac{81\left(1-\left(\frac{2}{3}\right)^{10}\right)}{1-\frac{2}{3}}$$

$$x = \frac{81\left(1-\frac{1024}{59049}\right)}{1-\frac{2}{3}}$$

$$x = 238.786$$

PC 11 1.4 con't....2

1. g.) $t_2=-12$
$t_5=\frac{3}{2}$
$S_6=x$

$\underset{t_1}{\_\_}, \underset{t_2}{-12}, \_\_, \_\_, \underset{t_5}{\frac{3}{2}}$

$\times r \quad \times r \quad \times r$

$-12r^3=\frac{3}{2}$

$\frac{-12r^3}{-12}=\frac{3}{2\cdot -12}$

$r^3=\frac{3}{-24}$

$r=\sqrt[3]{\frac{-1}{8}}$

$r=-\frac{1}{2}$

$t_1=-12\div -\frac{1}{2}$
$=-12\cdot -2$
$=24$

$S_n=\frac{t_1(1-r^n)}{1-r}$

$S_6=\frac{24\left(1-\left(-\frac{1}{2}\right)^6\right)}{1-\left(-\frac{1}{2}\right)}$

$=\frac{24\left(1-\frac{1}{64}\right)}{1+\frac{1}{2}}$

$=\frac{24\left(\frac{63}{64}\right)}{\frac{3}{2}}$

$=15.75$

h.) $r=3$
$S_8=6560$
$t_1=x$

$S_n=\frac{t_1(1-r^n)}{1-r}$

$6560=\frac{x(1-3^8)}{1-3}$

$6560=\frac{x(1-6561)}{2}$

$\frac{2\cdot 6560}{-6560}=\frac{x(-6560)}{-6560}$

$2=x$

i.) $r=\frac{3}{2}$
$S_7=2059$
$t_1=x$

$S_n=\frac{t_1(1-r^n)}{1-r}$

$2059=\frac{x\left(1-\left(\frac{3}{2}\right)^7\right)}{1-\frac{3}{2}}$

$2059=\frac{x\left(1-\frac{2187}{128}\right)}{-\frac{1}{2}}$

$\frac{-\frac{1}{2}\cdot 2059}{\left(-\frac{2059}{128}\right)}=\frac{x\left(-\frac{2059}{128}\right)}{\left(-\frac{2059}{128}\right)}$

$-\frac{1}{2}(\cancel{2059})\cdot\left(\frac{-128}{\cancel{2059}}\right)=x$

$64=x$

j.) $r=\frac{1}{3}$
$S_6=-40.\overline{4}$
$t_4=x$

$t_n=t_1r^{n-1}$
$x=t_1\left(\frac{1}{3}\right)^{4-1}$
$x=t_1\left(\frac{1}{3}\right)^3$
$x=t_1\left(\frac{1}{27}\right)$
$27x=t_1$

$S_n=\frac{t_1(1-r^n)}{1-r}$

$-40.\overline{4}=\frac{27x\left(1-\left(\frac{1}{3}\right)^6\right)}{1-\frac{1}{3}}$

$-40.\overline{4}=\frac{27x\left(1-\frac{1}{729}\right)}{\frac{2}{3}}$

$-40.\overline{4}\cdot\frac{2}{3}=27x\left(\frac{728}{729}\right)$

$\frac{-40.\overline{4}(2)}{3}=x\left(\frac{728}{27}\right)$

$\rightarrow \frac{-40.\overline{4}(2)}{3}\left(\frac{27}{728}\right)=x$

$x=1$

PC11 1.4 con't...3

1\. k.) $t_1=2$

$t_3=18$

$S_x=19682$

$t_n=t_1 r^{n-1}$

$\frac{18}{2}=\frac{2r^{3-1}}{2}$

$9=r^2$

$\pm\sqrt{9}=r$

$r=\pm 3$

$S_n=\frac{t_1(1-r^n)}{1-r}$

$19682=\frac{2(1-(3)^x)}{1-(3)}$

$19682=\frac{2(1-(3)^x)}{-2}$

$-19682=1-3^x$

$-1 \quad -1$

$-19683=-3^x$

$19683=3^x$

$3^9=3^x$ Guess & Check

$x=9$

$S_n=\frac{t_1(1-r^n)}{1-r}$

$19682=\frac{2(1-(-3)^x)}{1-(-3)}$

$19682=\frac{2(1-(-3)^x)}{4}$

$19682\left(\frac{4}{2}\right)=1-(-3)^x$

$39364=1-(-3)^x$

$-1 \quad -1$

$39363=-(-3)^x$

$-39363=(-3)^x$

Guess & Check shows that no integer for "x" will give the answer so $r\neq -3$

l.) $t_2=6$

$t_5=-48$

$S_x=4095$

$\underline{\quad}, \underline{6}, \underline{\quad}, \underline{\quad}, \underline{-48}$

$t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5$

$\times r \quad \times r \quad \times r$

$\frac{6\cdot r^3}{6}=\frac{-48}{6}$

$r^3=-8$

$r=\sqrt[3]{-8}=-2$

$t_1=6\div -2$

$=-3$

$S_n=\frac{t_1(1-r^n)}{1-r}$

$4096=\frac{-3(1-(-2)^x)}{1-(-2)}$

$4096=\frac{-3(1-(-2)^x)}{3}$

$-4096=1-(-2)^x$

$-1 \quad -1$

$-4097=-(-2)^x$

$4097=(-2)^x$

$4097=(-2)^x$

$(-2)^{12}=(-2)^x$

$12=x$

← note x must be even for the answer to be positive. Guess & Check.

PC11 1.4 con't...

1\. m.) $t_2 = 500$ $\quad r = \frac{400}{500} = 0.8$ $\quad t_1 = \frac{500}{0.8} = 625$

$t_3 = 400$

$S_x = 2600.712$

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

$$2600.712 = \frac{625(1-0.8^x)}{1-0.8}$$

$$2600.712 = \frac{625(1-0.8^x)}{0.2}$$

$$\frac{2600.712(0.2)}{625} = \frac{625(1-0.8^x)}{625}$$

$$0.83222784 = 1-0.8^x$$

$-1 \qquad -1$

$$-0.167772216 = -0.8^x$$

$$0.16777216 = 0.8^x$$

$$0.8^8 = 0.8^x$$

$$8 = x$$

Guess & check.

2\. Day 1: 70 000 If production decreases by 10% /day, then each consecutive day produces 90% of previous day.

$t_1 = 70\,000 \qquad r = 0.9$

a.) Day 26: $t_{26} = ?$

$t_n = t_1 r^{n-1}$

$t_{26} = 70\,000(0.9)^{26-1}$

$= 70\,000(0.9)^{25}$

$= 5025.286$

∴ 5025 barrels were produced on day 26.

b.) Sum on Day 20

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

$$S_{20} = \frac{70\,000(1-0.9^{20})}{1-0.9}$$

$= 614\,896.34$

∴ Total number of barrels on day 20 was 614 896.

3\. Rises 120m, each minute, it rises to 97% of previous height

$t_1 = 120, \quad r = 0.97, \quad S_{10} = ?$

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

$$S_{10} = \frac{120(1-0.97^{10})}{1-0.97}$$

$= 1050.3$

∴ The balloon is 1050.3m high 10min after release

PC11 1.4 con't...5

4 \$50.. $\underbrace{\text{Increases 4\% each year.}}$ 20 years.

Each subsequent year, it is still worth 100% but add 4% $\therefore$ $r = 104\%$
$r = 1.04$.

$$\underset{\substack{\text{First \$50 earns}\\ \text{interest for}\\ \text{20 years}}}{50(1.04)^{20}} + \underset{\substack{\text{Second \$50}\\ \text{earns interest}\\ \text{for 19 years}}}{50(1.04)^{19}} + 50(1.04)^{18} + \ldots + \underset{\substack{\text{Last \$50 earns}\\ \text{interest for}\\ \text{1 year.}}}{50(1.04)} =$$

$$= 50(1.04) + 50(1.04)^2 + 50(1.04)^3 + \ldots + 50(1.04)^{20}$$

$t_1 = 50(1.04)$ $\qquad S_n = \dfrac{t_1(1-r^n)}{1-r}$

$r = 1.04$

$n = 20$ $\qquad S_{20} = \dfrac{50(1.04)(1-1.04^{20})}{1-1.04}$

$= \$1548.46$

The value of the annuity at the end of 20 years is \$1548.46

5. $S_3 = 228$, $S_4 = 390$, $S_5 = 633$

$$\underbrace{\underbrace{\underbrace{\_\_ + \_\_ + \_\_}_{S_3 = 228} + t_4}_{228 + t_4 = 390} + t_5}_{390 + t_5 = 633}$$

$228 + t_4 = 390 \rightarrow 228 + t_4 = 390 \rightarrow t_4 = 162$

$390 + t_5 = 633 \rightarrow 390 + t_5 = 633 \rightarrow t_5 = 243$

$r = \dfrac{243}{162} = \dfrac{3}{2} = 1.5$

PC 11 1.4 con't... 6.

6. $t_1 = -6$ $\quad$ $t_1 + t_2 + t_3 = S_3$

$S_3 = -42$ $\quad$ $\underline{-6} + \underline{-6r} + \underline{-6r^2} = -42$

$0 = 6r^2 + 6r + 6 - 42$

$6r^2 + 6r - 36 = 0$

$6(r^2 + r - 6) = 0$

$6(r+3)(r-2) = 0$

$r + 3 = 0 \qquad r - 2 = 0$

$r = -3 \qquad r = 2$

$t_2 = t_1 r$

If $r = -3 \rightarrow$ $t_2 = -6(-3)$ $\qquad$ If $r = 2 \rightarrow$ $t_2 = -6(2)$

$t_2 = 18$ $\qquad$ $t_2 = -12$

$$S_\infty = \frac{t_1}{1-r}$$

Pre-Calculus 1.6

1. a) $t_1 = 24$, $r = -\frac{1}{2}$, $S_\infty = x$

$S_\infty = \frac{t_1}{1-r}$

$x = \frac{24}{1 - -\frac{1}{2}}$

$= \frac{24}{\frac{2}{2}+\frac{1}{2}}$

$= 24 \div \frac{3}{2}$

$= 24 \cdot \frac{2}{3}$

$x = 16$

b) $t_1 = 625$, $r = \frac{4}{5}$, $S_\infty = x$

$S_\infty = \frac{t_1}{1-r}$

$x = \frac{625}{1-\frac{4}{5}}$

$= \frac{625}{\frac{5}{5}-\frac{4}{5}}$

$= 625 \div \frac{1}{5}$

$= 625 \cdot \frac{5}{1}$

$x = 325$

c.) $t_1 = 81$, $r = -\frac{1}{9}$, $S_\infty = x$

$S_\infty = \frac{t_1}{1-r}$

$x = \frac{81}{1 - -\frac{1}{9}}$

$= \frac{81}{\frac{9}{9}+\frac{1}{9}}$

$= \frac{81}{\frac{10}{9}}$

$= 72.9$

d.) $t_1 = 27$, $t_3 = 3$, $S_\infty = x$

$t_n = t_1 r^{n-1}$

$\frac{3}{27} = \frac{27 r^{3-1}}{27}$

$\frac{3}{27} = r^2$

$r = \pm\sqrt{\frac{1}{9}}$

$r = \pm \frac{1}{3}$

$r = +\frac{1}{3}$

$S_\infty = \frac{t_1}{1-r}$

$x = \frac{27}{1-\frac{1}{3}}$

$= \frac{27}{\frac{2}{3}}$

$= 27\left(\frac{3}{2}\right)$

$= 40.5$

$r = -\frac{1}{3}$

$S_\infty = \frac{t_1}{1-r}$

$x = \frac{27}{1 - -\frac{1}{3}}$

$= \frac{27}{\frac{4}{3}}$

$= 27\left(\frac{3}{4}\right)$

$= 20.25$

e.) $t_1 = 96$, $t_2 = 144$, $S_\infty = x$

$r = \frac{144}{96} = \frac{3}{2}$

$= 1.5$

Note: $r > 1$ so there is no sum.

$S_\infty = \infty$

f.) $t_1 = 81$, $S_\infty = \frac{729}{8}$, $r = x$

$S_\infty = \frac{t_1}{1-r}$

$\frac{729}{8} = \frac{81}{1-x}$

$729(1-x) = 8(81)$

$729 - 729x = 648$

$-729 \qquad -729$

$-729x = -81$

$x = \frac{-81}{-729} = \frac{1}{9}$

PC 11 1.6 con't...2

1. g) $r = -\frac{2}{7}$

$S_\infty = \frac{117\,649}{9}$

$t_1 = x$

$S_\infty = \frac{t_1}{1-r}$

$\frac{117\,649}{9} = \frac{x}{1 - -\frac{2}{7}}$

$\frac{117\,649}{9} = \frac{x}{\frac{7}{7}+\frac{2}{7}}$

$\frac{117\,649}{9} = \frac{x}{\frac{9}{7}}$

$\frac{9}{7}\left(\frac{117\,649}{9}\right) = x$

$x = 16\,807$

h.) $t_1 = 40\,960$

$S_\infty = \frac{163\,840}{3}$

$r = x$

$S_\infty = \frac{t_1}{1-r}$

$\frac{163\,840}{3} = \frac{40\,960}{1-x}$

$163\,840(1-x) = 40\,960(3)$

$163\,840 - 163\,840x = 122\,880$

$-163\,840 \qquad -163\,840$

$\frac{-163\,840x}{-163\,840} = \frac{-40\,960}{-163\,840}$

$x = \frac{1}{4}$

i.) $r = -\frac{1}{4}$

$S_\infty = \frac{40\,960}{5}$

$t_1 = x$

$S_\infty = \frac{t_1}{1-r}$

$\frac{40\,960}{5} = \frac{x}{1 - -\frac{1}{4}}$

$\frac{40\,960}{5} = \frac{x}{\frac{5}{4}}$

$\frac{5}{4}\left(\frac{40\,960}{5}\right) = x$

$x = 10\,240$

j.) $t_1 = 20\,480$

$S_2 = 25\,600$

$S_\infty = x$

$20\,480 + t_2 = 25\,600$

$t_2 = 5120$

$\therefore r = \frac{5120}{20\,480} = \frac{1}{4}$

$S_\infty = \frac{t_1}{1-r}$

$x = \frac{20\,480}{1-\frac{1}{4}}$

$= \frac{20\,480}{\frac{3}{4}}$

$= 20\,480 \div \frac{3}{4}$

$= 20\,480\left(\frac{4}{3}\right)$

$= \frac{81\,920}{3}$

k.) $t_2 = -2560$

$t_5 = 320$

$S_\infty = x$

$-2560\, r^{5-2} = 320$

$r^3 = \frac{320}{-2560} = -\frac{1}{8}$

$r = \sqrt[3]{-\frac{1}{8}} = -\frac{1}{2}$

$t_1 = -2560 \div -\frac{1}{2}$

$= -2560 \cdot -2$

$= +5120$

$S_\infty = \frac{t_1}{1-r}$

$x = \frac{+5120}{1 - -\frac{1}{2}}$

$= \frac{5120}{\frac{2}{2}+\frac{1}{2}}$

$= 5120 \div \frac{3}{2}$

$= 5120 \cdot \frac{2}{3}$

$= \frac{10\,240}{3}$

l.) $t_1 = 3840$

$S_2 = 1920$

$S_\infty = x$

$3840 + t_2 = 1920$

$t_2 = -1920$

$r = \frac{-1920}{3840} = -\frac{1}{2}$

$S_\infty = \frac{t_1}{1-r}$

$x = \frac{3840}{1 - -\frac{1}{2}}$

$= \frac{3840}{\frac{2}{2}+\frac{1}{2}}$

$x = 3840 \div \frac{3}{2}$

$= 3840\left(\frac{2}{3}\right)$

$= 2560$

PC11 1.6 con't...3

1. m) $r = -\frac{2}{3}$

$S_2 = 27$

$S_\infty = x$

$t_1 + t_1\left(-\frac{2}{3}\right) = 27$

$t_1\left(1 + -\frac{2}{3}\right) = 27$

$t_1\left(\frac{3}{3} + -\frac{2}{3}\right) = 27$

$t_1\left(\frac{1}{3}\right) = 27$

$t_1 = 27(3)$

$t_1 = 81$

$S_\infty = \frac{t_1}{1-r}$

$x = \frac{81}{1 - -\frac{2}{3}}$

$= \frac{81}{\frac{3}{3} + \frac{2}{3}}$

$= 81 \div \frac{5}{3}$

$= 81 \cdot \frac{3}{5}$

$= \frac{243}{5}$

n) $r = \frac{1}{2}$

$S_3 = 140$

$S_\infty = x$

$t_1 + t_1\left(\frac{1}{2}\right) + t_1\left(\frac{1}{2}\right)^2 = 140$

$t_1\left(1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2\right) = 140$

$t_1\left(\frac{4}{4} + \frac{2}{4} + \frac{1}{4}\right) = 140$

$t_1\left(\frac{7}{4}\right) = 140$

$t_1 = 140 \div \frac{7}{4}$

$t_1 = 140\left(\frac{4}{7}\right)$

$t_1 = 80$

$S_\infty = \frac{t_1}{1-r}$

$x = \frac{80}{1 - \frac{1}{2}}$

$= \frac{80}{\frac{1}{2}}$

$x = 160$

o) $t_1 = 27$

$S_3 = 57$

$S_\infty = x$

$t_1 + t_2 + t_3 = S_3$

$\underline{27} + \underline{27r} + \underline{27r^2} = 57$

$27r^2 + 27r + 27 - 57 = 0$

$27r^2 + 27r - 30 = 0$

$3(9r^2 + 9r - 10) = 0$

$3(9r^2 + 15r - 6r - 10) = 0$

$3(3r(3r+5) - 2(3r+5)) = 0$

$3(3r-2)(3r+5) = 0$

$3r - 2 = 0 \qquad 3r + 5 = 0$

$r = \frac{2}{3} \qquad r = -\frac{5}{3}$

$\uparrow$

$r < -1$ so there is no finite sum.

$S_\infty = \infty$

$S_\infty = \frac{t_1}{1-r}$

$= \frac{27}{1 - \frac{2}{3}}$

$= \frac{27}{\frac{1}{3}}$

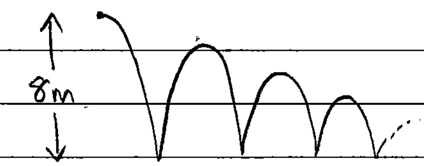
$= 27 \div \frac{1}{3}$

$= 27 \times 3$

$= 81$

PC11 1.6 con't...4

2.



$8+2(8)(0.8)+2(8)(0.8)^2+2(8)(0.8)^3+\ldots$

$8+\underbrace{16(0.8)+16(0.8)^2+16(0.8)^3+\ldots}$

$t_1=16(0.8) \qquad r=0.8$

$S_\infty=\dfrac{t_1}{1-r}$

$=\dfrac{16(0.8)}{1-0.8}$

$=\dfrac{12.8}{0.2}$

$=64$

Total distance $=8+64=72$m.

3. Day 1: 30 000 barrels → Production decreases 8%/day

∴ Production is 92% of previous day

$30000+30000(0.92)+30000(0.92)^2+\ldots$

a) $t_1=30\,000$

$r=0.92$

$t_{14}=?$

$t_n=t_1r^{n-1}$

$t_{14}=30\,000(0.92)^{14-1}$

$=30\,000(0.92)^{13}$

$=10\,147.6$

∴ There were 10 147 full barrels on day 14

b) $S_{17}=?$ $\quad S_n=\dfrac{t_1(1-r^n)}{1-r}$

$S_{17}=\dfrac{30\,000(1-0.92^{17})}{1-0.92}$

$=284\,129.2$

∴ There were a total of 284 129 full barrels on day 17

c) $S_\infty=\dfrac{t_1}{1-r}$

$=\dfrac{30\,000}{1-0.92}$

$=\dfrac{30\,000}{0.08}$

$=375\,000$

∴ There were a total of 375 000 full barrels.

PC 11 1.6 con't...5

4 1st b-day 2nd b-day 3rd bday
\$0.01 + \$0.02 + \$0.04 + ...

a) $t_1 = 0.01$
$r = 2$
$t_{10} = ?$
$n = 10$

$t_n = t_1 r^{n-1}$
$t_{10} = 0.01(2)^{10-1}$
$= 0.01(2)^9$
$= 5.12$

Carl gets \$5.12 on his 10th bday.

b) $t_{30} = ?$

$t_n = t_1 r^{n-1}$
$t_{30} = 0.01(2)^{30-1}$
$= 0.01(2)^{29}$
$= 5\,368\,709.12$

Carl gets \$5 368 709.12 on his 30th bday.

c) $S_{35} = ?$

$S_n = \dfrac{t_1(1-r^n)}{1-r}$
$= \dfrac{0.01(1-2^{35})}{1-2}$
$= 343\,597\,383.7$

Carl will have gotten \$343 597 383.70

5. $t_1 = 70$
$r = 0.88$

Rises 12% less than previous minute
∴ Rises to 88% of previous minute.

$S_\infty = \dfrac{t_1}{1-r}$
$S_\infty = \dfrac{70}{1-0.88}$
$= 583.\overline{3}$

∴ In total, the balloon rises $583.\overline{3}$ m.

PC11 1.6 con't...6

6a) $0.9\overline{8} = 0.9 + \underbrace{0.08 + 0.008 + 0.0008 + \ldots}_{\text{Infinite Geometric Series.}}$

$r = \dfrac{0.008}{0.08} = 0.1$

$S_\infty = \dfrac{t_1}{1-r} = \dfrac{0.08}{1-0.1} = \dfrac{0.08}{0.9} \dfrac{\cdot 100}{\cdot 100} = \dfrac{8}{90}$

$= \dfrac{9}{10} + \dfrac{8}{90}$

$= \dfrac{81}{90} + \dfrac{8}{90} = \dfrac{89}{90}$ $\quad \therefore 0.9\overline{8} = \dfrac{89}{90}$

b) $0.\overline{23} = \underbrace{0.23 + 0.0023 + 0.000023 + \ldots}_{\text{Inf. Geo. Series.}}$

$r = \dfrac{0.0023}{0.23} = 0.01$

$S_\infty = \dfrac{t_1}{1-r} = \dfrac{0.23}{1-0.01} = \dfrac{0.23}{0.99} = \dfrac{23}{99}$

$\therefore 0.\overline{23} = \dfrac{23}{99}$

c) $3.5\overline{76} = 3.5 + \underbrace{0.076 + 0.00076 + 0.0000076 + \ldots}_{\text{Inf. Geo. Series}}$

$r = \dfrac{0.00076}{0.076} = 0.01$

$S_\infty = \dfrac{t_1}{1-r} = \dfrac{0.076}{1-0.01} = \dfrac{0.076}{0.99} \dfrac{\times 1000}{\times 1000} = \dfrac{76}{990}$

$= \dfrac{35}{10} + \dfrac{76}{990}$

$= \dfrac{3465}{990} + \dfrac{76}{990} = \dfrac{3541}{990}$ $\quad \therefore 3.5\overline{76} = \dfrac{3541}{990}.$

PC 1.6 con't...

7. a) $3 + 3(x+2) + 3(x+2)^2 + \ldots$

The series will have a finite sum if $-1 < r < 1$

$r = x+2$

$\therefore -1 < (x+2) < 1$

$-1 < x+2 \qquad x+2 < 1$

$-2 \quad -2 \qquad -2 \quad -2$

$-3 < x \qquad x < -1$

$\therefore -3 < x < -1$

b.) $5 + (2x-3) + \frac{(2x-3)^2}{5} + \ldots$

$r = \frac{2x-3}{5}$ → finite sum if $-1 < r < 1$

$\therefore -1 < \frac{2x-3}{5} < 1$

$-1 < \frac{2x-3}{5} \qquad \frac{2x-3}{5} < 1$

$-1(5) < 2x-3 \qquad 2x-3 < 1(5)$

$-5 < 2x-3 \qquad 2x-3 < 5$

$+3 \quad +3 \qquad +3 \quad +3$

$\frac{-2}{2} < \frac{2x}{2} \qquad \frac{2x}{2} < \frac{8}{2}$

$-1 < x \qquad x < 4$

$\therefore -1 < x < 4.$

Pre-Calculus 11 1X

1. a) $\sum_{k=1}^{5} k = 1+2+3+4+5$
$= 15$

b) $\sum_{k=1}^{7} (2k-1) = (2(1)-1) + (2(2)-1) + (2(3)-1) + \ldots (2(7)-1)$
$= 1 + 3 + 5 + \ldots + 13$

Arithmetic
$t_1 = 1$ $\quad S_n = \frac{n(t_1+t_n)}{2}$
$d = 2$
$t_7 = 13$ $\quad S_7 = \frac{7(1+13)}{2} = 49$

c) $\sum_{k=1}^{10} 2^k = 2^1 + 2^2 + 2^3 + \ldots + 2^{10}$
$= 2 + 4 + 8 + \ldots + 1024$

Geometric
$t_1 = 2$ $\quad S_n = \frac{t_1(1-r^n)}{1-r}$
$r = \frac{4}{2} = 2$
$S_{10} = \frac{2(1-2^{10})}{1-2} = \frac{2(-1023)}{-1} = 2046.$

d) $\sum_{k=1}^{8} 2 \cdot 3^{k-1} = 2(3)^{1-1} + 2(3)^{2-1} + 2(3)^{3-1} + \ldots + 2(3)^{8-1}$
$= 2(3)^0 + 2(3)^1 + 2(3)^2 + \ldots + 2(3)^7$

Geometric
$t_1 = 2(3)^0 = 2$ $\quad S_n = \frac{t_1(1-r^n)}{1-r}$
$r = 3$
$S_8 = \frac{2(1-3^8)}{1-3} = \frac{2(-6560)}{-2} = 6560.$

e) $\sum_{k=1}^{7} 160\left(\frac{1}{2}\right)^k = 160\left(\frac{1}{2}\right)^1 + 160\left(\frac{1}{2}\right)^2 + 160\left(\frac{1}{2}\right)^3 + \ldots + 160\left(\frac{1}{2}\right)^7$

Geometric $\quad S_n = \frac{t_1(1-r^n)}{1-r}$
$t_1 = 160\left(\frac{1}{2}\right) = 80$
$r = \frac{1}{2}$
$= \frac{80\left(1-\left(\frac{1}{2}\right)^7\right)}{1-\frac{1}{2}} = \frac{80\left(1-\frac{1}{128}\right)}{\frac{1}{2}} = 158.75$

PC11 1x con't...2.

1. f.) $\sum_{k=3}^{9} (0.4)^k = (0.4)^3 + (0.4)^4 + (0.4)^5 + \ldots + (0.4)^9$

Geometric

$t_1 = (0.4)^3 = 0.064$

$r = 0.4$

$n = 9-3+1 = 7$

$S_n = \frac{t_1(1-r^n)}{1-r}$

$S_7 = \frac{0.064(1-0.4^7)}{1-0.4}$

$= 0.106492.$

g.) $\sum_{k=1}^{10} 6(-2)^{k+2} = 6(-2)^{1+2} + 6(-2)^{2+2} + 6(-2)^{3+2} + \ldots + 6(-2)^{10+2}$

$= 6(-2)^3 + 6(-2)^4 + 6(-2)^5 + \ldots + 6(-2)^{12}$

Geometric

$t_1 = 6(-2)^3 = -48$

$r = (-2)$

$n = 10$

$S_n = \frac{t_1(1-r^n)}{1-r}$

$S_{10} = \frac{-48(1-(-2)^{10})}{1-(-2)}$

$= 16368.$

h.) $\sum_{k=3}^{8} 5\left(-\frac{1}{3}\right)^{k-2} = 5\left(-\frac{1}{3}\right)^{3-2} + 5\left(-\frac{1}{3}\right)^{4-2} + 5\left(-\frac{1}{3}\right)^{5-2} + \ldots + 5\left(-\frac{1}{3}\right)^{8-2}$

$= 5\left(-\frac{1}{3}\right)^1 + 5\left(-\frac{1}{3}\right)^2 + 5\left(-\frac{1}{3}\right)^3 + \ldots + 5\left(-\frac{1}{3}\right)^6$

Geometric

$t_1 = 5\left(-\frac{1}{3}\right)^1 = -\frac{5}{3}$

$r = -\frac{1}{3}$

$n = 8-3+1 = 6$

$S_n = \frac{t_1(1-r^n)}{1-r}$

$S_6 = \frac{-\frac{5}{3}\left(1-\left(-\frac{1}{3}\right)^6\right)}{1-\left(-\frac{1}{3}\right)} = -1.248253.$

i.) $\sum_{j=2}^{10} (-3+5j) = (-3+5(2)) + (-3+5(3)) + (-3+5(4)) + \ldots + (-3+5(10))$

$= (-3+10) + (-3+15) + (-3+20) + \ldots + (-3+50)$

$= 7 + 12 + 17 + \ldots + 47$

Arithmetic

$t_1 = 7$

$d = 5$

$n = 10-2+1 = 9$

$t_9 = 47$

$S_n = \frac{n(t_1+t_n)}{2}$

$S_9 = \frac{9(7+47)}{2} = \frac{9(54)}{2}$

$= 243$

PC11 1X con't...3.

1. j.) $\sum_{j=-1}^{7}(-6k+10) = (-6(-1)+10)+(-6(0)+10)+(-6(1)+10)+\ldots+(-6(7)+10)$

$= (6+10)+(0+10)+(-6+10)+\ldots+(-42+10)$

$= 16+10+4+\ldots+-32$

Arithmetic

$t_1 = 16$

$d = -6$

$n = 7--1+1$

$= 7+1+1 = 9$

$t_9 = -32$

$S_n = \frac{n(t_1+t_n)}{2}$

$S_9 = \frac{9(16+-32)}{2}$

$= \frac{9(-16)}{2}$

$= -72$

k.) $\sum_{k=1} 8\left(\frac{1}{2}\right)^k = 8\left(\frac{1}{2}\right)^1 + 8\left(\frac{1}{2}\right)^2 + 8\left(\frac{1}{2}\right)^3 + 8\left(\frac{1}{2}\right)^4 + \ldots$

Infinite Geometric

$t_1 = 8\left(\frac{1}{2}\right) = 4$

$r = \frac{1}{2}$

$S_\infty = \frac{t_1}{1-r}$

$= \frac{4}{1-\frac{1}{2}}$

$= \frac{4}{\frac{1}{2}}$

$= 8$

l.) $\sum_{k=2} 243\left(-\frac{1}{3}\right)^{k-1} = 243\left(-\frac{1}{3}\right)^{2-1} + 243\left(-\frac{1}{3}\right)^{3-1} + 243\left(-\frac{1}{3}\right)^{4-1} + \ldots$

$= 243\left(-\frac{1}{3}\right)^1 + 243\left(-\frac{1}{3}\right)^2 + 243\left(-\frac{1}{3}\right)^3 + \ldots$

Infinite Geometric

$t_1 = 243\left(-\frac{1}{3}\right) = -81$

$r = -\frac{1}{3}$

$S_\infty = \frac{t_1}{1-r}$

$= \frac{-81}{1--\frac{1}{3}}$

$= \frac{-81}{\frac{4}{3}}$

$= -81\left(\frac{3}{4}\right)$

$= \frac{-243}{4} = -60.75$

m.) $\sum_{k=3} 15625\left(\frac{4}{5}\right)^{k-1} = 15625\left(\frac{4}{5}\right)^{3-1} + 15625\left(\frac{4}{5}\right)^{4-1} + \ldots$

$= 15625\left(\frac{4}{5}\right)^2 + 15625\left(\frac{4}{5}\right)^3 + 15625\left(\frac{4}{5}\right)^4 + \ldots$

Infinite Geometric

$t_1 = 15625\left(\frac{4}{5}\right)^2$

$= 10\,000$

$r = \frac{4}{5}$

$S_\infty = \frac{t_1}{1-r}$

$= \frac{10\,000}{1-\frac{4}{5}}$

$= \frac{10\,000}{\frac{1}{5}}$

$= 10\,000\left(\frac{5}{1}\right)$

$= 50\,000$

PC11 IX con't...4

1. n.) $\sum_{k=3} 3(2)^k = 3(2)^3 + 3(2)^4 + 3(2)^5 + \ldots$

Infinite Geometric Series.

$t_1 = 3(2)^3 = 24$

$r = 2$

* Because $r > 1$, there is no finite sum.

$\therefore$ The sum is infinite... ∞

2. a.) $-4+8-16+\ldots$

Geometric

$t_1 = -4$

$r = -2$

$n = \infty$

$t_n = t_1 r^{n-1}$

$t_n = -4(-2)^{n-1}$

$\sum_{k=1} -4(-2)^{k-1}$

b.) $5+8+11+\ldots+35$

Arithmetic

$t_1 = 5$

$d = 3$

$t_n = 35$

$n = ?$

$t_n = t_1 + (n-1)d$

$t_n = 5 + (n-1)(3)$

$35 = 5 + (n-1)(3)$

$-5 \quad -5$

$30 = 3(n-1)$

$= \frac{30}{3} = n-1$

$10 = n-1$

$n = 11$

$\sum_{k=1}^{11} (5+3(k-1))$

c.) $80+40+20+\ldots$

Geometric

$t_1 = 80$

$r = \frac{40}{80} = \frac{1}{2}$

$n = \infty$

$t_n = t_1 r^{n-1}$

$t_n = 80(\frac{1}{2})^{n-1}$

$\sum_{n=1} 80(\frac{1}{2})^{n-1}$

d.) $1 - \frac{1}{6} + \frac{1}{36} - \ldots$

Geometric

$t_1 = 1$

$r = \frac{-\frac{1}{6}}{1} = -\frac{1}{6}$

$n = \infty$

$t_n = t_1 r^{n-1}$

$t_n = 1(-\frac{1}{6})^{n-1}$

$\sum_{k=1} (-\frac{1}{6})^{k-1}$

PC11 1X con't...5

2. e.) $-2-6-10-\ldots-34$

Arithmetic

$t_1=-2$ $\quad t_n=t_1+(n-1)d$ $\quad \dfrac{-32}{-4}=\dfrac{(n-1)(-4)}{-4}$ $\quad \sum_{k=1}^{9}(-2-4(k-1))$

$d=-4$ $\quad t_n=-2+(n-1)(-4)$ $\quad 8=n-1$

$t_n=-34$ $\quad -34=-2+(n-1)(-4)$ $\quad n=9$

$n=?$ $\quad +2 \quad +2$

f.) $3+6+12+\ldots+3072$

Geometric

$t_1=3$ $\quad t_n=t_1r^{n-1}$ $\quad 1024=2^{n-1}$ $\quad \sum_{k=1}^{11}3(2)^{k-1}$

$r=\frac{6}{3}=2$ $\quad t_n=3(2)^{n-1}$ $\quad 2^{10}=2^{n-1}$

$t_n=3072$ $\quad \dfrac{3072}{3}=\dfrac{3(2)^{n-1}}{3}$ $\quad n=11$

$n=?$

g.) $100+10+1+\ldots+10^{-6}$

Geometric

$t_1=100$ $\quad t_n=t_1r^{n-1}$ $\quad 10^{-8}=\left(\frac{1}{10}\right)^{n-1}$

$r=\frac{10}{100}=\frac{1}{10}$ $\quad t_n=100\left(\frac{1}{10}\right)^{n-1}$ $\quad \left(\frac{1}{10}\right)^{8}=\left(\frac{1}{10}\right)^{n-1}$

$t_n=10^{-6}$ $\quad \dfrac{10^{-6}}{100}=\dfrac{100\left(\frac{1}{10}\right)^{n-1}}{100}$ $\quad n=9$

$n=?$ $\quad \sum_{k=1}^{9}100\left(\frac{1}{10}\right)^{k-1}$

h.) $a+3a+5a+\ldots+23a$

Arithmetic $\quad t_n=t_1+(n-1)d$ $\quad \dfrac{22a}{2a}=\dfrac{2a(n-1)}{2a}$

$t_1=a$ $\quad t_n=a+(n-1)(2a)$ $\quad 11=n-1$

$d=3a-a=2a$ $\quad 23a=a+2a(n-1)$ $\quad n=12$

$t_n=23a$ $\quad -a \quad -a$

$n=?$ $\quad 22a=2a(n-1)$ $\quad \sum_{k=1}^{12}(a+2a(k-1))$

PC 11 IX con't....6

2. i.) $a - b + \frac{b^2}{a} - \ldots$

Geometric (Inf.)

Checked both "r" values are same

$\frac{t_3}{t_2} \rightarrow r = \frac{b^2}{a} \div -b$

$= \frac{b^2}{a} \times \frac{1}{-b} = -\frac{b}{a}$

$\frac{t_2}{t_1} \rightarrow = \frac{-b}{a}$

$t_1 = a$

$t_n = t_1 r^{n-1}$

$t_n = a\left(-\frac{b}{a}\right)^{n-1}$

$\sum_{k=1} a\left(\frac{-b}{a}\right)^{k-1}$

j.) $a^6 + a^4 + a^2 + \ldots + a^{-10}$

Geometric

$r = \frac{a^4}{a^6} = a^{-2} = \frac{1}{a^2}$

$t_1 = a^6$

$t_n = a^{-10}$

$n = ?$

$t_n = t_1 r^{n-1}$

$t_n = a^6\left(\frac{1}{a^2}\right)^{n-1}$

$a^{-10} = a^6\left(\frac{1}{a^2}\right)^{n-1}$

$\frac{a^{-10}}{a^6} = \left(\frac{1^{n-1}}{a^{2n-2}}\right)$

$= a^{-16} = \frac{1}{a^{2n-2}}$

$a^{-16} = a^{-(2n-2)}$

$-16 = -(2n-2)$

$-16 = -2n+2$

$-2 \qquad -2$

$\frac{-18}{-2} = \frac{-2n}{-2}$

$9 = n$

$\sum_{k=1}^{9} a^6\left(\frac{1}{a^2}\right)^{k-1}$

k.) $\frac{1024}{a^4} - 1280a + 1600a^6 - \ldots + 3906.25a^{26}$

Geometric

Checked both "r" values are same to confirm it's Geometric

$\frac{t_3}{t_2} \rightarrow r = \frac{1600a^6}{-1280a} = \frac{-5a^5}{4}$

$\frac{t_2}{t_1} \rightarrow = \frac{-1280a}{\frac{1024}{a^4}}$

$= -1280a \cdot \frac{a^4}{1024}$

$= -\frac{5a^5}{4}$

$t_1 = \frac{1024}{a^4}$

$t_n = 3906.25a^{26}$

$n = ?$

$t_n = t_1 r^{n-1}$

$3906.25a^{26} = \frac{1024}{a^4}\left(\frac{-5a^5}{4}\right)^{n-1}$

$3906.25a^{26}\left(\frac{a^4}{1024}\right) = \left(\frac{-5a^5}{4}\right)^{n-1}$

$\frac{15\,625\,a^{30}}{1024} = \left(\frac{-5}{4}\right)^{n-1}\left(a^5\right)^{n-1}$

$\left(\frac{5}{4}\right)^6 a^{30} = \left(\frac{-5}{4}\right)^{n-1} a^{5n-5}$

$6 = n-1$

$n = 7$

and

$30 = 5n-5$

$35 = 5n$

$7 = n$

$\sum_{k=1}^{7} \frac{1024}{a^4}\left(\frac{-5a^5}{4}\right)^{k-1}$

Pre-Calculus 11 - Problem Solving Lab

$$t_n = \sum_{k=1}^{n} \left(\frac{1}{x}\right)^{k-1} + \sum_{k=1}^{n} \left(-\frac{1}{x}\right)^{k-1}$$

a.) $t_1 = \sum_{k=1}^{1} \left(\frac{1}{x}\right)^{k-1} + \sum_{k=1}^{1} \left(-\frac{1}{x}\right)^{k-1}$

$= \left(\frac{1}{x}\right)^{1-1} + \left(-\frac{1}{x}\right)^{1-1}$

$= \left(\frac{1}{x}\right)^{0} + \left(-\frac{1}{x}\right)^{0}$

$= 1 + 1$

$\boxed{t_1 = 2}$

$t_2 = \sum_{k=1}^{2} \left(\frac{1}{x}\right)^{k-1} + \sum_{k=1}^{2} \left(-\frac{1}{x}\right)^{k-1}$

$= \left[\left(\frac{1}{x}\right)^{1-1} + \left(\frac{1}{x}\right)^{2-1}\right] + \left[\left(-\frac{1}{x}\right)^{1-1} + \left(-\frac{1}{x}\right)^{2-1}\right]$

$= \left(\frac{1}{x}\right)^{0} + \left(\frac{1}{x}\right)^{1} + \left(-\frac{1}{x}\right)^{0} + \left(-\frac{1}{x}\right)^{1}$

$= 1 + \frac{1}{x} + 1 - \frac{1}{x}$

$\boxed{t_2 = 2}$

$t_3 = \sum_{k=1}^{3} \left(\frac{1}{x}\right)^{k-1} + \sum_{k=1}^{3} \left(-\frac{1}{x}\right)^{k-1}$

$= \left[\left(\frac{1}{x}\right)^{1-1} + \left(\frac{1}{x}\right)^{2-1} + \left(\frac{1}{x}\right)^{3-1}\right] + \left[\left(-\frac{1}{x}\right)^{1-1} + \left(-\frac{1}{x}\right)^{2-1} + \left(-\frac{1}{x}\right)^{3-1}\right]$

$= \left(\frac{1}{x}\right)^{0} + \left(\frac{1}{x}\right)^{1} + \left(\frac{1}{x}\right)^{2} + \left(-\frac{1}{x}\right)^{0} + \left(-\frac{1}{x}\right)^{1} + \left(-\frac{1}{x}\right)^{2}$

$= 1 + \frac{1}{x} + \frac{1}{x^2} + 1 - \frac{1}{x} + \frac{1}{x^2}$

$\boxed{t_3 = 2 + \frac{2}{x^2}}$

PC11 - Ch. 1 Lab con't... 2

1. a) con't...

$$t_4 = \sum_{k=1}^{4}\left(\frac{1}{x}\right)^{k-1} + \sum_{k=1}^{4}\left(-\frac{1}{x}\right)^{k-1}$$

$$= \left(\frac{1}{x}\right)^{1-1} + \left(\frac{1}{x}\right)^{2-1} + \left(\frac{1}{x}\right)^{3-1} + \left(\frac{1}{x}\right)^{4-1} + \left(-\frac{1}{x}\right)^{1-1} + \left(-\frac{1}{x}\right)^{2-1} + \left(-\frac{1}{x}\right)^{3-1} + \left(-\frac{1}{x}\right)^{4-1}$$

$$= \left(\frac{1}{x}\right)^{0} + \left(\frac{1}{x}\right)^{1} + \left(\frac{1}{x}\right)^{2} + \left(\frac{1}{x}\right)^{3} + \left(-\frac{1}{x}\right)^{0} + \left(-\frac{1}{x}\right)^{1} + \left(-\frac{1}{x}\right)^{2} + \left(-\frac{1}{x}\right)^{3}$$

$$= 1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + 1 - \frac{1}{x} + \frac{1}{x^2} - \frac{1}{x^3}$$

$$\boxed{t_4 = 2 + \frac{2}{x^2}}$$

b.) Following the pattern:

$$\underbrace{2}_{t_1},\ \underbrace{2}_{t_2},\ \underbrace{2 + \frac{2}{x^2}}_{t_3},\ \underbrace{2 + \frac{2}{x^2}}_{t_4},\ \underbrace{2 + \frac{2}{x^2} + \frac{2}{x^4}}_{t_5},\ \underbrace{2 + \frac{2}{x^2} + \frac{2}{x^4}}_{t_6},$$

$$\underbrace{2 + \frac{2}{x^2} + \frac{2}{x^4} + \frac{2}{x^6}}_{t_7},\ \underbrace{2 + \frac{2}{x^2} + \frac{2}{x^4} + \frac{2}{x^6}}_{t_8},\ \underbrace{2 + \frac{2}{x^2} + \frac{2}{x^4} + \frac{2}{x^6} + \frac{2}{x^8}}_{t_9}$$

$$\therefore\ t_{10} = 2 + \frac{2}{x^2} + \frac{2}{x^4} + \frac{2}{x^6} + \frac{2}{x^8}$$

* note: The odd exponents cancel each other out since one term is positive & the other negative (zero pairs)

If $x = 2$:

$$t_{10} = 2 + \frac{2}{2^2} + \frac{2}{2^4} + \frac{2}{2^6} + \frac{2}{2^8}$$

$$= 2 + \frac{2}{4} + \frac{2}{16} + \frac{2}{64} + \frac{2}{256}$$

$$= \frac{512}{256} + \frac{128}{256} + \frac{32}{256} + \frac{8}{256} + \frac{2}{256}$$

$$= \frac{682}{256}$$

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c. $t_\infty = 2 + \frac{2}{2^2} + \frac{2}{2^4} + \frac{2}{2^6} + \frac{2}{2^8} + \frac{2}{2^{10}} + \ldots$

t_∞ = the sum of the infinite geometric series where $t_1 = 2$, $r = \frac{1}{2^2}$

$$t_\infty = S_\infty = \frac{t_1}{1-r}$$

$$t_\infty = \frac{2}{1-\frac{1}{2^2}} = \frac{2}{1-\frac{1}{4}} = \frac{2}{\frac{4}{4}-\frac{1}{4}} = \frac{2}{\frac{3}{4}} = 2 \div \frac{3}{4} = 2 \times \frac{4}{3} = \frac{8}{3}$$

$t_\infty = \frac{8}{3}$

d. $t_\infty = \frac{9}{4}$

$t_\infty = 2 + \frac{2}{x^2} + \frac{2}{x^4} + \frac{2}{x^6} + \frac{2}{x^8} + \ldots$

t_∞ = the sum of the infinite geometric series where $t_1 = 2$, $r = \frac{1}{x^2}$

$$t_\infty = S_\infty = \frac{t_1}{1-r}$$

$$t_\infty = \frac{2}{1-\frac{1}{x^2}} = \frac{9}{4}$$

$2(4) = 9\left(1-\frac{1}{x^2}\right)$

$8 = 9 - \frac{9}{x^2}$

$-9 \quad -9$

$-1 = \frac{-9}{x^2}$

$-1x^2 = -9$

$x^2 = 9$

$x = \pm\sqrt{9}$

$x = \pm 3$

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e) $t_{\infty} = p$

$$t_{\infty} = \frac{2}{1 - \frac{1}{x^2}} = p$$

$$2 = p\left(1 - \frac{1}{x^2}\right)$$

$$2 = p - \frac{p}{x^2}$$

$$2 - p = -\frac{p}{x^2}$$

$$x^2(2-p) = -p$$

$$x^2 = \frac{-p}{2-p} = \frac{-p}{-1(-2+p)} = \frac{-p}{-1(p-2)} = \frac{p}{p-2}$$

looks better ☺

$$x = \pm\sqrt{\frac{p}{p-2}}$$